PROCEEDINGS

OF

THE PHYSICAL SOCIETY

From December 1927 to August 1928

VOL. XL.

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Group Photograph taken on the Occasion of the Visit to Bristol, July, 7, 1928 *Frontispiece*

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY.

SESSION 1927-1928.

Except where the contrary is stated, the Meetings were held at the Imperial College of Science, the President being in the Chair.

October 28, 1927.

E. H. RAYNER, M.A., Sc.D., in the Chair.

A DEMONSTRATION of an Improved Mercury Vapour Trap was given by Prof. G. I. FINCH, M.B.E.

The following Papers were read :—

"Theory of Liquid Flow Through Cones," by W. N. BOND, M.A., D.Sc., F.Inst.P.

"The Structure of Smoke Particles from a Cadmium Arc," by H. P. WALMSLEY, M.Sc.

November 11, 1927.

The following were elected Fellows of the Society: R. P. G. Denman, M.A., A.M.I.E.E.; Rupert Woodville Hardisty; Prof. Arnold Crowther, M.A., Sc.D.; Prof. Harold John Taylor, B.Sc.; Raymond William Crabtree, B.Sc. The President announced that the Council had elected the following Student Members: Frederick G. Filby; Arthur Stanley Radford; Ernest Gordon Cox; Frank Gill; Cyril Alfred Sinfield; R. A. Fereday; John Bernard Charters; Morris Ernest Sions.

The following Papers were read :—

"The Power Factor and Capacity of the Electrodes and Base of Triode Valves, with Special Reference to their use in Thermionic Rectifiers," by G. W. SUTTON, B.Sc.

"The Refraction and Dispersion of Air, Oxygen and Gaseous Chloroform; and New Determinations of the Gaseous Refractivities of Acetone, Methyl Ether, and Ethyl Ether," by H. LOWERY, M.Sc.

"Theory of the Elastic Pianoforte Hammer," by PANCHANON DAS, M.Sc.

A DEMONSTRATION was given by Mr. C. O. BROWNE, of Messrs. Adam Hilger, Ltd., of an experiment due to Prof. Kerr Grant, in which a beam of light is made to fluctuate in intensity at radio frequencies in synchronism with the piezo-electric oscillations of a quartz crystal.

November 25, 1927.

E. H. RAYNER, M.A., Sc.D., in the Chair.

The following were elected Fellows of the Society: Geo. E. Harrison, M.Sc.; M. O. Pelton, B.Sc.

The following Papers were read:—

"On the Measurement of the Electrical Resistance of Porous Materials," by A. E. KNOWLER, M.Sc., A.Inst.P.

"Regularities in the Spectrum of Ionised Neon," by P. K. KICHLU, M.Sc.

"A Calorimeter for the Determination of the Heat Developed by Fruit," by EZER GRIFFITHS, D.Sc., F.R.S.

A series of Lantern Slides illustrating Air Conditioning Experiments, with especial reference to Fruit Storage, were shown by EZER GRIFFITHS, D.Sc., F.R.S.

December 9, 1927.

The following was elected a Fellow of the Society: Percy Wilson, M.A. The President announced that the Council had elected the following a Student Member of the Society: Robert William Sutton.

The following Papers were read:—

"The Scattering of Light by Individual Particles in Smokes," by H. P. WALMSLEY.

"On the Construction and Standardisation of an Interferometer Pressure Gauge," by J. J. MANLEY, M.A., Research Fellow, Magdalen College, Oxford.

"The Dielectric Constant of Liquid Bromine," by ANNE I. ANDERSON, B.Sc.

January 10, 11 and 12, 1928.

The Eighteenth Annual Exhibition of Electrical, Optical and other Physical Apparatus was held by the Physical and Optical Societies at the Imperial College of Science.

January 27, 1928.

The following was elected a Fellow of the Society: Charles Edwin Foster. The President announced that the Council had elected the following a Student Member of the Society: Solomon Zeidenfeld.

The following Papers were read :—

"The Swan Band Spectrum of Carbon," by W. E. PRETTY, A.R.C.S., B.Sc.

"On Some Misapplications of the Law of Errors, and on the Intrinsic Error in Focometry," by T. SMITH, M.A., F.Inst.P.

"Some Acoustical Phenomena Illustrated by Ripples—Transmission through Quincke Filters, Curved Conduits and Vibrating Partitions," by A. H. DAVIS, D.Sc.

February 10, 1928.

The following were elected Fellows of the Society: Harold Ashley Daynes, Reginald Carr Brown, George Morris.

The following Papers were read :—

"A Simple Graphical Method for the Determination of Galvanometer and Fluxmeter Constants; with a Note on the Measurement of Intense Magnetic Fields," by ALLAN FERGUSON, D.Sc., and E. J. IRONS, B.Sc.

"On a Method of Constructing the Caustic Curve formed by Refraction at a Plane Surface," by C. J. SMITH, Ph.D.

"The Application of Electrical Resistance Measurements to the Study of Atmospheric Corrosion of Metals," by J. C. HUDSON.

February 24, 1928.

The following Papers were read :—

"Some Methods of Estimating the Intensities of Spectral Lines," by W. H. J. CHILDS, B.Sc., Demonstrator in Physics, King's College, London.

"Electrical Methods of Hygrometry," by P. W. BURBIDGE, M.Sc., B.A., Professor of Physics, and N. S. ALEXANDER, M.Sc., Auckland University College, New Zealand.

March 9, 1928.

The President announced that the Council had elected the following a Student Member of the Society: H. Harrison Macey.

The Thirteenth Guthrie Lecture was delivered by Prof. Sir J. J. THOMSON, O.M., F.R.S., who took as his subject "Electrodeless Discharges through Gases."

A vote of thanks to the lecturer was proposed by Sir OLIVER LODGE, D.Sc., F.R.S., seconded by ALEXANDER RUSSELL, M.A., D.Sc., F.R.S., and carried by acclamation.

March 23, 1928.

Annual General Meeting.

The Report of the Council and that of the Hon. Treasurer were presented and unanimously adopted.

REPORT OF THE COUNCIL.

During the year fourteen Ordinary Science Meetings have been held at the Imperial College of Science. At these meetings 39 Papers were presented and 13 Demonstrations were given. A Special Meeting was held on June 3, at which Dr. E. H. Rayner gave a lecture on "The Total Eclipse of the Sun." About 75 Fellows and Visitors were present. About 250 Fellows, Students and Friends of the Society went to Richmond (Yorkshire) for the purpose of seeing the Total Eclipse of the Sun on the morning of June 29. On the return journey some hours were spent in York.

Prof. Sir Ernest Rutherford, O.M., P.R.S., delivered the Twelfth Guthrie Lecture on February 25, the subject being "Atomic Nuclei and their Disintegration." About 160 Fellows and Visitors were present.

The average attendance at the meetings of the Society was 57.

The Seventeenth Annual Exhibition, arranged jointly by the Physical and Optical Societies, was held, through the courtesy of the Governing Body, at the Imperial College, on January 4, 5 and 6. Trade Exhibits were arranged by seventy-four firms, and the recently instituted Research and Experimental Section contained exhibits contributed from thirty-two sources. Discourses were given by Dr. C. V. Drysdale on "Progress in Electrical Instrument Design and Construction"; by Mr. J. L. Baird on "Television";

and by Prof. E. N. da C. Andrade, whose discourse consisted of a full-dress impersonation of Francis Hawksbee giving an experimental lecture in the time of Queen Anne. On the third day the general public was admitted. The total attendance at the Exhibition was estimated at over four thousand.

A Joint Committee of the Physical and Optical Societies has been formed to deal with all matters concerning the Annual Exhibition, with Mr. T. Martin (Secretary of the Institute of Physics) as Secretary.

Dr. D. Owen and Dr. J. H. Vincent have been reappointed as representatives of the Society on the Board of the Institute of Physics; and Mr. J. Guild and Dr. D. Owen on the Science Abstracts Committee. Mr. T. Smith has been appointed as the Society's delegate to the Organising Committee of the Seventh International Congress of Photography; Dr. A. Ferguson has been appointed as the Society's representative on the National Council for Physics; and Dr. E. H. Rayner will represent the Society on the National Committee for Radio-telegraphy.

Prof. H. A. Wilson and Prof. W. F. G. Swann represented the Society at the Bicentenary Celebrations of the American Philosophical Society, held at the end of April, and presented a letter of congratulation from the Society. The President, Prof. O. W. Richardson, represented the Society and presented an address at the International Congress of Physicists, held in September at Como in honour of Alessandro Volta. Mr. F. Twyman represented the Society at the Fresnel Centenary Celebrations, held in Paris at the end of October, and conveyed an address to the Société Française de Physique. Prof. A. Fowler represented the Society at the opening of the new Physics Laboratories at Bristol in October.

The Council has awarded the Fifth Duddell Medal to Dr. F. E. Smith, F.R.S. This will be presented at the Annual General Meeting.

The Report on Series in Line Spectra, by Prof. A. Fowler, has been reprinted with an additional index.

A Special Sub-committee has been formed to consider all questions dealing with publications issued by the Society. In consequence of its preliminary reports, the Council has given notice of the termination of its printing and publishing agreement with the Fleetway Press, and has made arrangements with the Cambridge University Press for the printing of its publications in the future. A leaflet prepared by the Sub-committee, entitled "Instructions to Authors," has been issued, which, it is hoped, will effect at the same time an improvement in the accuracy and a reduction in the cost of printing Papers in the Proceedings. The recommendations contained in the leaflet have been adopted by the Optical Society, and are under consideration by other societies.

As a result of the negotiations with the Institute of Physics, which were referred to in the last Annual Report, it has been arranged that some of the routine business of the Society shall be performed by the Institute. The registered address of the Society has been changed to 1, Lowther Gardens, South Kensington, S.W.7.

In view of the difficulty in getting the accounts audited as early as the second Friday in February, the Council has arranged to hold the Annual General Meeting on the fourth Friday in March. It is anticipated that a similar date will be selected in future years.

Prof. Sir Arthur Schuster, K.B.E., F.R.S. (Past-President, 1912-1914), has tendered his resignation as Foreign Secretary. The Council has accepted with regret his resignation of the duties which he has so ably fulfilled since 1920, and records the great indebtedness of the Physical Society to him for his valued service.

The Council records with regret the death of Prof. Svante Arrhenius, who was elected an Honorary Fellow of the Society in 1910.

It is necessary also to record with regret the deaths of Prof. W. H. Coleman (elected 1890), Prof. R. A. Lehfeldt (elected 1895), and Prof. A. Liversidge (elected 1876), all Life Fellows of the Society; also of Mr. E. F. Etchells (elected 1904), Major J. R. Erskine-Murray (elected 1902), and Prof. A. W. Scott (elected 1883).

The number of Honorary Fellows on the Roll on December 31, 1927, was 11. At the same date Ordinary Fellows numbered 680 and Students 27.

The changes in the membership of the Society during the year are shown in the appended Table :—

—	Total Dec. 31, 1926.	Changes during 1927.			Total Dec. 31, 1927.
Honorary Fellows...	12	Deceased		1	
		Net decrease		1	11
Ordinary Fellows ...	666	Elected		26	
		Student transfers		0	
				26	
		Deceased ...	6		
		Resigned or lapsed	6		
				—12	
		Net increase		14	680
Students	12	Elected		16	
		Resigned		1	
		Net increase		15	27
Total Membership...	690	Net increase		28	718

REPORT OF THE HON. TREASURER.

The Income for the year 1927 exceeded the Expenditure by £29 14s. 8d. This is satisfactory when it is considered that the Society has had to reprint three Parts of the Proceedings of the Society which were out of print, and the report of Prof. Fowler on "Series in Line Spectra," which was also out of print. The Society is indebted to the Council of the Royal Society for the grant of £50 towards the cost of printing Prof. Fowler's report, and also for a further grant of £100 towards the cost of its publications. The valuation of the publications of the Society in the Balance Sheet has been increased from £400 (the figure at which it has stood for the last few years) to £500. It has been considered advisable to do this, as the addition of the Parts of the Proceedings which have been reprinted, and the recently instituted increase in the price of the Proceedings, have materially increased the value of the publications in stock.

It is gratifying to notice that by slightly stricter control of the length of Papers, and the number and size of the illustrations, a reduction of over £100 has been made in the cost of printing Volume 39 of the Proceedings as compared with Volume 38.

The Society has not received any remission of taxes on its investment income for 1926, and no amount has been included in the Balance Sheet for a refund in the 1927 accounts. This is due to the whole question of the remission of taxes to scientific societies being *sub judice*. It is hoped that on some future occasion it will be possible to report that the taxes have been refunded.

The sum of £112 10s. 0d. was paid to the Institute of Physics for secretarial and other work carried out on behalf of the Society by the Institute during the latter part of the year, and this sum includes the preparatory work for the 1928 Exhibition. The item "other expenses" under the heading of "Administration" includes the secretarial and accountancy expenses of the Society before the offices were transferred to 1, Lowther Gardens. It also includes some new furniture and expenditure connected with the transfer.

A legacy of £250, free of legacy duty, was received from the Executors of the late Prof. A. W. Scott, of St. David's College, Lampeter, in July, 1927, and was invested in 5 per cent. War Loan Stock.

At the request of the Treasurer, the Council appointed a Finance Committee to advise on the various financial questions that may arise.

The investments have been valued at market prices through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank.

INCOME AND EXPENDITURE ACCOUNT.

For the Year ended 31st December, 1927.

1926. £ s. d.	EXPENDITURE.	£ s. d.	£ s. d.
	To		
	Stock of Publications 1st January, 1927 ...		400 0 0
434 0 0	"Science Abstracts"		475 0 0
	Ordinary Publications :—		
1,053 6 1	"Proceedings"	922 0 8	
73 3 3	"Bulletin"	73 15 4	
92 17 10	General	101 17 3	
			1,097 13 3
	Reprints		188 12 7
115 9 4	Postage of Publications		105 0 10
61 12 3	Reporting		61 8 5
37 12 0	Refreshments and Expenses of Meetings...		33 15 2
369 12 7	Exhibition Expenses		485 15 10
8 8 4	Periodicals and Library		5 0 0
23 16 9	*Secretary and Treasurer's Expenses.*		
	Administration :		
	Payment to Institute of Physics as from April 1st, 1927	112 10 0	
15 9 5	Other Expenses	100 8 7	
			212 18 7
20 0 0	Guthrie Lecture (Honorarium)		20 0 0
21 0 0	Optical Society—Exhibition Account		10 4 5
	Expenses of Eclipse Excursion		384 15 0
	Expenses of Delegates to Volta Congress and Fresnel Centenary		26 13 6
10 6 0	*Participation in "Coming-of-Age" Celebrations, Sheffield University.*		
	A. W Scott Bequest: Purchase of £250 5% War Loan		255 11 7
	Balance being excess of Income over Expenditure carried to Accumulated Fund		29 14 8
£2,366 13 10			£3,792 3 10

1926. £ s. d.	INCOME.	£ s. d.	£ s. d.
	By		
	Stock of Publications, 31st December, 1927		500 0 0
	Subscriptions :		
1,063 3 11	Fellows*	1,113 17 1	
9 9 0	(Voluntary†)	9 9 0	
8 18 6	Students	8 5 0	
52 18 6	for "Science Abstracts" and Advance Proofs	64 5 0	
1,134 9 11		1,195 16 1	
	Less Subscriptions written off	4 4 0	
			1,191 12 1
484 10 8	Sale of Publications (Fleetway Press, Ltd.)		630 16 2
	Dividends from Investments	191 8 6	
169 11 4	*Less :* Transferred to Duddell Memorial Trust Fund	20 0 0	
			171 8 6
12 16 11	Interest on Deposit Account		6 9 7
29 10 11	*Income Tax repayable.*		
434 8 0	Exhibitors' Payments on account of Exhibition Catalogue		507 2 6
100 0 0	Royal Society Grant for Publications...		150 0 0
	Eclipse Excursion: Receipts from Sale of Tickets		384 15 0
	A. W. Scott Bequest		250 0 0
1 6 1	*Balance being excess of Expenditure over Income.*		
£2,366 13 10			£3,792 3 10

* Sixty-four Fellows paid reduced subscriptions by the arrangement with the Institute of Physics, the total rebate amounting to £23 4s. 6d.

† "Voluntary Subscriptions" are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT DECEMBER 31ST, 1927.

LIABILITIES.	£	s.	d.	£	s.	d.
Accumulated Fund :—						
Balance as per last account	1,878	17	11			
Entrance Fees, 1927	30	9	0			
	1,909	6	11			
Add Increased Value of Investments	57	0	0			
Excess of Income over Expenditure, 1927	29	14	8			
				1,996	1	7
Life Compositions :—						
As per last Balance Sheet				2,289	10	0
Duddell Memorial Trust Fund :—						
As per last Balance Sheet	429	12	9			
Add Increased Value of Investments	1	0	0			
				430	12	9
W. F. Stanley Trust Fund :—						
As per last Balance Sheet	428	0	0			
Less Decreased value of Investments	42	0	0			
				386	0	0
A. W. Scott Bequest				250	0	0
Sundry Creditors				304	18	9
Subscriptions paid in advance				19	8	6
				£5,676	11	7

ASSETS.	£	s.	d.	£	s.	d.
Investments at Market Value on date :—						
£399 London Midland & Scottish Railway 4 per cent. Debenture Stock	319	0	0			
£1,000 London Midland and Scottish Railway 4 per cent. Preference Stock	750	0	0			
£200 Metropolitan Board of Works 3½ per cent. Stock	194	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	268	0	0			
£254 2s. 9d. New South Wales 5 per cent. Stock, 1935-55	248	0	0			
£300 Southern Railway Preferred Ordinary Stock	221	0	0			
£442 Southern Railway Deferred Ordinary Stock	165	0	0			
£500 London & North Eastern Railway 4 per cent. Debenture Stock	392	0	0			
£500 India 3½ per cent. Stock	360	0	0			
£650 4% Funding Loan, 1960/90	573	0	0			
£550 5% War Loan 1929/47 inscribed, "A" Account	556	0	0			
£400 5% War Loan 1929/47 inscribed, "B" Account (Duddell Memorial Fund)	404	0	0			
£150 Southern Railway 5 per cent. Debenture Stock	150	0	0			
				4,600	0	0
Stock of Publications (as per Treasurer's valuation)				500	0	0
Subscriptions due				57	15	0
Due from Fleetway Press				282	17	1
Due from Inland Revenue—Income Tax Reclaimed (as per last Balance Sheet)				27	10	10
Cash at Bank	203	13	3			
Cash in Hand	4	15	5			
				208	8	8
				£5,676	11	7

ROBERT S. WHIPPLE.—*Honorary Treasurer.*

We have audited the above Balance Sheet and have obtained all the information and explanations we have required. We have verified the Bank Balance and the Investments. In our opinion such Balance Sheet is properly drawn up so as to exhibit a true and correct view of the state of the Society's affairs according to the best of our information and the explanations given to us and as shown by the books of the Society.

Spencer House,
South Place, E.C.2.
March 5th, 1928.

KNOX, CROPPER & CO.,
Chartered Accountants.

Examined and approved on behalf of the Society { (Signed) **G. H. Baillie.**
(Signed) **L. Hartshorn.**

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1927.

	£	s.	d.
128 Fellows paid £10	1,280	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20	20	0	0
1 Fellow paid £20 10s.	20	10	0
14 Fellows paid £21	294	0	0
20 Fellows paid £31 10s.	630	0	0
167	£2,289	10	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN").

	£	s.	d.		£	s.	d.
£300 Southern Railway Preferred Ordinary Stock	221	0	0	Carried to Balance Sheet	386	0	0
£442 Southern Railway Deferred Ordinary Stock	165	0	0				
	£386	0	0		£386	0	0

DUDDELL MEMORIAL TRUST FUND.

CAPITAL

	£	s.	d.		£	s.	d.
£400 War Loan 5% 1929/47 inscribed Stock	404	0	0	Carried to Balance Sheet	404	0	0

REVENUE.

	£	s.	d.		£	s.	d.
Balance at 31st December, 1926	26	12	9	Honorarium to Medallist	20	0	0
Dividends	20	0	0	Balance carried to Balance Sheet	26	12	9
	£46	12	9		£46	12	9

PRESENTATION OF THE DUDDELL MEDAL.

The Duddell Memorial Medal for 1928 was presented to F. E. Smith, C.B., C.B.E., D.Sc., F.R.S.

Prof. O. W. **RICHARDSON**, President of the Society, in making the presentation, said that, after considering the terms of the Duddell Medal Foundation, the Council had unanimously chosen Dr. Smith as the medallist for this year, in view of the distinguished part he has played in promoting the accuracy of electrical measurements. He became head of the Electrical Standards Department of the National Physical Laboratory on its foundation in 1901, and soon achieved world-wide fame for his work on the British Association Standards of Resistance. His research at the N.P.L. included important developments in precision bridge methods and standard cells, an absolute current balance accurate within 2 parts in 100,000, devices for measuring resistance and the earth's magnetic field in C.G.S. units, a new magnetograph and a new silver voltameter, the latter having come into international use. During the war he devised a number of instruments for the use of the fighting services, and he was appointed first Director of Research at the Admiralty in 1919. He had contributed to the progress of science no less ably on the administrative than on the experimental side, not only as Hon. Secretary of the British Association and in similar offices of a general character, but, as Fellows would remember with gratitude, as Hon. Secretary (Business) for many years and ultimately President (1924/1925) of the Physical Society, whose flourishing condition was in no small measure due to his enthusiasm. In his Presidential Address in 1925 he had urged the international adoption of the C.G.S. electrical units and the gradual abolition of the present international standards, and it was pleasing to know that this proposal had been endorsed by the International Committee on Weights and Measures which met at Sèvres in 1927. He had advocated in season and out of season the intensive improvement of instrument design and construction in order that the advance of knowledge might not be retarded by the present limits of accurate measurement. He was also an enthusiastic advocate of a stronger emphasis on experimental accuracy in the teaching of science.

Dr. F. E. **SMITH** expressed his gratitude for the award. He appreciated it particularly because of his friendship with Duddell and of his regard for the Physical Society, with which he had been so intimately associated. Duddell was not only a brilliant designer, but an exceptionally clever hand-worker, and he had been a source of inspiration to all who had the interests of experimental accuracy at heart. Dr. Smith added that he had first become interested in precision measurements when as a student of the Royal College of Science he had worked under Prof. Watson on absolute electromagnetic determinations, and in those days he had learned much from Sir A. Rücker and Prof. Boys, whose measurements of minute quantities he had always followed with great interest. The Physical Society had always attached due importance to experimental methods, and he had followed with satisfaction the growth of the Annual Exhibition of Apparatus, which had increased from a small beginning until it now attracted 200 exhibitors and 5,000 visitors. He believed that as it continued to grow it would play an important part in focussing interest on the development of instrument design, as regards both the devising of new instruments and the improvement of old models, with the view of obtaining increased accuracy.

ELECTION OF COUNCIL.

Dr. J. S. Anderson and Mr. J. H. Coste having been appointed scrutators, the Officers and Council for 1928-1929 were elected as follows :—

President.—W. H. Eccles, D.Sc., F.R.S.

Vice-presidents who have filled the office of President.—Sir Oliver J. Lodge, D.Sc., F.R.S., Sir Richard Glazebrook, K.C.B., D.Sc., F.R.S., C. Chree, Sc.D., LL.D., F.R.S., Prof. H. L. Callendar, M.A., LL.D., F.R.S., Sir Arthur Schuster, Ph.D., Sc.D., F.R.S., Sir J. J. Thomson, O.M., D.Sc., F.R.S., Prof. C. Vernon Boys, F.R.S., Prof. C. H. Lees, D.Sc., F.R.S., Prof. Sir W. H. Bragg, K.B.E., M.A., F.R.S., Alexander Russell, M.A., D.Sc., F.R.S., F. E. Smith, C.B., C.B.E., D.Sc., F.R.S., Prof. O. W. Richardson, M. A., D.Sc., F.R.S.

Vice-presidents.—R. W. Paul, Prof. E. A. Owen, M.A., D.Sc., D. Owen, B.A., D.Sc., Prof. F. L. Hopwood, D.Sc.

Hon. Secretaries.—Prof. A. O. Rankine, O.B.E., D.Sc., J. Guild, A.R.C.S., D.I.C.

Hon. Foreign Secretary.—Prof. O. W. Richardson, M.A., D.Sc., F.R.S.

Hon. Treasurer.—R. S. Whipple.

Hon. Librarian.—J. H. Brinkworth, M.Sc., A.R.C.S.

Ordinary Members of Council.—Miss A. C. Davies, D.Sc., Prof. A. M. Tyndall, D.Sc., T. Smith, M.A., A. Ferguson, M.A., D.Sc., J. S. G. Thomas, D.Sc., Prof. E. V. Appleton, M.A., D.Sc., F.R.S., Sir Richard Paget, Bart., Prof. E. N. da C. Andrade, D.Sc., Ezer Griffiths, D.Sc., F.R.S., A. B. Wood, D.Sc.

ELECTION OF HONORARY FELLOW.

Prof. ALBERT EINSTEIN was unanimously elected an Honorary Fellow of the Society.

VOTES OF THANKS.

A vote of thanks to the Hon. Auditors was proposed by Mr. Robert Whipple and seconded by Prof. F. L. Hopwood. A vote of thanks to the retiring Officers and Council was proposed by Dr. W. H. Eccles and seconded by Dr. J. H. Vincent. A vote of thanks to the Governors of the Imperial College was proposed by Mr. T. Smith and seconded by Dr. J. S. G. Thomas. The votes of thanks were carried by acclamation.

Ordinary Meeting following the Annual General Meeting.

The following were elected Fellows of the Society: John William Avery, A.R.C.S., D.Sc., D.I.C. (previously a Student Member), Martin W. Strong, B.Sc. The President announced that the Council had appointed the following to be Student Members: Hugh Charles Tett, Sidney T. Newing, John Rankine.

The following Papers were read :—

"The Terminal Velocity of Drops," by W. D. FLOWER, B.Sc.

"On the Longitudinal Waves along a Rod," by SATYANDRA RAY, M.Sc., Government College, Ajmer, India."

"On the Damping of Mercury Ripples,'" by J. J. **Manley**, M.A., Research Fellow, Magdalen College, Oxford.

A **Demonstration** of *Die Rastermethode* (the testing of spherical aberration by means of shadows in an astigmatic beam) was given by J. E. **Calthrop**, M.A., M.Sc.

April 27, 1928.

The following were elected Fellows of the Society: S. N. Banerji, M.Sc.; Maurice Milbourn, A.R.C.S., B.Sc., D.I.C.

The President announced that Mr. J. Guild had been obliged to resign the Honorary Secretaryship (Papers) of the Society for reasons of health. He expressed the deep regret felt by the Council, and moved a vote of thanks to Mr. Guild for the invaluable services which he has rendered to the Society during the past seventeen years. The vote of thanks was carried by acclamation.

The President announced that Dr. Allan Ferguson had been appointed Honorary Secretary for Papers, and heartily welcomed Dr. Ferguson in the name of the Society.

The following Papers were read:—

"Experiments with Mercury Jets and the Phenomena exhibited at their Impact with Steel and Glass," by Prof. **Will C. Baker**, M.A., Queen's University, Kingston, Ont., Canada.

"The Elastic Constants of Glass," by E. P. **Perman** and W. D. **Urry**, University College, Cardiff.

"A Valve-maintained High-frequency Induction Furnace and some Notes on the Performance of Induction Furnaces," by G. E. **Bell**, B.Sc., A.Inst.P., National Physical Laboratory.

A **Demonstration** of the Blowing of Selenium Bubbles was given by Major C. E. S. **Phillips**, F.R.S.E.

May 11, 1928.

The following were elected Fellows of the Society: Elizabeth Mary Collins, B.Sc.; Norman Leslie Harris, B.Sc., A.Inst.P.

The following Papers were read:—

"The Amplitude of Sound Waves in Resonators," by E. G. **Richardson**, B.A., M.Sc., Ph.D., University College, London.

"The Focus of a Gas-filled X-ray Tube," by R. E. CLAY, M.Sc.

A DEMONSTRATION of an Electric Harmonic Analyser was given by R. THORNTON COE, The British Thomson-Houston Co., Ltd.

May 25, 1928.

Special General Meeting.

The following resolution was proposed by Mr. R. W. PAUL, seconded by Prof. F. L. HOPWOOD, and carried *nem. con.* :—

"That the following words be added at the end of Article 37 of the Articles of Association of the Society: 'The Council, may, however, in its discretion, reduce or remit the subscription of any individual Fellow, in special circumstances.'"

Ordinary Meeting following the Special General Meeting.

The following were elected Fellows of the Society: W. H. J. Childs, K. L. Goodall, B.Sc. The President announced that the Council had elected the following to be Students of the Society: F. E. Hoare, W. Goldsmith, John J. Green, M. Farrell.

The following Paper was read :—

"Some New Methods of Linking Mechanical and Electrical Oscillations," by W. H. ECCLES, D.Sc., F.R.S., and Miss W. A. LEYSHON, B.Sc., Ph.D.

The meeting then adjourned to the Royal School of Mines, where a Demonstration of "Some Recent Studies in Alternating Magnetism with Some Possible Applications" was given by W. M. MORDEY, M.I.E.E.

June 8, 1928.

Special General Meeting.

The resolution passed at the Special General Meeting held on May 25, 1928, was unanimously confirmed.

Ordinary Meeting following the Special General Meeting.

The following Papers were read :—

"Contact Potential in the Dolezalek Electrometer Connected Idiostatically," by L. F. RICHARDSON, F.R.S., and STUDENTS of Westminster Training College.

"Some Experiments on the Light-Sensitivity of Commercial Selenium Cells," by GEO. P. BARNARD, The National Physical Laboratory.

"The Vapour Pressure of Water over Sulphuric Acid-Water Mixtures at 25°C. and its Measurement by an improved Dew-point Apparatus," by J. R. I. HEPBURN, M.Sc., Ph.D., Northern Polytechnic.

A DEMONSTRATION of a Simple Practical Application of the Properties of Selenium Cells was given by Mr. GEO. P. BARNARD, the National Physical Laboratory.

June 22, 1928.

The following Papers were read :—

"The Stability of a Valve Amplifier with Tuned Circuits and Internal Reaction," by R. T. BEATTY, M.A., B.E., D.Sc.

"The Scattering of Light by Particles of Metallic Oxides Dispersed in Dry Air," by T. C. NUGENT, M.Sc., and H. P. WALMSLEY, M.Sc.

"A Note on the Transmission of Cathode Rays through Thin Films," by B. L. WORSNOP, B.Sc.

A DEMONSTRATION of Various Methods for the Determination of Poisson's Ratio was given by J. P. ANDREWS, M.Sc.

July 7, 1928.

A visit was paid to the University of Bristol. The morning was devoted to a tour of the neighbourhood and to lunch. During the afternoon a meeting was held in the Henry Herbert Wills Physical Laboratory, and the following Papers were read in abstract :—

"The Mobility of Positive Ions in Air," by A. M. TYNDALL, L. H. STARR and P. A. SHEPPARD.

"The Polarisation of Impact Radiation in Mercury and Helium," (with DEMONSTRATION), by H. W. B. SKINNER.

"The Structure and Magnetic Properties of Single Crystals of Heusler Alloy," by H. H. POTTER.

"The Angular Distribution of Photo-electrons from a Single Crystal," by S. H. PIPER and H. W. B. SKINNER.

"A Method for the Measurement of the Gyromagnetic Ratio for Paramagnetic Substances," by W. SUCKSMITH.

The following papers were read in title :—

"Some Recent Developments of Statistical Mechanics" (lecture), by J. E. LENNARD-JONES.

"The X-Ray Analysis of the Fatty Acids and their Salts," by S. H. PIPER.

"Apparatus for the Determination of the Principal Magnetic Susceptibilities of Crystals at Low Temperatures," by L. C. JACKSON.

"An Integrating Photometer for Sun and Sky Light" (with DEMONSTRATION), by M. G. BENNETT.

A vote of thanks to Professor Tyndall and the Physics staff of the University, as hosts of the Society, was proposed by the President and carried by acclamation.

An exhibition of apparatus and experiments was then held and tea was served in the Laboratory.

I.—THEORY OF LIQUID FLOW THROUGH CONES

(An approximate General Solution for Conical Tubes of Small Angle).

By W. N. Bond, *M.A., D.Sc., F.Inst.P.*, Lecturer in Physics, University of Reading.

Abstract.

This Paper gives an approximate general solution of the hydro-dynamical equations for liquid flow through conical tubes of circular section, the errors due to the approximation being small for converging cones of small angle and for flow through similar diverging cones till near the speed at which the theory predicts turbulent motion. The theory is compared with experimental results previously published.

Introduction.

It is notorious that the general hydrodynamical equations are of such a form that in few cases (if indeed in any) has a general solution of them been obtained covering the whole range between the stage where the pressure-gradient is almost independent of the acceleration of the fluid and the stage where it is almost entirely dependent on the acceleration. In these circumstances, an approximation may be worth consideration. The case which is solved approximately in this Paper consists of the steady flow of liquids through small-angled conical tubes of circular section. The approximation is such that the magnitude of the errors introduced can be estimated with the help of the approximate solution obtained.

The Two Approximations.

Choosing rectangular axes, with that of x coinciding with the axis of the cone, we have the three usual hydrodynamical equations such as

$$\frac{\partial p}{\partial x}=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)\quad \ldots\ldots\quad (1)$$

where p denotes the pressure, u, v, w are components of the velocity at a point, μ and ρ are the viscosity and density of the fluid. (The hydrostatic pressure gradients have for convenience been considered zero.)

Let us *first assume* that the angle of the cone is small. Denoting the radius of the transverse section by r_m, the distance from the (imaginary) apex of the cone by R and the volume of fluid passing through the cone per second by Q, we can find the order of magnitude of the terms involved. Thus, u is of order Q/r_m^2, v and w of order Q/Rr_m. Each differentiation with respect to x produces one higher power of R in the denominator, and each with respect to y and z introduces one higher power of r_m in the denominator. Omitting, then, the second order terms in equations (1), we obtain

$$\frac{\partial p}{\partial x}=\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho\frac{Du}{Dt};\ \frac{\partial p}{\partial y}=\frac{\partial p}{\partial z}=0\quad \ldots\ldots\ldots\quad (2)$$

This result shows that for cones of small angle the transverse pressure gradients may be neglected, and the only way in which the equations differ from those for parallel stream-lines is in the term Du/Dt. This result might have been anticipated. The closeness of this first approximation depends only on how small the angle of the cone may be. In the extreme cases of very fast and very slow flow, the errors introduced for a cone of semi-angle 10° are about 1 per cent. and 4 per cent. respectively.*

If the terms in ρ may be omitted from the general equation (1), the flow is in straight stream-lines that when produced would intersect at the apex of the cone.† (This case, that of very slow flow, can be realised to a very close approximation indeed in practice.) If, on the other hand, the terms in μ may be omitted from the general equations (1), the flow would again be in straight stream-lines that when produced would intersect at the apex of the cone. (This case can only be roughly approximated to in practice. Close to the walls of the tube the terms in μ always have an appreciable effect in the case of a converging cone; and in the case of a diverging cone the flow becomes turbulent.) The velocity variation over the transverse section is in these two cases respectively approximately parabolic and approximately constant except near the walls. In the case of a long converging cone, these cases would occur very far from and very near to the apex respectively; and it is evident that the stream-lines in the intervening parts of the tube must be curved. However, as *the second approximation*, for any short part of the cone considered, let us assume that the flow is in straight stream-lines that pass when produced through the apex of the cone. This assumption will introduce a small error in the term Du/Dt (see below), and this error remains even for indefinitely small angles of cone.

With the second approximation, equations (2) reduce to

$$\frac{dp}{dx} = 2\frac{\mu}{r} \cdot \frac{du}{dr} + 2\,\frac{\alpha\rho}{r_m} u^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where the cone has a semi-angle α (α positive if the tube diverges in the direction of motion); and there is a pressure-gradient dp/dx (positive when the pressure increases in the direction of motion), at a place where the transverse section of the tube has a radius r_m.

By proceeding with the calculation using the approximate equation (3) (*see* next section of the Paper), the velocity distribution over the transverse section can be deduced, and knowing this at different sections of the cone, the slightly curved stream-lines may be plotted. If this is done, it is found that for a converging cone, in the worst case ($|\alpha|\rho Q/r_m\mu=12$ and $r=0{\cdot}4\ r_m$ approximately), the inclination of the stream-line differs by about 11 per cent. from that assumed in the approximation (and is, therefore, still about 1 per cent. in error). As the assumed inclinations are correct at $r=0$ and $r=r_m$, and as the approximation only affects one of the terms in equation (3), the maximum error in the estimates of u and Q will probably be not more than 5 per cent. (For larger and smaller values of $|\alpha|\rho Q/r_m\mu$ the error will decrease steadily to zero.) For diverging cones the error will only become appreciable as the stage of instability is neared, and as the prediction of instability will not be affected by the removal of the approximation, and is affected by the nature of the entrance to the cone, the error is not of much importance.

* Bond, Proc. Phys. Soc., Vol. 34, p. 188 (August, 1922).

† Harrison, Proc. Camb. Phil. Soc., Vol. 19, pp. 311-12; and Bond, Phil. Mag., Vol, 50, p. 1,059 (1925).

THE APPROXIMATE GENERAL SOLUTION.

In the solution of equation (3) three cases arise :—

Case I.—a and dp/dx both negative. Converging Cones.

If the velocity at the walls be assumed zero we have

$$u=\sqrt{\frac{r_m}{2a\rho}\cdot\frac{dp}{dx}}\cdot\tanh\left\{\frac{1}{2\mu}\sqrt{\frac{a\rho}{2r_m}\cdot\frac{dp}{dx}}\cdot(r_m{}^2-r^2)\right\} \quad . \quad . \quad . \quad . \quad (4)$$

This equation may further be integrated to give the volume of liquid passing through the tube per second, Q,

$$Q=-\frac{2\pi\mu r_m}{a\rho}\cdot\log\cosh\left\{\frac{r_m{}^2}{2\mu}\sqrt{\frac{a\rho}{2r_m}\cdot\frac{dp}{dx}}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Case II.—a positive and dp/dx negative. Slow flow through Diverging Cones.

Again assuming zero velocity at the walls of the tube, we obtain

$$u=\sqrt{-\frac{r_m}{2a\rho}\cdot\frac{dp}{dx}}\cdot\tan\left\{\frac{1}{2\mu}\sqrt{-\frac{a\rho}{2r_m}\cdot\frac{dp}{dx}}\cdot(r_m{}^2-r^2)\right\} \quad . \quad . \quad . \quad (6)$$

$$Q=\frac{2\pi\mu r_m}{a\rho}\cdot\log\sec\left\{\frac{r_m{}^2}{2\mu}\sqrt{-\frac{a\rho}{2r_m}\cdot\frac{dp}{dx}}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

According to these equations the velocity should become infinite at the axis of the tube, and the rate of flow become infinite at a pressure-gradient given by

$$\frac{dp}{dx}=-\frac{2\pi^2\mu^2}{ar_m{}^3\rho}$$

The flow would then, presumably, become turbulent.

Case III.—a and dp/dx both positive. Fast flow through Diverging Cones.

The solution of equation (3) corresponding to an increase of pressure along a diverging cone (such as is observed experimentally) takes the form

$$u=\sqrt{\frac{r_m}{2a\rho}\cdot\frac{dp}{dx}}\cdot\tanh\left\{\frac{1}{2\mu}\sqrt{\frac{a\rho}{2r_m}\cdot\frac{dp}{dx}}\cdot r^2+\text{constant}\right\}$$

If the velocity of the liquid at the walls be assumed zero, this equation gives negative values of u, the whole becoming the convergent case (equation 4) already considered. Hence (unless slip be assumed to occur at the walls, and the very peculiar solution yielded by the above equation accepted as feasible), we must conclude that stream-line motion is impossible under the conditions of Case III.

DISCUSSION OF THE EQUATIONS.

The solution obtained for convergent flow, and the absence of a solution for stream-line flow in the diverging case except at low speeds, are in general agreement with experiment. The fact that turbulence may sometimes occur in converging cones* is not inconsistent with this theory.

In discussing the solutions of equation (3) it is convenient to note that equations

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* Gibson, Proc. Roy. Soc., A., 83, p. 376 (1910).

(5) and (7) represent relationships between two non-dimensional functions, such as $(\alpha \rho Q)/(r_m \mu)$ and $(r_m{}^4 \,.\, dp/dx)/(\mu Q)$; and that the form of the variation of velocity with distance from the axis, expressed by equations (4) and (6), depends on only one non-dimensional function, which may be either of the above or any product of powers of them.

In Fig. 1, equations (5) and (7) are used to plot a relationship between the above non-dimensional functions. (For convenience, the sign of α has here been called positive.) This diagram may be compared with similar diagrams of experimental results given in an earlier Paper,* showing the relationship between Q and h/Q (h being the pressure-difference in centimetres of water column, between the liquid at two parts of the tube).

For slow motion, equations (4) and (6) represent the velocity varying parabolically over the transverse section; and equations (5) and (7) reduce to Poiseuille's equation. This is in agreement with experiment, and is represented by the point A in Fig. 1. For convergent flow at large rates, equation (4) denotes a velocity which is almost constant over the transverse section, except close to the walls of the tube. This is in agreement with experiment, as is also the limiting rate of flow predicted, equation (5) reducing to Bernoulli's expression for frictionless flow. This state corresponds to the limit B in Fig. 1.

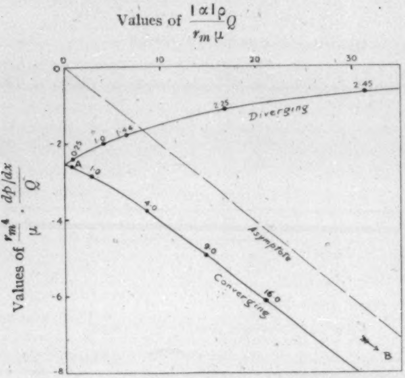

FIG. 1.

For moderately slow flow, when the kinetic energy of the liquid is just starting to be of importance, the slope of the lines at A in Fig. 1 can be shown to be equal to two-thirds of the final slope of the lower line at B. The experimental results† gave the slope at A roughly equal to that at B. (When those results were published the slope at A had been expected to be about double that at B.)

Fig. 2 illustrates for converging flow the velocities and the velocity distribution across the transverse section for various pressure-gradients (the numbers by the curves being proportional to the pressure-gradient). The corresponding points on Fig. 1 are indicated.

For diverging flow, corresponding curves are shown in Fig. 3, the outermost curve representing the limiting case. When the very large velocities predicted for points near the axis are attained, a large rate of flow corresponds to a small pressure-gradient, and the curve in Fig. 1 asymptotes to the axis. In the experiment previously referred to, such a state was found, except that the pressure seemed to rise slightly in the direction of motion, whereas according to the theory it would be expected to fall slightly.

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* Bond, loc. cit., Figs. 3, 4 and 5.

† Bond, loc. cit., p. 194.

At yet higher speeds turbulence would be expected to set in near the axis of the tube, the liquid moving in a narrow jet near the centre of the diverging cone. This effect is well known, and was suggested as an explanation in the author's Paper previously mentioned. This turbulence should occur first at the narrow entrance to the cone, and, as the direction of the walls changes at this point, the theory cannot be supposed to apply accurately and predict the exact critical pressure-gradient.

When the turbulent state is reached, the theory described cannot yield numerical results. However, if the liquid be supposed to move very fast and turbulently in a small and roughly parallel-sided jet from the entrance down the centre of the diverging cone, and this jet at some point beyond spreads out at an angle larger than that of the cone, till it fills the whole cone, the increase in pressure with distance may, over this diverging part of the jet, be larger than for a jet of angle equal to the cone. If, as the rate of flow is increased, the point of divergence moves nearer to the entrance of the cone, a large upward pressure-gradient would be observed when the point of divergence reached the part of the tube at which the pressure-gradient was being measured. This would correspond to the humps in Figs. 4 and 5 in the Paper previously referred to. Finally, when the point of divergence had almost reached the entrance to the cone, the liquid in the part of the tube experimented on would move turbulently in a cone nearly filling the tube, and the upward pressure-gradient might be expected to be about that deduced for frictionless flow, as was found to be the case in the experiments quoted.

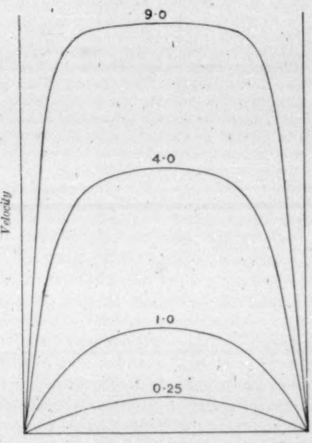


FIG. 2.—LONGITUDINAL SECTION OF TUBE.

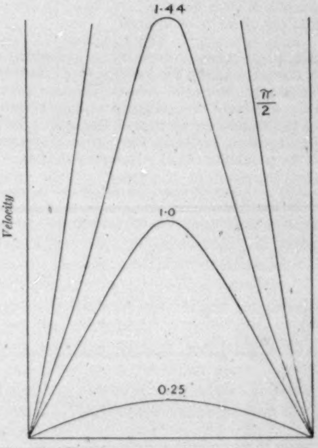


FIG. 3.—LONGITUDINAL SECTION OF TUBE.

DISCUSSION.

Dr. A. **Ferguson**: The Society is indebted to Dr. Bond for the solution which he has given of this very interesting problem. It would add much to the interest of his Paper if he would compare his solution with one given by Professor Gibson in the Philosophical Magazine about 1909. Professor Gibson, if I remember rightly, puts

$$v=w=\frac{\partial p}{\partial y}=\frac{\partial p}{\partial z}=0$$

so that the fundamental equations reduce to

$$-\frac{\partial p}{\partial x}=-\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho u\,.\,\frac{\partial u}{\partial x}$$

which, transferred to polar co-ordinates x, θ, integrates quite readily. This solution was not discussed in such detail as Dr. Bond's, and a comparison of the simplifying assumptions made would not be without interest.

Author's reply: The equation in Professor Gibson's Paper (Phil. Mag., July, 1909), to which Dr. Ferguson refers, differs somewhat from equation (2) of the present Paper, and I am also doubtful about the validity of its derivation (*see* Harrison, Proc. Camb. Phil. Soc., Vol. 19, pp. 311-12). Professor Gibson assumes the stream-lines to be straight lines through the cone apex; and later in solving the equation seems to assume that the velocity varies parabolically over the transverse section of the cone. In the present Paper the former is assumed as a first approximation, enabling the (curved) stream-lines to be plotted to the next order of accuracy; and the parabolic velocity variation is definitely not assumed. These differences in assumption make the results of the present Paper considerably different from those of Professor Gibson, some of whose conclusions I think are definitely in error.

II.—THE STRUCTURE OF THE SMOKE PARTICLES FROM A CADMIUM ARC.

By H. P. Walmsley, *M.Sc.*

Received April 21, 1927.

(*Communicated by* Dr. A. B. Wood.)

Abstract.

Using the powder method of X-ray analysis, the particles dispersed in air from a cadmium arc are found to be isometric crystals of cadmium oxide. From X-ray data, a density of 8·16 was obtained for the primary particles in the smokes—the normal density of cadmium oxide. Photometric measurements of the breadth of the lines showed that the primary crystals were of colloidal dimensions, values of $5\cdot8 \times 10^{-6}$ cms. and $4\cdot9 \times 10^{-6}$ cms. being obtained in two cases. On aggregation, the ultramicroscopic crystals are shown to grow along binary axes of symmetry—i.e., they tend to unite on their 110 faces. Photomicrographs of the deposits obtained from cadmium oxide smokes are shown, together with models of aggregates built on these lines. The models give many of the characteristic structures which are observed in the complex particles. Previous measurements on the smoke particles are discussed in the light of these results.

I.

By means of the powder method of X-ray analysis, an attempt has been made to determine the density and average size of the primary particles dispersed as a cloud, by arcing between cadmium electrodes in air. The arc was enclosed in a box from which the cloud could be withdrawn, by means of an aspirator, through an intense electric field, where the cloud particles were precipitated. A sample of the dark brown precipitate when examined by the Debye-Scherrer method using copper $K\alpha$ radiation gave a photograph characteristic of a crystalline powder. The position and intensities of the lines were in good agreement with those recorded by Scherrer (1) for cadmium oxide. This identified the material.

To determine the lattice dimensions, a second photograph was taken of the powder mixed with a little nickel oxide.(2) The lines due to the latter were employed to calibrate the film and avoid corrections due to errors of adjustment, such as imperfect centering of the specimen in the camera. The results were reduced by the method of least squares and are tabulated below.

Cadmium Oxide.

Cu $K\alpha_1 = 1\cdot537$Å, Cu $K\alpha_2 = 1\cdot541$Å, NiO $a = 4\cdot705$Å.

No.	*hkl*	θ	$\sin\theta/\sqrt{\Sigma h^2}$	No.	*hkl*	θ	$\sin\theta/\sqrt{\Sigma h^2}$
4	113	65·44°	$0\cdot1630_0$	9	224	106·39°	$0\cdot1634_8$
5	222	68·79°	$0\cdot1634_5$	$10\alpha_1$	$\left\{\begin{matrix}115\\333\end{matrix}\right\}$	116·78°	$0\cdot1639_0$
6	004	81·55°	$0\cdot1632_6$	α_2	...	117·31°	$0\cdot1639_3$
7	133	90·80°	$0\cdot1633_5$	$11\alpha_1$	044	136·03°	$0\cdot1639_0$
8	024	93·88°	$0\cdot1633_8$	α_2	...	136·76°	$0\cdot1639_1$
						Mean ...	0·16355

The lattice constant $a = \frac{\lambda}{2} \cdot \frac{\sqrt{\Sigma h^2}}{\sin\theta} = \frac{1\cdot537\text{ Å}}{2 \times 0\cdot16355} = 4\cdot699 \pm 0\cdot002$ Å.

Assuming four molecules to the unit cell, the density

$$\rho=4\times\frac{128{\cdot}4\times1{\cdot}649\times10^{-24}}{(4{\cdot}699\times10^{8})^{3}}=8{\cdot}164\pm0{\cdot}004$$

According to Scherrer, the crystal structure of cadmium oxide is similar to that of rock salt with a value of $\alpha=4{\cdot}72$Å. Its mean density according to Landolt Börnstein's tables is 8·15.

It therefore appears that the molecules of cadmium volatilized by the arc, after oxidation, reassemble with the symmetry of the same isometric space lattice, as is found in laboratory specimens of cadmium oxide. Since there is nothing to hinder the growth of the particles in the presence of the supersaturated vapour, the internal structure will be bounded by facets. The smoke particles are therefore tiny crystals.

The size of the particles was determined from the equation

$$B=2\sqrt{\frac{\log_e 2}{\pi}}\cdot\frac{\lambda}{D}\cdot\frac{1}{\cos\theta}+b,$$

where D is the depth of the layers contributing to the reflection at the glancing angle θ. To determine B (Halbwertsbreite), photometric measurements of opacity were taken across the lines on the photographic film, excluding those occurring at very small angles and very large angles on account of their curvature. The coefficient of $1/\cos\theta$ is obtained from the slope of the straight line obtained by plotting the values of B as ordinates and those of $1/\cos\theta$ as abscissæ. This gave a value of $D=5{\cdot}8\times10^{-6}$ cms.

The homogeneous particles of the cloud—the primary particles—are, therefore, ultramicroscopic crystals, precisely similar in density and internal structure to macroscopic crystals of cadmium oxide.

II.

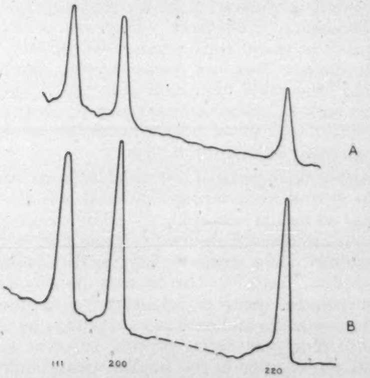
After dispersion, the primary particles in smoke clouds form secondary or complex particles by aggregation. With Cd O there is a marked tendency to produce chain-like structures. In sparse clouds the structure is occasionally seen in the ultramicroscope, but it may be observed under the microscope (with suitable illumination) in deposits collected on a glass plate by sedimentation (Fig. 3). With dense smokes, particles diffuse and adhere to the roof of the smoke box and to projections on its walls, and grow as fine threads under the action of gravity. In an electric field these tend to lie along the lines of force, and the chains grow as arborescent structures from each electrode. The growth may continue until they stretch completely across the field. If a horizontal sheet of paper is just pierced from below by the lips of charged wires, the threads as they deposit under gravity mark out the lines of force of the electric field, just as iron filings do in the analogous magnetic fields.

As the structures are aggregations of crystals, it may be expected that the space lattices of adjacent particles will exhibit the usual tendency to set themselves parallel on linking up—the phenomenon of parallel grouping. This occurs frequently with isometric crystals; the crystallisation of native copper and gold are usually quoted as examples. On this hypothesis, the long chains or threads will form along the direction of an axis of symmetry, and if we employ such a thread instead of a cylinder of powder in the Debye-Scherrer method, it should be possible to identify the axis. The restricted orientation of the particles changes the intensities of the lines on the film relative to those obtained with irregularly distributed particles for only planes whose normals are perpendicular to the axis of the camera are able to reflect X-rays. Compared with the case where the same particles are distributed irregularly, the mathematical expressions for the intensities of a given reflection will differ only in the factor which represents the number *n* of possible reflecting faces. For example, if a tetragonal axis lies parallel to the axis of the camera, *n* for the 100 and 110 faces is 4, and for the 111 faces is zero. When the particles are irregularly oriented, the corresponding values are 6, 12 and 8. Thus, the ratios of the intensities are as 4/6 : 4/12 : 0—i.e., as 2 : 1 : 0. Table II gives the relative changes in the intensities of the reflections from the principal planes for each of the three types of axes along which aggregation may occur.

TABLE II.

Plane.	Vertical axis in camera.		
	Tetragonal.	Trigonal.	Digonal.
100	2	0	2
110	1	3	1
111	0	0	3

Some of the fine thread-like structures formed from a dense cloud were attached lengthwise along a hair and a powder photograph taken. The film was measured photometrically and compared with one of similar density taken with irregularly distributed particles. The results for the three principal reflections are shown graphically in Fig. 1. The intensity of reflection is proportional to the area of the curve above the general background. With the oriented particles, it is clear that the 111 reflection is enhanced relatively to the 200, and the 220 is weakened. The direction of growth of the chain-like structures is therefore the direction of a binary axis of symmetry.



A Chain-like structures placed along axis of camera.
B Precipitated material placed along axis of camera.

FIG. 1.

Let I^n be the intensity of the reflections when the particles are oriented at random and I_0 that due to the thread-like structures on the hair. In Table III are given the ratios I_n/I_0, which would have been observed if the 220 reflection had been equal in each case. The fourth column gives the theoretical values of the ratio on the assumption that 20 per cent. of the particles are lying with a binary axis of symmetry parallel to the axis of the camera, and 80 per cent. are irregularly distributed. The agreement of these values with the observed seems tolerably good when it is remembered that, owing to uncertainty in assigning values for the intensity of the background of the film at each line, accurate measurements of the intensities of the weaker ones are difficult to obtain.

TABLE III.

hkl	I_n	Ratio of intensities.		*hkl*	I_n	Ratio of intensities.	
		Found.	Calc.			Found.	Calc.
111	77	1·40	1·40	400	12	1·26	1·20
200	96	1·20	1·20	331	22	1·08	1·00
220	100	1·00	1·00	420	32	1·05	1·00
311	53	1·12	1·00	422	35	1·00	1·00
222	24	1·38	1·40	511 333	33	1·00	1·10

The value 20 per cent. is perhaps as high as can be expected. For obvious reasons, it is impossible to pull the chain-like structures taut when attaching them to the hair in the camera, and so get the maximum effect. Further, for each crystal unit of the chain there are five other binary axes, all inclined to the direction of the hair along which growth is possible. From the widths of the lines, the average dimensions D of the primary particles in the chain-like structures was found to be $4{\cdot}9 \times 10^{-6}$ cms. Thus, the structures are built up of ultramicroscopic primary crystals united on their rhombic dodecahedral (110) faces. They are skeleton crystals. Similar skeleton forms composed of macroscopic crystals are familiar in crystallography as crystallites (3) or dendrites. They are forms showing parallel growth. The skeleton structure will not be entirely destroyed when the complex particles are obtained from the cloud in bulk by either sedimentation or electrical precipitation. Thus, although the individual units in the deposit possess the normal density of cadmium oxide, that of the material in bulk will be less. (4)

There is an interesting analogy between the growth of the skeleton forms from the tiny crystals and the growth of the crystal from vapour. Natural crystals of CdO occur rarely. They have been found as regular octahedra. (5) They have been prepared artificially by Florence (6) as microscopic octahedra by fuzing CdO with borax. Prepared by burning metallic cadmium in a stream of oxygen, they appear as cubes, and as cubes showing octahedran faces. (5) The crystals occasionally found in the muffles in zinc smelting processes occur as octahedra (7) ; at times modified by 100, 110 and 210 faces. The octahedra and cube faces appear to be the dominant forms. The 110 faces tend to disappear during growth, so occur less frequently, and it is on these faces that the crystals in the smokes usually unite. Thus, when two primary crystalline particles of cadmium oxide aggregate, the colliding crystal particle tends to attach itself, preferentially to precisely the same face as would a colliding molecule during the growth of the crystal from vapour'. Repetition of this process naturally produces a skeleton form. From the point of view of the Gibbs-Helmholtz equation, the 110 faces, growing more rapidly than either the 100 or 111 faces, possess the maximum adhesional surface energy.

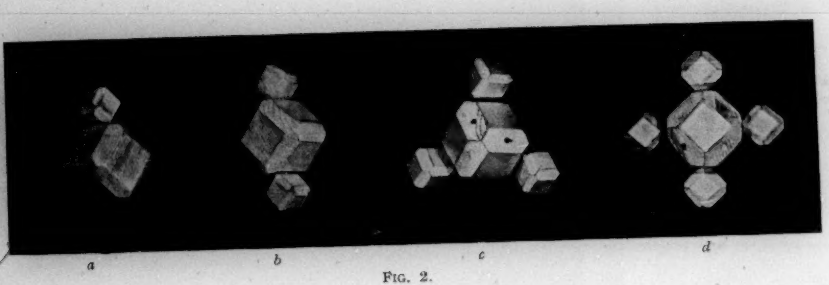


FIG. 2.

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III.

The general configuration of the crystalline complex resulting from aggregation of the particles depends upon the conditions under which the building material is supplied. The production of long tree-like forms from the upper walls of the smoke-box and from the electrodes of an electric field through which a flow of smoke is maintained, obviously arises by growth along a concentration gradient. In the absence of such gradients one would expect the skeleton forms to develop more symmetrically; but, once development has commenced, the subsequent growth from primary particles may be considered as before as a growth along a concentration gradient. Complex particles like the one illustrated in Fig. 3*a* clearly arise from the union of two or more skeleton forms. The general appearance of the dendritic forms is therefore governed by the disposition of those particles which united to form the centre of growth.

The primary particles of the clouds are not of uniform size. In fact, the sizes seem to range between comparatively wide limits. As the collision frequency is greatest between particles of extreme sizes,(8) the most frequently occurring type of binary particle will be one containing a large and a small primary, united on a 110 face. This is represented by the model in Fig. 2*a*, where the cadmium oxide particles are represented as cubes showing 110 faces (in conformity with their shape when produced by burning cadmium in oxygen). The large particle may be considered as the centre from which the dendrite grows by union with the small one. Growth will continue for a time mainly by the addition of the smaller particles. A second small particle uniting with the binary *may* join on to any one of the 22 exposed 110 faces, or may even unite on the vacant part of the 110 face of the large particle which already carries the first small particle. Considerations of size make a union on another 110 face of the large particle more probable, but do not indicate which face. Independent evidence exists that the motions of the particles are not entirely governed by the laws of chance, so that union with any one of the vacant 110 faces of the larger particle is not equally likely. If we may extend the previous analogy between crystal growth from vapour and the dendritic growth of aggregates, it is perhaps not unreasonable to suppose that just as the molecules of a crystal assemble with rhythmic symmetry, a tendency to produce symmetrical arrangements exists with the particles. Crystals frequently show a tendency on association to produce symmetrical groupings. With crystals of low symmetry this often result in the simulation of higher forms (mimetic twinning).

Assuming, therefore, that the small crystal particles attach themselves, if possible, symmetrically about the large one, which is acting as a centre of growth combinations with two, three and four particles will tend to associate as in the models of Fig. 2. These should be commonly occurring basic forms on which the dendritic structures may be conceived to develop by growth along concentration gradients.

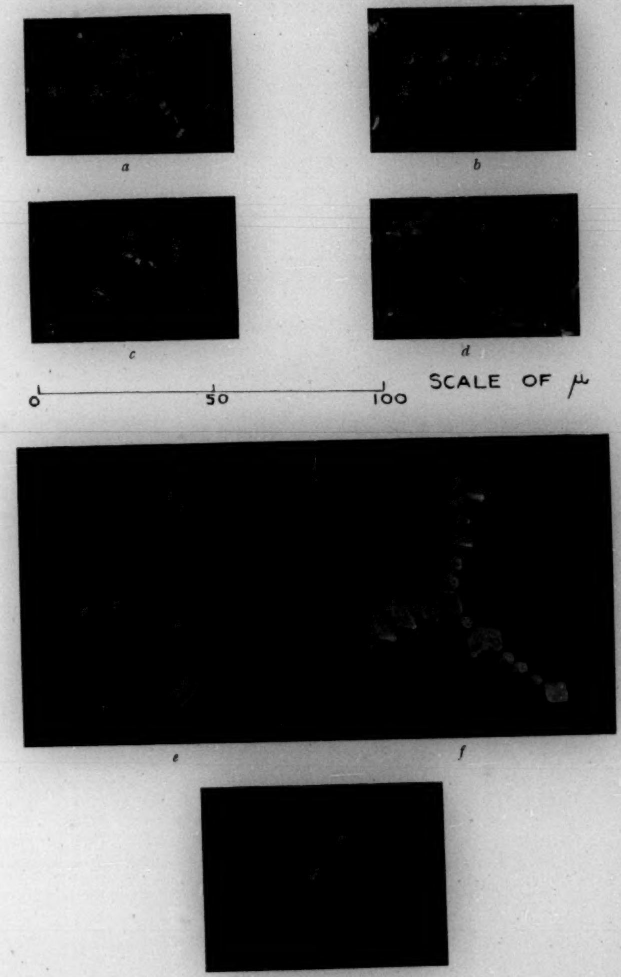
The cruciform structure of Fig. 2*d* is found in the complex shown in the photomicrograph of Fig. 3*c*. The spots there are diffraction images formed by oblique illumination of the particles under the ultramicroscope, using the violet line from a mercury vapour arc. The average distance along the axes between the centres of the spots is $740\mu\mu$. Thus, if the crystal dimensions D were $740\mu\mu$ the structure would be represented by the model of Fig. 2*d*, with five particles equal in size. A cube edge of $7{\cdot}4 \times 10^{-5}$ cms. is a very large but not impossible size for a primary particle. As there is no definite information of the actual sizes of the particles forming the cross, the presence of much smaller particles in the cloud would suggest that the structure is more like the model of Fig. 3*e*, where the five large particles, now each less than $7{\cdot}4 \times 10^{-5}$ cms. edge, are united by smaller ones. If the dimensions of the large particles are but two or three times greater than those of the small ones, the ratio of the amount of light scattered, according to Rayleigh's law, will be 2^6 or 3^6 respectively—i.e., the smaller particles will scatter only 1·56 per cent., or 0·14 per cent. of the amount scattered by the larger ones. This could hardly have been detected; so making the links uniting the large particles invisible.

The three-limbed complex in the photomicrograph of Fig. 3*d* is more interesting. The sharpness of focus for all the spots shows that the three arms are coplanar, or nearly so. This arrangement is only possible with growth along binary axes of symmetry. The arms show trigonal symmetry about a perpendicular through the centre of growth, which immediately makes growth along a tetragonal axes impossible. The other possible growth axis—a trigonal axis—is equally not responsible for the structure, for in this case, although threefold symmetry is possible, the arms would not be coplanar, but the centre of growth would be supported by the arms in the manner of a tripod above the glass surface carrying the deposit. With the microscope objective used, the central spots would be out of focus, just as are parts of the neighbouring large complexes. The structure arises from the basic model of Fig. 2*c*, and the crystallographic evidence of this quite common type of complex confirms the X-ray data. Fig. 3*f* is a model of the complex built on similar lines to that in Fig. 3*e*. Fig. 3*g* is a reduced photograph of this model when illuminated obliquely as were the particles in the photomicrographs.

IV.

In a Paper (9) on the conductivity of cadmium oxide clouds, the author found it convenient to classify the ions present as small ones and large ones. The size of the large ions was found to increase as the clouds aged, and although it was not possible to obtain exact determinations of their mobilities and sizes, limiting values were obtained for the ions in a particular case. Treated as spherical in shape, the upper limit to the radius of the small ions was found to be $3{\cdot}35 \times 10^{-6}$ cms. After 20 minutes the lower limit for the radius of the large ions was $2{\cdot}2 \times 10^{-5}$ cms., and as the cloud aged some of the particles attained at the upper limit radii of $9{\cdot}11 \times 10^{-5}$ cms. and $17{\cdot}6 \times 10^{-5}$ cms. The dimensions of the primary crystals measured in this Paper—$D = 5{\cdot}8 \times 10^{-6}$ cms. and $D = 4{\cdot}9 \times 10^{-6}$ cms.—fall within the limits of size previously given for the small ions. Apparently the majority of the small ions are primary crystal particles carrying charges, and the large ones are dendrites or crystal clusters. This is confirmed by recent work of Patterson and Whytlaw Gray, (10) who have obtained estimates of the sizes and densities of individual particles in cadmium oxide smokes, using Millikan's method for oil drops. Determinations gave values for the radius in 10^{-5} cms. of 2·07, 2·42, 5·70, 6·38, 6·60, 7·60, 7·81 and 13·1. These mostly fall within the limits of size of the large ions given above. The mean value obtained for their density, 0·51, is consistent with the dendritic structure of the aggregates.

The estimate of size by both methods rests on the assumption that the resistance to the motion of the particles is governed by Stokes' law. Since the resistance


0
50
100
SCALE OF μ

a b c d e f g

FIG. 3

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is apparently of the same order when the particles are moving under gravity and when moving in an electric field, it would appear that in general the complex particles in sparse smokes must be fairly symmetrical in shape, and photomicrographs seem to confirm this (Fig. 3*d*). As a corollary, the chain-like structures are perhaps not to be looked upon as a free normal growth. Although they certainly may arise by the union of structures like the models of Fig. 2 (*a* and *b*), a very large number must arise by disruption from the larger complexes, many of which are inherently unstable.(11) The disruptive process is quite evident if the growth of dendritic forms is observed in an electric field. It will be noticed that as the structure grows particles and branches occasionally break away from the dendrite growing from one electrode, shoot across the field and attach themselves to the branches of the dendrite growing from the other electrode, and vice versa.

REFERENCES.

(1) Scherrer, Zeit für Kristallographie, 57, p. 186 (1922).
(2) Brentano and Dawson, Phil. Mag. (7), 3, p. 411 (1927).
(3) Dana, Textbook of Mineralogy. 3rd edition. , p. 180 (1922).
(4) Kohlschutter and Tüscher, Zeit. für Electrochem., 27, p. 1 (1921).
(5) Wittich and Neuman, Centr. Min., p. 550 (1901).
(6) Florence, Neues Jahrb. Min., 2, p. 137 (1898).
(7) Von Werther, Jour. für Prak. Chem., 55 (1852).
(8) See succeeding note.
(9) Walmsley, Phil. Mag. (7), 1, p. 1266 (1926).
(10) Patterson and Whytlaw Gray, Proc. Roy. Soc., A, 113, p. 312 (1926).
(11) Walmsley, Phil. Mag. (7), 3, p. 587 (1927).

DISCUSSION.

Dr. G. **Shearer** said that the author could have saved himself a great deal of trouble by using a large photographic plate, or a photographic film surrounding the hair, instead of a narrow axial strip. The equation giving the relation between brightness and the size of the particles was primarily applicable only to cubic crystals, and he was surprised that there should be a linear relation between B and $\sec \theta$ in the present case ; the point needed fuller investigation.

The **Author**, in reply, said that his chief technical difficulty had been to get the particles arranged on the hair. His results could scarcely be taken as proving a linear relation between B and $\sec \theta$; he had wanted a rough estimate of the size, and had obtained this by finding the mean straight line through the points plotted.

III.—THE POWER-FACTOR AND CAPACITY OF THE ELECTRODES AND BASE OF TRIODE VALVES WITH SPECIAL REFERENCE TO THEIR USE IN THERMIONIC VOLTMETERS.

By G. W. Sutton, *B.Sc.*

Received May 19, 1927.

ABSTRACT.

The Paper discusses the conditions under which a three-electrode valve-voltmeter should be operated to ensure a minimum power consumption, and at the same time to give indications closely proportional to the square of the input voltage. A simple method of adjusting the operating voltages to fulfil the necessary conditions is described.

THE requirements for a valve-voltmeter to be used for high-frequency resistance measurements may be summarised as follows :—

(1) That it shall impose the smallest possible load on tuned circuits to which it may be connected.

(2) That its indications shall be as closely proportional to the square of the applied voltage as possible.

(3) That it shall be rapid in indication.

(1) An inherent disadvantage of the voltmeter, when compared with the thermo-junction, is that it involves connection with tuned circuits at points at high potential as well as at " earthed " points. Thus the effective capacity of the tuned circuit is increased, the magnitude of the induced voltage is likely to be altered, and any losses in the voltmeter are added to those in the tuned circuit. The additional effective capacity of the order of 5 to $10\mu\mu F$ is troublesome at times, but is much less so with the valve-voltmeter than with the electrostatic voltmeter, since in the latter case it varies appreciably with the instrument reading.

The losses in the voltmeter are a much more serious point. In his Paper on a " Direct Reading Thermionic Voltmeter,"* E. B. Moullin shows that the effective resistance of his anode bend type voltmeter is of the order of 0·5 megohm at full-scale reading of about 1·5 volts (R.M.S.), and varies very considerably with the applied voltage. He quotes a case in which the instrument introduces an additional decrement of about 0·015 into a tuned circuit. Since the decrements of good inductance coils range from 0·005 to 0·1, his subsequent statement that " In cases where the decrement introduced by the voltmeter is of importance, it can be determined with sufficient accuracy to enable the necessary allowance to be made," stands in need of experimental demonstration. In any case, it may even double or treble the decrement of the circuit under investigation. By the use of larger values of mean anode and mean negative grid voltage, than those employed in the Moullin instrument, the effect of grid current can be very materially reduced, but it has been found from time to time that the usual form of valve-holder and base may introduce considerable additional loss, as might be expected from their material and construction.

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* Journal I.E.E. (December, 1922).

L. Hartshorn and T. I. Jones have investigated the losses associated with valve capacities.* In both Papers the Schering bridge is used, at audio-frequencies. With condensers of such high power-factor as these appear to possess, the influence of a frequency change of, from 1,000 cycles to, perhaps, 1,000 *kC.*, is likely to be very considerable. It seemed well worth while therefore, to make a few further measurements at radio-frequencies. Moreover, Messrs. Hartshorn and Jones have considered the general case in which both anode and grid potentials fluctuate. For the present purpose we need only be concerned with fluctuating grid potential, since as large a condenser as is necessary will invariably be connected from anode to filament. The effective capacity of the voltmeter is therefore, to a very close approximation, the sum of the mutual capacities from grid to filament and from grid to anode.

In one of the above Papers† it is stated that "It would not be safe to make a quantitative deduction of power-factor, but an examination of the results indicates that power-factor is higher in the glass (the pinch) than in the black insulating compound." It was thought that this point needed further investigation, since it indicates the necessity for the use of specially constructed valves, of the "V" type, for instance, for valve-voltmeters.

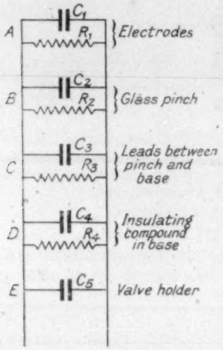

FIG. 1.

From the results on which this statement is based it is possible, by assuming the valve elements to constitute a circuit as illustrated in Fig. 1, to estimate the values of the various parallel capacities and resistances, and hence the corresponding power-factors.

Results of measurements on the Schering bridge from L. Hartshorn's Paper:

	$C\mu\mu F$	P.F.
$A+B+C+D+E$ ...	3·82 ...	0·21
$B+C+D+E$ ...	1·65 ...	0·31
$C+D+E$	1·11 ...	0·24
$D+E$	0·97 ...	0·23
E	0·26 ...	0·00

Assuming a frequency of $6{,}000/2\pi$ (for convenience in arithmetic; the actual value is not quoted), the following values may be deduced:—

TABLE I.

	C_1 etc. $\mu\mu F$	R_1 etc. Megohms	P.F.
A	2·17	578	0·13
B	0·54	676	0·47
C	0·14	3850	0·31
D	0·71	748	0·31
E	0·26	0	0

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* **L. Hartshorn and T. I. Jones, "The Inter-Electrode Capacities of Thermionic Valves." Exp. Wireless (February, 1925).**

† **L. Hartshorn, "The Input Impedances of Thermionic Valves at L.F." Proc. Phys. Soc. (February, 1927).**

This certainly indicates that the power-factor of the pinch is higher than that of the insulating compound in the base, and that both are exceptionally high. But it also allots a power-factor of 0·13 to the air-insulated electrodes and 0·31 to air-insulated leads; whilst the valve-holder has immeasurably small power-factor. It is true that the valve-holder used was of sound design and mounted on ebonite, but it is surprising to find its insulation resistance so much higher than that of a few centimetres of air-insulated conductors.

Some results obtained previously by the present writer had indicated that the losses to be expected, both in the glass pinch and also in the base, are of a much lower order than those just quoted. Before giving these results, it may be as well to describe the method by which they were obtained.

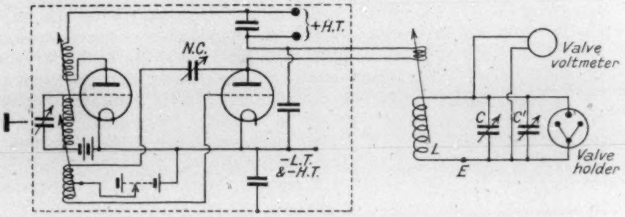

FIG. 2.

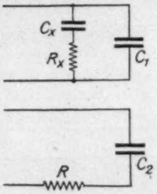

FIG. 2A.

C_x=effective capacity of valve, etc.
C_1=remaining tuning capacity.
C_2=capacity equivalent to C_x, C_1, R_x.
R_x=equivalent series resistance of valve losses.
R =resistance in series with C_2 equivalent to R_x, C_x, C_1.

Following E. Mallett and A. D. Blumlien,* the shielded generator unit used consisted of an oscillator and an amplifier (or preferably "separator") valve. Increased security against reaction between the circuit *L C* and the oscillator valve was obtained by balancing the grid-anode capacity of the separator valve by the neutralising condenser *N.C.* The circuit *LC* consisted of a low-resistance coil tuned to resonance by the condensers *C* and *C'*; *C'* being a small calibrated vernier enabling measurements of small changes of capacity to be made to an accuracy of about 0·02$\mu\mu F$. Across these condensers a valve-voltmeter and a well-designed valve-holder were connected. The two filament and the anode sockets of the latter were joined together and to the point "*E*" (the "earth" side of the circuit) and the grid to the high potential side.

When determining the power-factor of a valve or valve-base, the capacity was measured, first, by substitution, using *C'* alone for the purpose. The valve was then again placed in the holder (care being taken to see that it was always pressed home in the sockets to the same point), the circuit brought to resonance and the voltmeter reading noted. Next the valve was removed, the circuit retuned, and the new voltmeter reading, invariably larger than the first, was taken. Two or three resistances, of values found experimentally to be approximately equivalent to losses in the valve, were then inserted in turn at the point E. From these readings the resistance which, if placed in series with all the tuning capacity in the circuit, is exactly equivalent to the losses in the valve, may readily be calculated. Calling this value R, and that of the equivalent series resistance of the valve alone R_x (*see* Fig. 2A), we have, equating admittance operators,

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* Journal I.E.E., "A New Method of H.F. Resistance Measurement" (December, 1924).

$$j\omega C_1+\frac{1}{R_x-j\dfrac{1}{\omega C_x}}=\frac{j\omega C_2}{1+j\omega RC_2}$$

or
$$C_1+\frac{C_x-j\omega C_x^2R_x}{1+R_x^2\omega^2C_x^2}=\frac{C_2-j\omega C_2^2R}{1+R^2\omega^2C_2^2}$$

In this case $R_x^2\omega^2C_x^2$ and $R^2\omega^2C_2^2$ are $<<1$

So that $C_1+C_x-j\omega R_xC_x^2=C_2-j\omega C_2^2R$

Also $C_1+C_x=C_2$ (very nearly)

And hence $R_x=\left(\frac{C_2}{C_x}\right)^2R$ to a close approximation.

Thus, from R and the known capacities, R_x, and hence the power-factor of the valve, may be calculated.

In Table II the results of a number of tests carried out from time to time by this method are summarised.

TABLE II.

	(f=650kC.)	$C(\mu\mu F)$.	P.F.
(a)	Modern " low-loss " valve holder (sockets mounted on narrow insulating ring of large diameter)	$1{\cdot}4_8$	$0{\cdot}04$
(b)	French D.E. valve	$7{\cdot}0_0$	$0{\cdot}02_1$
(c)	Ditto base alone	$1{\cdot}1_0$	$0{\cdot}1_0$
(d)	Valve base found previously to have very low insulation resistance	$2{\cdot}4_2$	$0{\cdot}37$
(e)	English D.E. valve	$4{\cdot}9_6$	$0{\cdot}03_4$
	Ditto base alone	$1{\cdot}5_9$	$0{\cdot}05_5$
	Ditto remainder alone—i.e., pinch, electrodes and leads	$3{\cdot}0_6$	$0{\cdot}02_0$
	(f=545 kC.)		
(f)	" R " valve. Metal ring on cap	$5{\cdot}0_5$	$0{\cdot}06_3$
	Ditto base alone	$1{\cdot}8_3$	$0{\cdot}17$
	Ditto remainder alone	$2{\cdot}9_0$	$0{\cdot}00_3$
(g)	" R " valve. Another manufacturer	$4{\cdot}6_2$	$0{\cdot}04_9$
	Ditto base alone	$1{\cdot}7_2$	$0{\cdot}09_8$
	Ditto remainder alone	$2{\cdot}5_8$	$0{\cdot}02_8$*
(h)	D.E.R. valve. Bakelite cap	$4{\cdot}4_1$	$0{\cdot}01_8$
	Ditto base alone	$0{\cdot}7_5$	$0{\cdot}05_4$
	Ditto remainder alone	$3{\cdot}2_2$	$0{\cdot}00_{15}$
(i)	D.E.R. Old pattern, before the use of " gettering " ...	$5{\cdot}1_1$	$0{\cdot}03_8$
	Ditto base alone	$1{\cdot}6_1$	$0{\cdot}05_4$
	Ditto remainder alone	$3{\cdot}2_3$	$0{\cdot}02_7$

* It was noticed that the under surface of the glass pinch was oily. This may have accounted for the comparatively high value of the power-factor in this case as compared with the others.

From this it appears that, at least at radio-frequencies, the insulation in the base of the average receiving valve has a much higher power-factor than the glass pinch, and occasionally it may be very high indeed (*see* (*d*) in Table II). The power-factor of the pinch and electrodes, on the other hand, is frequently so low as to introduce completely negligible losses into an ordinary tuned circuit. In general, therefore, it should be sufficient merely to remove the cap from an ordinary receiving valve for use as a valve-voltmeter.

As an instance of the effect which neglect of this precaution may be expected to have when measuring H.F. resistance, the following experiment may be quoted. An anode-bend voltmeter was added to a circuit a portion at a time, the circuit consisting of an inductance of $140\mu H$ and a condenser of $400\mu\mu F$.

TABLE III.

(*a*) Resistance of tuned circuit alone	$3\cdot6_8$ ohms
(*b*) With valve-holder, galvanometer and batteries connected ; but no valve or shunting condenser from anode to filament	$3\cdot8_2$,,
(*c*) Valve added, but no filament or anode current	$4\cdot0_2$,,
(*d*) Filament current switched on...	$4\cdot0_1$,,
(*e*) Anode current switched on	$4\cdot3_5$,,
(*f*) Shunting condenser (1·5 μF) added added	$4\cdot0_3$,,

In view of the above results an ordinary D.E.R. valve was selected, and the base removed by clipping the connecting wires from the pins and soaking the cement with which the cap is held to the glass in hot water for a few hours. After cleaning the surface of the glass around the points where the electrodes emerge, the valve was fitted, in an inverted position, into a small wooden box. The grid lead was kept as short and as far removed from the remaining apparatus as possible. By this means the equivalent resistance of the valve was reduced to that due to the losses in the pinch, and to any grid current which might flow. The effective capacity was found to be $4\cdot5\mu\mu F$; and the power-factor, with no filament current, 0·005.

To limit the effect of grid current as far as possible it was decided to employ a grid bias of $-4\cdot5$ volts. A few measurements of the increase of series resistance of the circuit used above due to dielectric loss and grid current together were then carried out with decreasing values of negative grid bias, the applied voltage being adjusted in each case so that the positive peak value carried the instantaneous grid potential to $-0\cdot5$ volts. The results of this test were as follows :—

TABLE IV.

Mean grid potential	$-4\cdot2$	$-4\cdot1$	$-4\cdot0$	$-3\cdot9$	$-3\cdot8$	$-3\cdot0$	$-2\cdot6$ volts
Increase of series resistance of the circuit	$0\cdot04_3$	$0\cdot05_1$	$0\cdot06_8$	$0\cdot07_7$	$0\cdot06_8$	$0\cdot09_0$	0·10 ohms

The decrement of the circuit used in the above experiments is 0·014. Moullin's anode-bend voltmeter would add, on full-scale deflection (using his figure of 0·75 megohm), 0·0025, or about 18 per cent. The voltmeter discussed above would add, on full-scale deflection and using $-4\cdot2$ volts grid-bias, 0·00024, or about 1·8 per cent. Even with $-2\cdot6$ volts grid bias it only becomes $0\cdot0005_5$.

Against this improved performance must be set the fact that three circuit adjustments are necessary in place of one. It should be noticed, however, that the combined adjustment of filament emission and grid bias in the Moullin instrument is only satisfactory if the emission per ampere of filament current may be assumed to be constant. This is certainly not the case with dull-emitting filaments, where the voltage to give constant emission may change as much as from 3·42 to 2·75 volts in 200 hours of use.* This point has been borne in mind in designing the present instrument.

(2) The chief advantage of a square-law valve voltmeter is that it simplifies and accelerates the process of calculation in high-frequency resistance measurements. No adjustments additional to those which would be required in any case to maintain constancy of calibration are necessary, and the use of the somewhat cumbersome calibration graph is avoided.

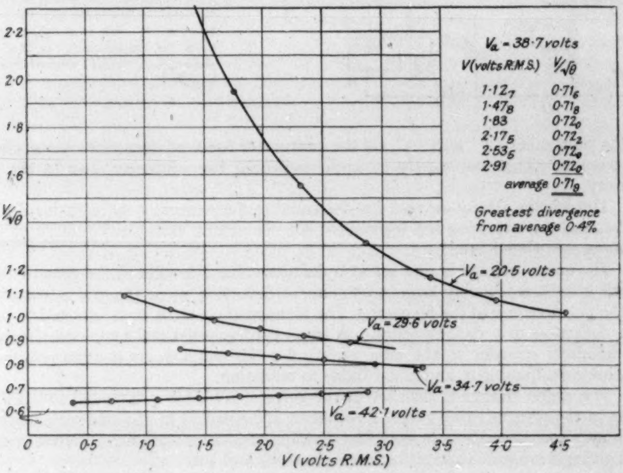

FIG. 3.

Several D.E.R. valves have been used from time to time, and little difficulty has been experienced in determining the adjustments necessary for compliance with the square-law within about 0·5 per cent.

From the results illustrated in Fig. 3 it is seen that for a given filament emission a grid bias and an anode voltage may be found for which the square root of the deflection on a good pivoted moving-coil galvanometer is proportional to the applied A.C. voltage within 0·4 per cent. The values of $V/\sqrt{\theta}$ given in the table for $Va = 38{\cdot}7$ are seen to rise and then fall slightly with increase of V, giving little hope that a better degree of constancy would be obtained for any neighbouring value of Va.

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* "Thermionic valves with dull-emitting filaments." M. Thompson and A. C. Bartlett, Journ. I.E.E. (April, 1924).

It is sufficient, however, for most measurements. This slight increase and decrease has been observed for all the valves so far examined.

It was subsequently decided to set the anode voltage at a definite value by means of the voltmeter shown in Fig. 4, and to adjust the grid-bias voltage by means of the resistances " *b* " and " *c* " till the instrument followed the square law as closely as possible. This was merely to avoid switching the voltmeter over to the grid battery.

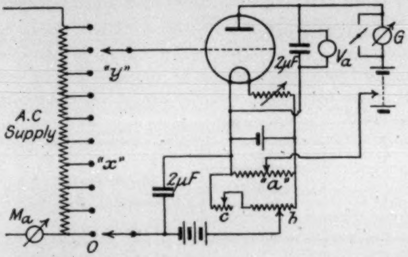

FIG. 4.—ILLUSTRATING THE ADJUSTMENTS NECESSARY TO RENDER $V/\sqrt{\theta}$ APPROXIMATELY CONSTANT.

" *a* " and " *b* " are 300 ohm potential dividers, " *c* " being a 30 ohm rheostat to give fine adjustment on the latter. *Ma* is a milliammeter used for calibration. *G*, M.C. galvanometer, with short-circuiting key. V_a, voltmeter.

The scheme finally devised for determining the necessary voltages for a given valve, and for adjusting the instrument for use at any time subsequently to calibration, is outlined below.

The anode voltage is first set at a definite value (15 volts in the present case), which gives a reasonably large galvanometer deflection when the grid is connected to the negative end of the filament. The filament rheostat is then adjusted to bring this deflection to a definite value. A small carbon pellet resistance was found to be definitely superior to the wire variety for this purpose, as it allows very fine continuous adjustment, and is not liable to oxidation.

The anode voltage is then set at the value at which it is intended to work (34 volts in the present case), and the voltmeter is connected to the calibration circuit. This may conveniently be a potential-dividing resistance carrying a known current and giving a range of voltages between about 0·5 and 3·0.

The next step is to determine the value of grid bias which will ensure the instrument following the square-law as nearly as possible. For this purpose two studs are selected on the potential divider giving galvanometer deflections of about 1/3 and 2/3 full-scale respectively. These have been marked " *x* " and " *y* " in Fig. 4. In the present case the ratio $\left(\frac{\text{resistance } y_0}{\text{resistance } x_0}\right)^2$ was 3·87. If the voltmeter conforms with the square law, therefore, the ratio of the two galvanometer deflections will have this value. A few readings are then taken with different settings of " *b* " and " *c*," and the resulting values of the ratio θ_y/θ_x are plotted to a base of θ_0, θ_0 being the deflection of the galvanometer corresponding to zero A.C. volts. The value of θ_0 corresponding to $\theta_y/\theta_x=3{\cdot}87$ is read off from the graph, and the grid bias is always adjusted on " *b* " and " *c* " to give this value when the voltmeter is set up for use. In taking these readings the current through the calibration potential-divider is adjusted in each case to give approximately the same value for θ_y.

Briefly, the sequence of adjustments necessary before using the instrument are as follows :—

(*a*) Switch on the filament current and allow about five minutes for the valve temperatures to steady down.

(*b*) Set the anode voltage to a recognised voltage (say 15), and connect the grid to negative filament lead. Set the filament emission to a recognised value, as indicated by the galvanometer.

(*c*) Set the anode voltage to the recognised operating value (say 34), and adjust the grid bias to give a second recognised galvanometer deflection.

Once the two standardising values of galvanometer deflection have been determined, the operation of setting up the voltmeter for use is only a matter of a few minutes. With a valve such as one of the D.E.R. type there is a range of voltage adjustment on the potential dividers of two volts, any larger adjustments in grid or anode battery voltage being effected by adding or subtracting unit dry cells.

The results of the calibration tests carried out at different times on this voltmeter may be summarised as follows :—

The mean value of $V/\sqrt{\theta}$ was deduced from seven readings corresponding to applied potentials of from about 0·6 to 2·8 volts.

TEST No. 1, mean value 0·725. Greatest departure from the mean, 0·004.

TEST No. 2, mean value 0·719. Greatest departure, 0·006.

TEST No. 3, mean value 0·721. Greatest departure, 0·002.

The differences between the mean values in the three tests is not necessarily due to changes in the valve-voltmeter, as the milliammeter used in calibration was an air-enclosed thermo-junction.

(3) The usefulness of rapid indication in the indicating instrument used will not be overlooked by anyone making a large number of high frequency resistance measurements. A pivoted and critically damped moving-coil galvanometer of short period should be used.

DISCUSSION.

Dr. E. MALLETT congratulated the author on having made some fairly exact measurements bearing on the accuracy of valve voltmeters, which are of increasing importance at wireless as well as at telephonic frequencies. The previous results quoted by the author had suggested the liability of these instruments to startling inaccuracies, and it was reassuring to learn that such inaccuracies could be avoided.

Mr. T. I. JONES: I welcome the results of Mr. Sutton's measurements of valve capacities, and I am struck with the general agreement shown between these results at 650 kilocycles per second, and those for the normal cases quoted by Mr. Hartshorn and me as measured at 800 cycles per second. Mr. Sutton has quoted in his Paper some figures of ours, and has proceeded in contravention of our express warning to apply to them the analysis erroneously implied in Fig. 1. This figure deliberately assumes that each of the portions *A*, *B*, *C*, etc., of the electrodes successively removed could be represented by a simple direct capacity in parallel with the others and that they were directly additive, which would only obtain if each was completely shielded. The simplest system to represent the situation approximately would require two cross capacities to the portions of the electrodes still remaining in each individual case. Even when the direct capacity was free from loss the cross capacities would not be, as the lines of force invariably penetrated into dielectrics other than air. The measure of agreement obtained in the normal cases between the results at radio and at audio frequencies would indicate that the losses are not to be too closely associated with a shunt or an insulation resistance. Thus we see that the results of the analysis as given in Table I are misleading, and that the subsequent comments are uncalled for. I would point out that the figures quoted were obtained on a burnt out valve which was broken open for the purpose. The P.F. is altogether higher than was found in the normal cases. Whether the surface of the glass pinch had been affected was not ascertained.

Dr. E. H. RAYNER referred to the table inset in Fig. 3, showing the variation of $V/\sqrt{\theta}$ with V, where V is the voltage to be measured and θ is the deflection observed. This variation appeared to be of the type which would result from the movement of the galvanometer coil in a field not strictly uniform.

The AUTHOR, in reply to the discussion, said that some of the valves tested had been broken open, while others had been intact. Possibly the effect of cross-capacity required fuller consideration than that which he had already given it. The galvanometer was a moving-coil Weston instrument; it had been calibrated with direct current, but the calibration curve did not confirm Dr. Rayner's suggestion.

Mr. R. P. FUGE (subsequent communication): I would like to ask Mr. Sutton if it is workable to put a very small condenser of the order of a fraction of a $\mu\mu$farad in series with the grid circuit of the valve in the voltmeter and so to reduce its capacity; and if the sensitivity of the voltmeter can be increased by amplifying by a second valve.

The AUTHOR (subsequent communication): In further reply to Mr. Jones, after giving further consideration to the effect of cross-capacities on the analysis suggested in Fig. 1, I am of the opinion that their neglect is permissible under the circumstances. If we consider, for instance, the cross-capacities between the electrodes and the leads in the pinch, to scale, it is obvious that the dielectric is almost entirely air. It seems most improbable that the small portion of the flux-path, which lies in the pinch itself, can account for such a high value of P.F. as 13 per cent.

The analysis was only intended to be a tentative one. The obvious assumptions made in replacing distributed capacities by localised condensers preclude any hope of precise deductions. It should represent the true conditions sufficiently accurately, however, to enable an estimate of the P.F. of the various portions to be made.

In reply to Mr. Fuge: The inclusion of a grid-condenser would be undesirable for two reasons. It would necessitate the use of a grid-leak; and any reduction in effective capacity would be accompanied by a corresponding reduction in sensitivity, the two condensers forming a potential divider. If previous amplifying valves are employed, the input impedance of the voltmeter depends on the circuit employed, owing to the "Miller" effect.

IV.—I. THE REFRACTION AND DISPERSION OF (1) AIR, (2) OXYGEN, (3) GASEOUS CHLOROFORM; II. NEW DETERMINATIONS OF THE GASEOUS REFRACTIVITIES OF (1) ACETONE, (2) METHYL ETHER, (3) ETHYL ETHER.

By H. LOWERY, *M.Sc.*, *F.Inst.P.*, Head of the Physics Department, Technical College, Huddersfield.

Received July 8, 1927.

(*Communicated by* PROF. W. L. BRAGG.)

ABSTRACT.

I. The refractivities of air, oxygen and gaseous chloroform have been found for the green mercury line (λ5461), and the dispersion studied over the range λ4800 to λ6700, with the following results :—

(1) Air : $(\mu-1)_{n.t.p.}\times 10^7 = 2931 (\lambda 5461)$

$$(\mu-1)_{n.t.p.} = \frac{4{\cdot}8286\times 10^{27}}{16774\times 10^{27}-\nu^2}$$

(2) Oxygen : $(\mu-1)_{n.t.p.}\times 10^7 = 2711 (\lambda 5461)$

$$(\mu-1)_{n.t.p.} = \frac{3{\cdot}4343\times 10^{27}}{12970\times 10^{27}-\nu^2}$$

(3) Gaseous chloroform : $(\mu-1)_d\times 10^6 = 1412 (\lambda 5461)$

$$(\mu-1)_d = \frac{14{\cdot}622\times 10^{27}}{10659\times 10^{27}-\nu^2}$$

where ν is the frequency of the light, $(\mu-1)_{n.t.p.}$ is the refractivity for N.T.P. conditions, and $(\mu-1)_d$ shows the refractivity by the same number of molecules of the gas as 1 cubic cm. of hydrogen contains at N.T.P.

II. The gaseous refractivities of acetone, methyl ether and ethyl ether for λ5461 were found to be :—

(1) Acetone : $(\mu-1)_d\times 10^6 = 1096$,
(2) Methyl Ether : $(\mu-1)_d\times 10^7 = 8876$,
(3) Ethyl Ether : $(\mu-1)_d\times 10^6 = 1509$.

I.

(1) AIR.

Introduction.

WHEN C. and M. Cuthbertson* published the results of their experiments on the refraction and dispersion of air in 1909 they reviewed the measurements of previous observers, and came to the surprising conclusion that " the accuracy of a determination is not a function of its date."

Meggers and Peters,† writing ten years later, also found occasion to draw attention to the " rather wide disagreements " between the observations up to that date, which, in addition to the results reviewed by C. and M. Cuthbertson, included work by Gruschke (1910), Siertsma (1913), Howell (1915), and Dickey (1917).

On examining the results of previous observers up to the present time (including the work of Meggers and Peters, Traub[*] and Stoll[†]), one still finds call to remark upon the considerable differences among the various determinations.

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* Proc. Roy. Soc., A, Vol. 83, p. 151 (1909).

† Bulletin, Bureau of Standards, Washington, XIV., No. 4, p. 697 (1919).

No apology is, therefore, necessary for publishing some additional results on the refraction and dispersion of air (stated for precisely known conditions), although the writer is fully conscious of the limited range of his dispersion curve and of the need for extending it as far as possible into the ultra-violet and infra-red regions, as was attempted, for example, by Meggers and Peters.

Experimental.

The refractive index of dry air (not deprived of carbon dioxide) for the green mercury line ($\lambda 5461$) was determined by means of a Jamin interferometer with the following results :—

$$(\mu-1)\times 10^7 \quad . \quad . \quad . \quad 2926,\ 2929,\ 2937,\ 2926,\ 2934,\ 2932.$$

Mean : 2931 ($\lambda 5461$).

Each of these values expresses the refractive index reduced to N.T.P. conditions by the formula

$$\mu-1=\frac{n\lambda}{l}\cdot\frac{760}{p}\cdot\frac{273+t}{273},$$

μ being the refractive index, n the number of interference bands counted, λ the wavelength of the light, l the length of the refraction tubes, p the difference in pressure between the tubes and t°C. the temperature of the experiment. About 350 bands were counted, reading to one-tenth of a band.

The dispersion was measured in relation to the value of the refractive index for $\lambda 5461$ by the method of C. and M. Cuthbertson (loc. cit., p. 152). (My thanks are due to Mr. Cuthbertson for loaning me his original compensator.)

The experimental results in the second column of Table I lead to the Sellmeier expression :—

$$\mu-1=\frac{4{\cdot}8286\times 10^{27}}{16774\times 10^{27}-\nu^2}$$

(ν being the frequency of the light), from which the values of the third column have been calculated by the method of least squares.

TABLE I.—*Dispersion of Air.*

λ in Å.U.	$(\mu-1)\times 10^7$.		Difference.
	Observed.	Calculated.	
Li 6708	2913, 4	2913, 4	0
Hα 6563	2914, 5	2914, 9	+4
Cd 6438	2916, 1	2916, 4	+3
Hg 5790	2925, 0	2925, 4	+4
Hg 5461	2931, 1	2931, 4	+3
Ag 5209	2936, 8	2936, 7	−1
Cd 5086	2940, 0	2939, 6	−4
Hβ 4861	2946, 1	2945, 5	−6
Cd 4800	2947, 9	2947, 3	−6

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[*] Ann. d. Phys., (4), LXI., p. 533 (1920).

[†] Ann. d. Phys., (4), LXIX., p. 81 (1922).

Previous Work.

This has been so thoroughly reviewed by C. and M. Cuthbertson and Meggers and Peters that it only remains to state the following most recent values of the refractive index for λ5461 and N.T.P. conditions for comparison with the above :—

1·0002925 (Meggers and Peters, 1919).
1·0002933 (Traub, 1920).
1·0002927 (Stoll, 1922).

C. and M. Cuthbertson's values of the constants in the Sellmeier formula (single term) are : $C=4{\cdot}6463\times10^{27}$ and $\nu_0^2=16125\times10^{27}$.

(2) OXYGEN.

Experimental.

The oxygen for the present work was prepared by heating pure recrystallised potassium permanganate. It was then washed by passage through a strong solution of caustic potash to remove traces of carbon dioxide, and dried over calcium chloride and phosphorus pentoxide.

Six experiments to determine the refractive index for λ5461 (Hg) and N.T.P. conditions yielded the following results :—

$(\mu-1)\times10^7$ . . . 2708, 2720, 2706, 2709, 2716, 2705.
Mean 2711 (λ5461).

The results of the dispersion measurements are shown in Table II, together with the values calculated from the Sellmeier expression :—

$$\mu-1=\frac{3{\cdot}4343\times10^{27}}{12970\times10^{27}-\nu^2}.$$

TABLE II.—*Dispersion of Oxygen.*

λ in Å.U.	$(\mu-1)\times10^7$		Difference.
	Observed.	Calculated.	
Li 6708	2688, 9	2689, 4	+5
Cd 6438	2693, 0	2693, 0	0
Hg 5770	2704, 8	2704, 2	−6
Hg 5461	2711, 2	2711, 0	−2
Ag 5209	2717, 4	2717, 4	0
Cd 5086	2720, 6	2720, 9	+3
Cd 4800	2729, 6	2730, 1	+5

Previous work.

The following values of the refractivity for λ 5461 serve for comparisons between the results of previous workers and the present :—

	$(\mu-1)\times10^7$
Rentschler (1908)	2725
Ahrberg (1909)	2706
C. and M. Cuthbertson (1909)	2717
Koch (1909)	2704
Howell (1915)	2728
Stoll (1922)	2718

[N.B.—The values of Howell and Stoll have been read from their dispersion curves.]

(3) CHLOROFORM.

Previous work.

Mascart,[*] working at a temperature "about 12°C.," found the refractivity of gaseous chloroform, relative to that of air, to be 4·98 for sodium light. Lorenz[†] found the refractive indices to be 1·001442 and 1·001435 for $\lambda 5893$ and $\lambda 6708$ respectively. These are the only recorded determinations.

Experimental.

The experimental details are exactly the same as those in the work on gaseous carbon disulphide already described.[‡] The greatest difference in pressure between the two refraction tubes was 20 cms. of mercury and about 250 interference bands were counted, reading to one-tenth of a band.

Pure liquid chloroform, supplied by Messrs. British Drug Houses, Ltd., was used in the experiments.

(*a*) *Refractive Index of Gaseous Chloroform.*—The refractive index of gaseous chloroform for the green mercury line ($\lambda 5461$) was determined in 10 experiments with the following results :—

$(\mu-1)\times 10^6$ . . 1409, 1418, 1408, 1411, 1419, 1410, 1408, 1409, 1416, 1410.

Mean : 1412 ($\lambda 5461$).

Each of these results was calculated from the expression

$$\mu-1=\frac{n\lambda}{l}\cdot\left(\frac{\text{Standard density}}{\text{Density observed in experiment}}\right).$$

Assuming C=12·000, H=1·008, Cl=35·457, and the density of hydrogen =0·08985 grams per litre, the standard density of gaseous chloroform is 5·321 grams per litre.

The mean experimental density obtained was 5·445 grams per litre. Dumas and Regnault found the density of gaseous chloroform, relative to that of air, to be 4·199 and 4·230 respectively. When reduced, these values become 5·429 and 5·469 grams per litre.

(*b*) *Dispersion of Gaseous Chloroform.*—The results of the dispersion measurements, together with the values of $(\mu-1)\times 10^6$ calculated from the expression

$$\mu-1=\frac{14\cdot 622\times 10^{27}}{10659\times 10^{27}-\nu^2},$$

are shown in Table III.

TABLE III.—*Dispersion of Gaseous Chloroform.*

λ in Å.U.	$(\mu-1)\times 10^6$		Difference.
	Observed.	Calculated.	
Li 6708	1397, 8	1398, 0	+2
Cd 6438	1400, 2	1400, 3	+1
Li 6104	1403, 7	1403, 6	−1
Hg 5770	1407, 8	1407, 5	−3
Hg 5461	1412, 0	1411, 8	−2
Cd 5086	1418, 1	1418, 1	0
Cd 4800	1423, 5	1424, 0	+5

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[*] Comptes Rendus, LXXXVI, p. 1182 (1878).

[†] Wied. Ann., XI, p. 70 (1880).

[‡] Proc. Phys. Soc., Vol. 38, p. 470 (1926).

(*c*) *The Refractive Index of Gaseous Chloroform for Sodium Light* ($\lambda 5893$) ***under*** *N.T.P. Conditions.*—This value may readily be obtained from the above data, thus the Sellmeier formula gives the value of $(\mu-1)$ for $\lambda 5893$ to be 0·001406. On altering this in the ratio,

$$\frac{\text{Experimental density at N.T.P.}}{\text{Standard density}}=\frac{5\cdot445}{5\cdot321},$$

the refractive index becomes 1·001439 which is the required result.

The corresponding values of Mascart and Lorenz are 1·001455 and 1·001442 respectively.

II.

(1) Acetone.

Previous Work.

Mascart* found the refractivity of gaseous acetone, relative to that of air, to be 3·74 for $\lambda 5893$, that is, assuming the refractive index of air for $\lambda 5893$ to be 1·0002923, his determination of the refractive index of acetone becomes 1·001093. Prytz† obtained the values 1·001085 and 1·001079 for $\lambda 5893$ and $\lambda 6708$ respectively. No other determinations are recorded.

Experimental.

Six experiments were performed in order to determine the refractive index in relation to the density for the green mercury line ($\lambda 5461$) with the following results :—

$(\mu-1)\times 10^6$ . . 1099, 1092, 1101, 1099, 1094, 1092.

Mean : 1096 ($\lambda 5461$).

These results were calculated as in the case of chloroform above. About 300 interference bands were counted in each experiment. The standard density of gaseous acetone is 2·587 grams per litre. The mean experimental density obtained was 2·594 grams per litre.

(2) Methyl Ether.

Previous Work.

The only recorded determination of the refractivity of this gas is that of Mascart (loc. cit.), who obtained the value 3·03 relative to that of air, leading to 1·000885 as the refractive index for λ 5893.

Experimental.

The methyl ether employed was prepared and purified by the method of Erlenmeyer and Kreichbaumer.‡ The refractive index was found in relation to the density from six experiments (about 400 bands being counted in each experiment), thus :—

$(\mu-1)\times 10^7$ . . . 8870, 8879, 8882, 8871, 8875, 8878.

Mean : 8876 ($\lambda 5461$).

*** Loc. cit.**

† Wied. Ann., XI, p. 104 (1880).

‡ Berichte, VII., p. 699 (1874).

The theoretical density of methyl ether is 2·052 grams per litre. The mean experimental density is 2·099 grams per litre.

(3) ETHYL ETHER.

Previous Work.

The following values are recorded by Dufet, Recueil des Données Numériques, Vol. I (1900). No other determinations appear to have been made.

λ	$(\mu-1)\times10^6$ at N.T.P.	Observer.
White.	1554	Arago.*
White.	1523	Dulong.†
D	1534	Mascart.‡
Li.	1517	Lorenz.§
D	1524	,,

Experimental.

The results of six experiments to determine the refractive index in relation to the density yielded the following results :—

$(\mu-1)\times10^6$ . . . 1506, 1513, 1505, 1507, 1515, 1506.

Mean : 1509 (λ5461).

The theoretical density of ethyl ether is 3·298 grams per litre. The mean density found in the present experiments is 3·342 grams per litre.

The author's thanks are again due to Prof. W. L. Bragg, F.R.S., and Mr. C. Cuthbertson, F.R.S., for the loan of apparatus and much advice.

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* Mém. Sci., II., p. 711 (1859).

† Ann. de Ch. et Phys. (2 Sér.), XXXI., p. 154 (1826).

‡ Loc. cit.

§ Loc. cit.

V.—THEORY OF THE ELASTIC PIANOFORTE HAMMER.

By Panchanon Das, *M.Sc.*

(*Communicated by Prof.* C. V. Raman, *F.R.S.*)

Abstract.

In previous Papers exact expressions have been obtained for the vibrations set up in a string by the impact of an inelastic and also of an elastic hammer. By making various approximations, the influence of which is examined in the Paper, formulæ are obtained from which it is easy to deduce the practical effect of the elasticity and velocity of the hammer, and of the position of its point of impact on the string.

Introduction.

SINCE Professor Bryan[*] first brought the subject before the Physical Society, numerous Papers have appeared on the theory of the pianoforte. Professor Raman[†] and Dr. Banerji investigated the case of a string excited by an inelastic hammer by considering the motion during the impact as equivalent to that of a loaded string. The present writer[‡] has discussed the subject in a series of theoretical Papers, giving an exact mathematical solution of the problem, first, of an inelastic hammer, and, later, of an elastic hammer, whose compression is proportional to pressure. On the experimental side, Datta[§] has investigated the harmonics of a struck string, while George,[‖] and, later, George and Beckett,[¶] have published a series of Papers on the dynamics of the hammer and the energy of the struck string. Papers have also appeared by Ghosh,[**] and Bhargava and Ghosh.[††]

It is proposed in this Paper to discuss a topic which has not yet been fully cleared up, namely, the purpose served by the elasticity of the hammer. In order to bring out the contrast between the rigid and the elastic hammers, it will be found convenient to consider a theory of both in simplified forms.

I. Simplified Theory of Inelastic Hammer.

Kaufmann,[‡‡] as is well known, assumed as an approximation that the shorter section of the string remains straight during the impact of the hammer, and in terms of the following symbols :—

a=shorter section of string
l=whole length of string
ρ=line-density of string
T=tension of string
c=wave-velocity of string
m=mass of hammer
V=initial velocity of hammer
P=pressure of impact of hammer

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[*] G. H. Bryan, Proc. Phys. Soc., Vol. 25 (1912-13).

[†] C. V. Raman and B. Banerji, Proc. Roy. Soc., Vol. 97, p. 108 (1920).

[‡] P. Das, Proc. Ind. Assoc. Cult. Sc., Vol. 7, Pt. 1 ; Vol. 9, Pt. 4 ; Vol. 10, Pt. 1. These will be referred to as No. A, B, C.

[§] Proc. Ind. Assoc. Cult. Sc., Vol. 8, p. 107.

[‖] Proc. Roy. Soc., 108, p 284 (1925). Phil. Mag. 49, p. 97, and 50, p. 491 (1925).

[¶] Proc. Roy. Soc., 114, p. 111 (1927).

[**] Phil. Mag., p. 875 (1926).

[††] Phil. Mag., 49, p. 121, and 47, p. 1141.

[‡‡] Ann. der Phys., Bd. 54 (1895).

obtained an expression for the pressure of the hammer in the following form :—

$$P=\frac{V}{c\eta}\cdot e^{\xi ct}\sin(\eta ct+\theta),\ \text{where } \xi=-\frac{\rho}{2m},$$

$$\eta=\sqrt{\frac{\rho}{ma}-\frac{\rho^2}{4m^2}}$$

$$\text{and } \theta=2\tan^{-1}\left(\frac{\eta}{\xi}\right).$$

In reality, however, this shorter section of the string is also set in vibration during the impact. This gives rise to an additional periodic term in the expression for pressure, the form of which is readily derived from the nature of the vibrations set up. Graphically, it is a periodic zig-zag, which can be expressed by a Fourier series. Thus the complete expression for the pressure of the hammer is

$$P=\frac{V}{c\eta}e^{\xi ct}\sin(\eta ct+\theta)+\frac{\rho Vc}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin\frac{n\pi ct}{a} \tag{1}$$

The curve of P against ct is drawn in Fig. 1, the dotted curve being derived from (1) and the continuous one from Das's exact formula.* It will be noted that formula (1) approximates very closely to the results of exact solution.

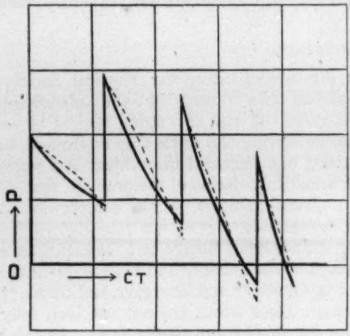

FIG.I

II. Elastic Hammer.

In this case, the natural vibrations of the section a of the string will later be shown to be of small amplitude. We may therefore at first follow Kaufmann and assume it to remain straight during the impact. Later, we shall also take into account the vibrations of the shorter section a.

Using the following notation :—

y_0=displacement of string at $x=0$
y_1= ,, ,, ,, for $x<0$
y_2= ,, ,, ,, ,, $x>0$
ζ= ,, ,, hammer
u=compression ,, ,,
E=elastic constant of hammer

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* Paper No. 1 (l.c.).

we are led to put down :—

$$\begin{cases} y_1=f(x+ct) \\ y_2=\dfrac{a-x}{a}y_0 \\ y_0=f(ct) \end{cases}$$

The equations of motion are easily seen to be :—

$$m_0\frac{d^2\zeta}{dt^2}=T\left(-\frac{\partial y_1}{\partial x}+\frac{\partial y_2}{\partial x}\right)_{x=0}=-Eu \quad \dots\dots \quad (2)$$

the term Eu being equal to the pressure P and m_0 being $m+\rho a/3$ (Kaufmann's correction).

Denoting by accents the differentiations with respect to time, we get

$$u=\frac{T}{E}\left(\frac{1}{a}y_0+y_0'\right) \quad \dots\dots \quad (3)$$

Again, since the displacement ζ of the hammer equals the sum of the displacement y_0 of the string and the compression u, we have

$$\zeta=y_0+u \quad \dots\dots \quad (4)$$

Thus, we have from (3) and (4) :—

$$m_0(y_0''+u'')=-Eu \quad \dots\dots \quad (5)$$

Substituting (3) in (5), we get :—*

$$\frac{m_0T}{Ec}y_0'''+\left(1+\frac{T}{aE}\right)y_0''+\frac{T}{c}y_0'+\frac{T}{a}y_0=0 \quad \dots\dots \quad (6)$$

The complete integral of (6) is

$$y_0=Ae^{qct}+Be^{\xi ct}\sin(\eta ct+\theta) \quad \dots\dots \quad (7)$$

where A, B, θ are constants of integration and q, $\xi \pm i\eta$ are the roots of

$$x^3+\left(\frac{E}{T}+\frac{1}{a}+\frac{\rho a}{3m_0}\right)x^2+\frac{\rho}{m_0}\cdot\frac{E}{T}\left(x+\frac{1}{a}\right)=0 \quad \dots\dots \quad (8)$$

From the initial conditions, $y_0=0$, $y_0'=0$, $\zeta'=V$, we get :—

$$\left.\begin{aligned} A&=\frac{V}{c}\cdot\frac{E}{T}\cdot\frac{1}{\eta^2+(q-\xi)^2} \\ B&=\frac{V}{c}\cdot\frac{E}{T\eta}\cdot\frac{1}{\sqrt{\eta^2+(q-\xi)^2}} \\ \theta&=\tan^{-1}\frac{\eta}{q-\xi} \end{aligned}\right\} \quad \dots\dots \quad (9)$$

*** Bhargava and Ghosh (l.c.) derived a formula in which they omitted the term y_0''' occurring in (6). Thus their value of pressure of impact at $t=0$ is finite, which cannot be true, showing that their formula is invalid.**

We note that the dimension of the roots of (8) is L^{-1}, and both ρ/m_0 and E/T also have the same dimension, and therefore these are comparable. If ρ/m_0 is small compared with the largest root, a further simplification arises. The equation (8) has, then, the following approximate roots :—

$$\left.\begin{array}{ll} & q=-\left(\frac{E}{T}+\frac{1}{a}\right) \text{ and } \xi\pm i\eta \\ \text{where} & \xi=-\frac{aE}{aE+T}\cdot\frac{\rho}{2m_0} \\ \text{and} & \eta=\sqrt{-\frac{2\xi}{a}-\xi^2} \end{array}\right\} \quad \ldots \ldots \quad (10)$$

The expression for the pressure of impact is then given by

$$P=\frac{V}{c}\cdot\frac{E^2}{Eq^2}\left(e^{\xi ct}\cos\eta ct-e^{qct}\right)-\frac{V}{c}\cdot\frac{E}{qa\eta}\cdot e^{\xi ct}\sin\eta ct \quad \ldots \quad (11)$$

which may also be written in the form

$$P=A_1e^{qct}+B_1e^{\xi ct}sin(\eta ct+\theta_1) \quad \ldots \ldots \quad (12)$$

III. Comparison with Exact Theory.

Taking the particular case of a string and hammer system for which $\rho/m_0=0{\cdot}01$ and $E/T=0{\cdot}4$, the curve (continuous one) of P against ct has been drawn in Fig. 2 (A, B), with the help of Das's exact formula* for $a=15$ cm. and $a=25$ cm. respectively.

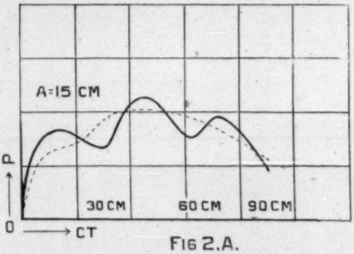

FIG 2.A.

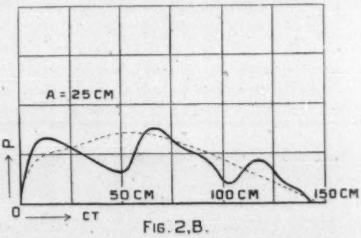

FIG. 2,B.

The dotted curves in the same figures are derived from the approximate formula (11).

It will be observed that the general trend of the exact curve is well indicated by the approximate curve, except that the former is characterised by a superimposed wave-curve due to the vibrations of a.

On comparing Fig. 1 with Fig. 2, we find that as the rigid hammer is replaced by an elastic one, the discontinuous periodic rises in pressure lose their sharp angularities and give place to well-rounded humps. In Fig. 3 the pressure-time curve *for*

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* Paper No. C, Art. I.

only one vibration of a has been drawn for different values of E/T, other things remaining the same. It will be seen that as the value of E is diminished these humps become less and less prominent.

IV. Harmonics of Struck String.

The s^{th} normal co-ordinate φ_s in the vibration of the string resulting from the impact is given by

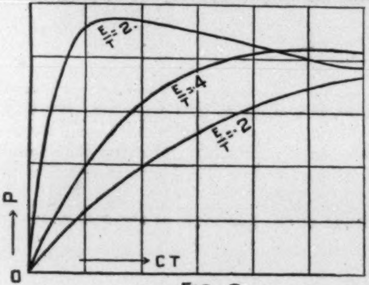

FIG. 3.

$$\varphi_s=\frac{2}{n\rho l}\int_o^t \sin n(t-t')\,\Phi_s dt',$$

where $n-s\pi c/l$, and Φ_s is the generalised component of force, here given by

$$\Phi_s=P\sin\frac{s\pi a}{l}$$

P being the expression (12). Now, the duration of impact is approximately given by

$$t_1=\frac{1}{\eta c}\tan^{-1}\frac{E}{T}\cdot\frac{a\eta}{q}\quad\ldots\ldots\ldots\ldots\quad(13)$$

Hence the time of action of the force Φ_s is given by (13). In order to simplify the integration involved in φ_s we take advantage of the fact that ξct is very small and qct_1 is a large negative quantity. So we at once put $e^{\xi ct}=1$, and after integration put $e^{qct_1}=0$, and get

$$\varphi_s=\frac{2}{n\rho l}\cdot\frac{V}{c}\sin\frac{s\pi a}{l}\cdot A_1\left[-\frac{qc\sin nt+n\cos nt}{q^2c^2+n^2}\right]$$

$$+\frac{2}{n\rho l}\cdot\frac{V}{c}\sin\frac{s\pi a}{l}\cdot B_1\left[\frac{\eta(\cos nt_1-\cos\theta_1)}{n^2-\eta^2c^2}\sin nt\right.$$

$$\left.-\frac{\eta\sin nt_1+n\sin\theta_1}{n^2-\eta^2c^2}\cos nt\right]\quad\ldots\ldots\ldots\ldots\quad(14)$$

$$=a_s\sin nt+C_s\cos nt,\ \text{say.}$$

Then the amplitude of the harmonic of order S is given by

$$\sqrt{a_s^2+b_s^2}$$

As experimental data on the elastic hammer are rather meagre, we content ourselves with a few general observations only on the formula (14).

It is obvious from the term having $n^2-\eta^2c^2$ as denominator that, if n or $s\pi c/l$ happens to be nearly equal to ηc, the corresponding harmonic will have a large amplitude. Since the value of $\pi/\eta c$ usually lies near half the period of vibration of the string, viz. l/c, it is apparent that the value of s satisfying this condition is unity. Hence it is generally the fundamental that has the largest amplitude.

In order to understand how the elasticity of the hammer modifies the tone-quality, it is also necessary to consider the vibrations of the section a of the string. The pressure-component arising out of these vibrations may be expressed as a Fourier series :—

$$\sum_{1}^{\infty} A_r \sin\frac{\pi rct}{a} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

From what has been remarked before, it is obvious that when we pass on to the limiting case of the rigid hammer by putting $E=\infty$ the series (15) should become identical with the additive series in (1). For finite values of E, the softer the hammer the smaller the amplitudes of these vibrations (*see* Art. III), hence the smaller are the values of A_r.

Now, the contribution to the amplitude of the harmonic of the string, say, of order s, by these terms is seen to be

$$\frac{2}{n\rho l}\sin\frac{s\pi a}{l}\int_0^t \sin n(t-t')\sum_{1}^{\infty} A_r \sin\frac{\pi rct'}{a}\,dt'.$$

If we retain only the most significant terms, it reduces to

$$\frac{1}{n\rho l}\sin\frac{s\pi a}{l}\sin nt\Sigma A_r \,.\, \frac{1}{n-\dfrac{\pi rc}{a}} \,.\, \left\{\cos\left(n-\frac{\pi rc}{a}\right)t_1-1\right\}$$

$$-\frac{1}{n\rho l}\sin\frac{s\pi a}{l}\cos nt\Sigma A_r \,.\, \frac{1}{n-\dfrac{\pi rc}{a}} \,.\, \sin\left(n-\frac{\pi rc}{a}\right)t_1 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

It is obvious from the denominator $n-\pi rc/a$ that if (barring absolute equality) n happens to be nearly equal to $\pi rc/a$, then the contribution to the corresponding harmonic is likely to be appreciable. Thus we find that if a harmonic of the string has nearly the same period as a harmonic of the section a, the former has considerable amplitude. If the elasticity of the hammer is varied under these circumstances, then, from what has been said about the coefficients A_r, it is obvious that the reinforçed harmonics in question will be most pronounced when the hammer is rigid (values of A_r being then maximum) and will be weakened as the hammer is replaced by softer ones. This explains why a hard hammer imparts a piercing quality to the general tone of the vibrating string.

In order to study the effect of altering the length a, let us, first, consider the following special values, $a=l/K$, where $K=s$, $s/2$, $s/3$, $s/4$, etc. One sees from the factor $\sin(s\pi c/l)$ that the expression (16) vanishes for these values of a. But if a is slightly different from these values the expression (18) is fairly large for the values of r, which make n nearly equal to $\pi rc/a$. Thus we see that as a is varied the overtones pass through a series of maxima* separated by small or minimum values.

Lastly, we examine the relation between the initial velocity of the hammer and the intensities of overtones. From the factor V occurring in (14) it is obvious that the *relative* intensities are unaffected by a change in the value of V. But it appears that, if the hammer undergoes a large compression, the corresponding pressure develops much more rapidly than is to be expected from Hooke's law, which is the basis of our theory. This is much the same as if the elasticity coefficient E does not remain constant, but rather increases with compression. Thus, if the hammer has a large initial velocity and hence undergoes a large compression, its effective hardness increases. Then from what has been said about the hard hammer one concludes that if the string is struck very hard the tone loses its softness and acquires a metallic ring.

My best thanks are due to Prof. C. V. Raman for his inspiring suggestions and continual guidance in this present work.

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* *See* Datta (l.c.).

DEMONSTRATION OF HIGH-FREQUENCY FLUCTUATIONS IN THE INTENSITY OF A BEAM OF LIGHT.

By C. O. Browne, Messrs. Adam Hilger, Ltd.

IN an experiment due to Prof. Kerr Grant, a beam of light was made to fluctuate in intensity at radio frequencies in synchronism with the piezo-electric oscillations of a quartz crystal.

The piezo-electric crystal was mounted between crossed nicol prisms, so that when stressed it enabled a pencil of light to pass through the system from a carbon arc. It was maintained in vibration by a valve, and the transmitted light pencil, narrowed by a slit after leaving the crystal, fell upon a polygonal reflecting drum which rotated about a horizontal axis at 132 revolutions per second. The streak of light thus produced on a cylindrical translucent screen surrounding the drum was found to be divided into bands 1·5 mm. apart, and from these data the frequency of the light beam was calculated as 168,800 cycles per sec., or approximately double the frequency (84,500 cycles per second) of the crystal as measured by an absorption frequency meter.

VI.—ON THE MEASUREMENT OF THE ELECTRICAL RESISTANCE OF POROUS MATERIALS.

By A. E. KNOWLER, *M.Sc., A.Inst.P.*, of the Building Research Station.

Received June 17, 1927.

(*Communicated by* A. F. DUFTON.)

ABSTRACT.

A method is described whereby the resistance of a material is measured independently of that at the surfaces in contact with it. The method is specially suited to the measurement of the resistance of ceramic, refractory, and building materials. Results show that the conductivity of a building stone is nearly proportional to its moisture content.

MANY measurements have been made of the resistance of ceramic, refractory and building materials. The published values, however, show great discrepancies, and for silica brick, for example, the figures of one authority are thirty times those of another.(1)

These discrepancies appear to be due mainly to lack of appreciation of the resistance at the ends of a specimen when it is inserted in an electrical circuit. Attempts have been made to reduce this resistance to a minimum. At the best, the resistance has been found by the author to be high and variable; a plaster-mercury contact was found to be as much as 50,000 ohms for 1 sq. cm.

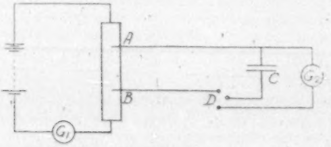


FIG. 1.

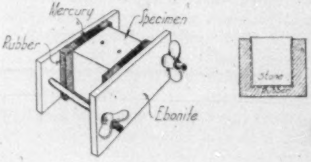


FIG. 2.

The potentiometer method, whereby the resistance of a good conductor is measured independently of the resistances at its ends, is not sufficiently sensitive for materials of high resistivity. The method now described has been devised for the measurement of the resistance of building materials.

A measured current is passed through the specimen in which are inserted two wires, *A* and *B*, which assume the potential of the material at points a measurable distance apart (Fig. 1). The potential difference between the points is measured by charging the condenser *C* and discharging it through the ballistic galvanometer G_2. With materials of high resistance the condenser takes a long time to become charged, so it is preferable to measure the potential difference with a quadrant electrometer.

Direct current can be used for dry materials; but when experiments were made with moist materials complications arose, which were traced to electroendosmosis (moisture movement caused by the current) and to electromotive force due to polarisation. These difficulties were avoided by the use of alternating current. This was taken from the mains at 200 volts, 50 cycles. It was found convenient to compare the potential drop in the material with that across a known resistance, the measurements being made with quadrant electrometers used idiostatically.

There is a possible source of error in the use of an electrometer for measurement of the A.C. potential. The quadrants, though small, have some capacity, and this has to charge up through the resistance between the material and the wires embedded in it. Increase in this resistance decreases the deflection. It was found, however, that the error involved was negligible, being only 0·5 per cent. when the contact resistance was as large as 1 megohm.

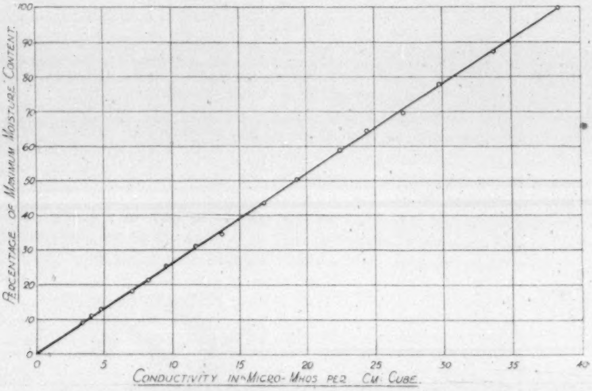


Fig. 3.—Corsham Stone Conductivity Concentration Curve.

The variation of the conductivity of a stone with its moisture content has been investigated. The specimens, which were 1″ × 1″ × 1½″, were held in the clamp shown in Fig. 2, the wires *A* and *B* being inserted in holes ½″ deep and 1/16″ diameter, which were filled with mercury. The moisture content was determined by weighing; the maximum moisture content is taken as that attained after the specimen has been soaking for three weeks. For the purposes of this Paper it is thought sufficient to show the results obtained with only one material, a limestone from Corsham Down. In Fig. 3 the conductivity of this stone is plotted against its moisture content. It will be seen that the curve is nearly a straight line and passes through the origin. The conductivity appears, therefore, to be almost entirely due to the moisture.

During the course of this work an interesting phenomenon was noticed. When the ends of a wet stone were connected to a galvanometer a current was observed which in some instances would change sign suddenly, but in others would remain approximately constant for week after week. It is suggested that this current is due to the non-uniform concentration of soluble salts in the stone. The electromotive force, which may be termed the "self-E.M.F." of the stone, is not large enough to affect the measurements ; with a specimen of plaster the greatest value observed was only 0·05 volts.

REFERENCES.

(1) Hartman, Sullivan, and Allen, J. Amer. Electrochem. Soc., 38, 279 (1920). Stansfield, MacLeod, and Mahon, J. Amer. Electrochem. Soc., 38, 574 (1920).

DISCUSSION.

Mr. J. H. **COSTE** : The author states, "The conductivity appears, therefore, to be almost entirely due to the moisture." Having regard to the very small conductivity of water, the author will doubtless agree that the actual conductivity is due to the ions dissolved by the water taken up. If, as one presumes, distilled water was used, these ions must be derived mainly from the stone itself, and in the case of limestone would be Ca'' and CO_3'' or HCO_3', unless, of course, other more soluble salts were present.

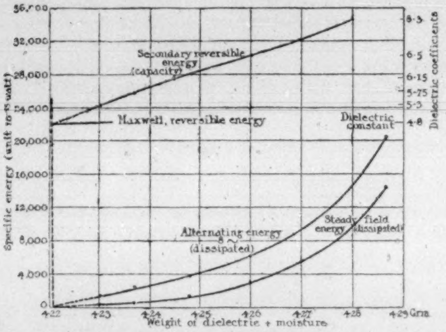
The author does not state the actual water content of the stone from Corsham Down, but limestones may contain from about 1 to 12 per cent. of water, according to their physical character. The specific conductivity of an approximately saturated solution of calcium carbonate in distilled water containing 0·0016 per cent. $CaCO_3$ was about 30×10^{-6} mho. Having regard to the conductivities found by Mr. Knowler and the probable percentage of water, one appears bound to assume that other ions than those mentioned contribute to the conductivity.

That the conductivity of stone and such materials is due to the effect of moisture seems a necessary conclusion, as no obvious means of producing detachable electrons except by ionic dissociation seems applicable.

The author's method may have some application in examining the hygroscopic state of stones in buildings.

Dr. B. A. **KEEN** : Many investigations similar to those described in the Paper have been made on wet plastic materials, such as soil and clays, and also on soils that have been ignited to destroy their colloidal properties. In the latter material a linear relation obtains between conductivity and moisture content over the greater part of the moisture range ; but when the moisture falls to 7 per cent. the curve bends sharply towards the conductivity axis. It is still linear, but cuts the moisture axis at 3·5 per cent. These results are quite reproducible. The interest of these discontinuities lies in the light they throw on the distribution of water in a granular mass. Calculations for an aggregation of spheres show that with cubical packing a continuous film of water over the spheres just ceases to be possible at a moisture content of 8 per cent., owing to the increasing curvature of the menisci in the annular wedge-shaped regions surrounding the points of contact. Similar calculations for close packing show a corresponding value of 3·5 per cent. The correspondence of these values with experimental determinations indicates the rupture of the film of water over the grains at 8 per cent., which allows a redistribution of the remaining water under surface tension. This process goes on until the last films break, and the remaining water is left in a series of isolated wedges throughout the mass. The author might try materials of a more granular structure—*e.g.*, oolitic limestone and sandstones—and see whether the theoretical results, confirmed for soil, also apply in these cases.

Mr. G. L. **ADDENBROOKE** sketched curves illustrating work described by him in *Nature*, April 5, 1924, in which the moisture content of celluloid was plotted as abscissa against (1) a function of the capacity (upper curve); (2) the energy dissipated by A.C. at 8 cycles; and (3) the energy dissipated by D.C., the R.M.S. voltage being 100 in each case, the temperature 13 degrees, the area of the electrodes 125 cm.², and the thickness of dielectric 0·037 cm. The celluloid was dried at 100 degrees and the moisture admitted to it step by step. Extrapolation near the origin suggested that the resistance of perfectly dry celluloid would be infinite. The capacity is independent of the frequency in the dry material, but decreases with increasing frequency in moist material.



Dr. F. E. Smith disagreed with the author's statement that the potentiometer method is less sensitive than that described in the Paper. He also thought that the potentiometer method is easier to carry out.

Dr. E. H. Rayner asked Dr. Keen why he had chosen to make an artificial material by igniting soil. How did the resistance of natural clay compare with that obtained by his theory and experiments?

Dr. B. A. Keen: In answer to Dr. Rayner, natural soils show very different types of curve from those shown by ignited soil. The measurements can in fact be used to throw light on the cause of this difference, which is associated with the presence of colloidal material in a kind of skin over the soil particles. This colloidal material is removed or rendered inert by ignition.

The Author: (In reply to Mr. J. H. Coste): The Corsham stone used was saturated by 11 per cent. of its weight by water. It was hoped to use these resistance measurements to determine the distribution of water in building stones, but it was found the method was not practicable. (In reply to Dr. B. A. Keen): Measurements have been made with a sandstone, Robinhood; and a limestone, Monks Park. Though the curves pass through the origin, they do not consist of single straight lines, but of two straight lines at an angle, and meeting at about 3 per cent. of moisture (60 per cent. of total moisture content). It is supposed that the first 3 per cent. goes to fill the fine pores in the cementing material, the remainder going to fill the interstices between the grains. The kink in the curve corresponds to the "fibre saturation point" of wood. Even when the moisture content is very low there should be a continuous water film due to adsorption, and this should be conducting. (In reply to F. E. Smith): The potentiometer method has been used; but, whereas the sensitivity of the method here described is the same whatever the resistance, the sensitivity of the potentiometer decreases with increase in resistance. It can be used in these experiments only for stones that are nearly saturated. If a stone is nearly dry the contact can be shifted over a wide range, the galvanometer giving no deflection. The galvanometer gave one mm. deflection for two micro-amps. With a dry stone there is a huge contact resistance between the mercury and stone, and this greatly decreases the sensitivity.

VII.—REGULARITIES IN THE SPECTRUM OF IONISED NEON.

II.

By P. K. KICHLU, *M.Sc.*, Research Scholar, University of Allahabad.

Received August 17, 1927.

(*Communicated by* Prof. A. O. RANKINE, D.Sc.)

ABSTRACT.

In a previous Paper on the same subject, the author traced a system of quartet terms in the known spectrum of ionised neon. In the present Paper the theory of Hund is applied in tracing doublet terms and intercombinations between doublets and quartets. Almost all the lines between 2,500 and 3,800 have been accounted for, with a few discrepancies.

IN the first Paper,[*] communicated to this Journal some time ago, the spectrum of ionised neon was discussed from the standpoint of modern theories of spectra, and it was shown that, in the list of ionised neon lines given by L. and E. Bloch[†] and Dejardin, a system of quartet terms is strongly developed. This work has been continued, and considerably more evidence has been brought forward to show the existence of further regularities in the spectrum.

In order to get a general idea of the whole spectrum, it is perhaps best to write out the terms that we might expect to find in Ne^+ on the theory of Hund. The arrangement of electrons in an atom of ionised neon is very well represented according to the following scheme due to Prof. M. N. Saha :—

Structure Diagram of Ne^+ (9).

K_1				
2	L_1	L_2		
	2	5		
		M_1	M_2	M_3
			N_1	N_2

The calculated terms are given below :—

Position of valence electrons.	Terms (calculated).	Terms (observed).
$5L_2$	a^2P	
↓		
$4L_2M_1$	$b^4\bar{P}$ b^2D b^2S $b^2\bar{P}$	$b^4\bar{P}$ $b^2\bar{P}$
↓		
$4L_2M_2$	$c^4\bar{D}$ c^4P $c^4\bar{S}$ $c^2\bar{D}$ c^2P $c^2\bar{S}$	$c^4\bar{D}$ c^4P $c^4\bar{S}$ $c^2\bar{D}$ c^2P $c^2\bar{S}$
↓		
$4L_2M_3$	d^4F d^4D $d^4\bar{P}$ $d^2\bar{F}$ d^2D $d^2\bar{P}$	$d^4\bar{F}$ d^4D $d^4\bar{P}$ d^2D $d^2\bar{P}$
$4L_2N_1$	$e^4\bar{P}$ e^2D e^2S $e^2\bar{P}$	

The quartet terms to be expected on this scheme have been derived in the first

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[*] P. K. Kichlu, Proc. Phys. Soc. After this Paper was published, Mr. de Bruin, of Amsterdam, published a partial analysis of the Ne^+ spectrum in the Zs. f. Physik., Vol. 93.

[†] L. et E. Bloch et Dejardin, Jour. de Phys., Vol. 7, p. 129 (1926).

Paper. In this Paper we will discuss the doublet terms, and intercombinations between doublets and quartets.

THE DOUBLET SPECTRUM.

Considerable difficulties were experienced in unravelling the doublet nature of the spectrum which is predicted on Hund's theory. No starting point like the multiplet utilised in the analysis of the quartet terms was available. The terms that have been obtained agree well with the theoretical considerations, though in certain cases discrepancies with regard to values and strengths have been noted.

As in the case of the quartet system, the deepest doublet terms so far observed originate from the configuration $4L_2M_1$. From this combination we expect $b^2\overline{P}$ and b^2D, b^2S, b^2P, being the deepest, because it arises from ^3P of $4L_2$, while ^2D and ^2S arises from $^1\overline{D}_2$, $^1\overline{S}_0$, others decreasing in the regular order. But only $b^2\overline{P}$

The Multiplets.

—	b^2P_1	612·3	$b^2\overline{P}_2$
$c^2\overline{D}_2$ 511·0	26823·0 (9*R*)		27435·3 (5)
$c^2\overline{D}_2$ 1787·0			26924·2 (10*R*)
$c^2\overline{S}_1$ 1494·0	28098·8 (4)		28711·2 (6)
c^2P_1 127·0	29592·4 (5)		30204·8 (3)
c^2P_2	29465·7 (7)		30077·5 (7)

Multiplets between the Doublet Terms of the Combination $4L_2M_2$ and $4L_2M_3$.

	$c^2\overline{D}_2$	511·0	$c^2\overline{D}_3$	$c^2\overline{S}_1$	c^2P_1	127·0	c^2P_2
d^2D_2 204·4	28951·6 (3)		[29462·5]* 	27675·0 (4)	26181·5 (6)		26308·4 (4)
d^2D_3	28746·8 (3)		29257·3 (2)				26104·0 (7)
d^2P_1	22372·4 (3)				29601·9 (1)		
d^2P_2	31800·4 (2)		32311·4 (4)	30524·2 (2)	29029·8 (2)		29157·1 (5)
B				28922·6 (3)	27428·0 (3)		27554·4 (4)
C				28534·1 (5)	27040·0 (0)		27167·7 (2)

* This line falls very near the strong line 29465·7 (7) and is probably masked by it.

have been observed. It may be, under different conditions, b^2D and b^2S will also appear, but there is no possibility of their being represented so strongly. All the terms of the combination $4L_2M_2$, and some of the combination $4L_2M_3$, have been observed.

It will be seen that of the terms which we expect from the combination $4L_2M_3$ only d^2D have been identified with certainty. The reality of the term designated $^2P_2$ is unmistakable, but the want of a sure companion has made the identification rather obscure. The term here called $^2P_1$ explains only two lines, though more or less exactly. The natures of B and C have not been determined. B appears to be of the nature of $^2S_1$. No such place from the theoretical scheme can, however, be given to it. An S term is certainly to be expected from $4L_2M_3$, but this will probably be very weak, coming from remote combinations.

All the quartet and doublet terms so far observed are inverted—i.e., those with higher quantum numbers have also the higher value. The rule laid down by the Hund that of the terms arising from the same configuration those with the highest azimuthal quantum numbers are the deepest, is not being strictly obeyed. The order in many cases has been upset.

INTERCOMBINATION LINES.

A reference to the author's first Paper will show that there are many levels, designated variously as α, β, γ . . . whose presence has been well established, but whose natures could not be exactly ascertained. As was suspected, some of the lines that these terms give rise to have turned out to be intercombination lines between doublet and quartet systems. The terms called α and β are actually d^2D_2 and d^2D_3 respectively, and combine with the quartet terms of the configuration $4L_2M_3$, and give the multiplets noted below :—

——	d^2D_2	204·4	d^2D_3
c^4P_1	33875·2 (1)		
c^4P_2	34058·2 (0)		33852·8 (0)
c^4P_3	34280·9 (1)		34076·6 (1)
c^4D_1	30634·1 (2)		
$c^4\overline{D}_2$	30777·8 (3*dr*)		30573·7 (2)
c^4D_3	?		30822·8 (2)
c^4D_4			?
$c^4\overline{S}_2$	27519·6 (1)		27315·3 (2)

The intercombination lines are occurring frequently. The $c^2\bar{D}-d^4\bar{F}$ multiplet given below is quite prominent.

—	$c^2\bar{D}_2$	511·0	$c^2\bar{D}_3$
$d^4\bar{F}_2$	29649·3 (3)		30160·7 (1)
$d^4\bar{F}_3$	29503·4 (5)		30014·2 (1)
$d^4\bar{F}_4$			29689·3 (6)
$d^4\bar{F}_5$			

All the lines explained by the doublet terms and intercombinations between doublets and quartets have been collected in Table I.

TABLE I.

λ	I	ν (vac.)	Combination	λ	I	ν (vac.)	Combination
2692·47	0	37129·7	$c^4P_3-d^2\bar{P}_2$	3443·75	2	29029·8	$c^2P_1-d^2\bar{P}_2$
2722·2	0	36724·2	$c^4P_1-d^2\bar{P}_2$	3453·05	3	28951·6	$c^2\bar{D}_2-d^2D_2$
2916·22	1	34280·9	$c^4P_3-d^2D_2$	3456·52	3	28922·6	$c^2\bar{S}_1-B$
2933·71	1	34076·6	$c^4P_3-d^2D_3$	3477·66	3	28746·8	$c^2\bar{D}_2-d^2D_3$
2935·29	0	34058·2	$c^4P_2-d^2D_2$	3481·97	6	28711·2	$b^2\bar{P}_2-c^2\bar{S}_1$
2951·15	1	33875·2	$c^4P_1-d^2D_2$	3503·58	5	28534·1	$c^2\bar{S}_1-C$
2953·10	0	33852·8	$c^4P_2-d^2D_3$	3522·7	0	28379·1	c^4S_2-C
2973·03	1	33625·9	$c^4D_2-d^2P_2$	3537·98	3	28256·7	$b^2\bar{P}_1-c^4\bar{S}_2$
2986·0	0	33479·0	$c^4\bar{D}_1-d^2\bar{P}_2$	3546·06	1	28192·3	$c^2\bar{S}_1-\varepsilon$
3088·16	3	32372·4	$c^2\bar{D}_2-d^2\bar{P}_1$	3554·39	1	28126·2	$c^2\bar{D}_3-d^4D_1$
3093·99	4	32311·4	$c^2D_3-d^2\bar{P}_2$	3557·86	4	28098·8	$c^2P_1-c^2S_1$
3135·79	1	31880·7	$b^4P_3-c^2D_3$	3574·23	0	27970·1	$c^2\bar{S}_1-\gamma$
3143·70	2	31800·4	$c^2\bar{D}_2-d^2\bar{P}_2$	3612·35	4	27675·0	$c^2\bar{S}_1-d^2D_2$
3160·0	0	31636·2	$c^4\bar{D}_2-C$	3628·09	4	27554·9	c^2P_1-B
3187·61	2	31362·4	$b^4\bar{P}_2-c^2\bar{D}_3$	3632·74	1	27519·6	$c^4\bar{S}_2-d^2D_2$
3243·42	2	30822·8	$c^4\bar{D}_3-d^2D_3$	3643·90	5	27435·3	$b^2\bar{P}_2-c^2\bar{D}_2$
3248·16	3*dr*	30777·8	$c^4\bar{D}_2-d^2D_2$	3644·87	3	27428·0	c^2P_1-B
3263·40	2	30634·1	$c_1\bar{D}_1-d^2D_2$	3659·91	2	27315·3	$c^4\bar{S}_2-d^2D_3$
3269·84	3	30573·7	$c^4D_2-d^2D_3$	3679·79	2	27167·7	c^2P_2-C
3275·15	2	30524·2	$c^2\bar{S}_1-d^2P_2$	3697·1	0	27040·0	c^2P_1-C
3309·78	3	30204·8	$b^2\bar{P}_2-c^2P_1$	3701·79	3	27006·3	$c^2P_2-d^4\bar{F}_2$
3314·62	1	30160·7	$c^2\bar{D}_3-d^4\bar{F}_2$	3713·07	10*R*	26924·2	$b^2\bar{P}_2-c^2\bar{D}_3$
3323·79	7	30077·5	$b^2P_2-c^2P_2$	3721·87	2	26860·6	$c^2P_2-d^4\bar{F}_3$
3330·80	1	30014·2	$c^2\bar{D}_3-d^4\bar{F}_3$	3727·09	9*R*	26823·0	$b^2P_1-c^2\bar{D}_2$
3345·49	3	29882·4	$c^2\bar{S}_1-d^4\bar{P}_1$	3744·66	3	26697·1	$c^2P_1-\varepsilon$
3367·25	6	29689·3	$c^2\bar{D}_3-d^4\bar{F}_4$	3753·79	4	26632·2	$c^2P_2-\pi$
3371·80	3	29649·3	$c^2\bar{D}_3-d^4\bar{F}_2$	3799·99	4	26308·4	$c^2P_2-d^2D_2$
3377·19	1	29601·9	$c^2P_1-d^2\bar{P}_1$	3818·41	6	26181·5	$c^2P_1-d^2D_2$
3378·28	5	29592·4	$b^2\bar{P}_1-c^2P_1$	3829·75	7	26104·0	$c^2P_2-d^2D_3$
3388·47	5	29503·4	$c^2\bar{D}_2-d^4\bar{F}_3$	3942·19	3	25359·4	$b^2P_2-c^4D_3$
3392·81	7	29465·7	$b^2\bar{P}_1-c^2P_2$	3999·41	1	24996·6	$b^2P_1-c^4\bar{D}_2$
3414·83	1	29275·7	$c^2\bar{D}_2-\pi$	4522·65	4	22104·7	$b^2\bar{P}_2-c^4P_3$
3416·97	2	29257·3	$c^2\bar{D}_3-d^2D_3$	4565·3	0	21898·0	$b^2\bar{P}_1-c^4P_1$
3428·71	5	29157·1	$c^2P_2-d^2\bar{P}_2$				

CONCLUDING REMARKS.

The present analysis brings out all the essential details of the spectrum of Ne^+, and has been shown to fit in remarkably well with Hund's theory. The available data extend only from 2500 Å to 5000 Å, and almost all the lines in the ultra-violet part of the spectrum below 3800 Å, with few exceptions, have been explained. This part of the spectrum occurs as a distinct group of lines, lying removed from the rest. All attempts to classify lines in the higher wavelength region have proved unfruitful. The present analysis does not seem to afford any clue to their ordering, though it may be remarked that there are many pairs of lines with strong intensities, having intervals of c^4P, c^2P terms. Some of these are shown in the list of lines given in the first Paper. The lines with c^2P intervals are given below :—

c^2P_1	127	c^2P_2
22611·0 (4)		22483·7 (2)
22762·6 (7)		22634·6 (2)
22798·7 (2)		22672·9 (7)

Nothing can be said about their origin, and it is possible that these are purely chance coincidences.

As has already been remarked, the deepest terms so far found come from $4L_2M_1$ combination. The lines coming from the b^2P terms of the $5L_2$ combination obviously lie in the far Schumann region. It is hoped in the near future to examine this part of the spectrum experimentally with a vacuum spectrograph.

The work outlined in this Paper was carried out under the direction of Prof. M. N. Saha, F.R.S., and in concluding the Paper the author records his best thanks to Prof. Saha for his interest and encouragement during the progress of the research.

VIII.—A CALORIMETER FOR THE DETERMINATION OF THE HEAT DEVELOPED BY FRUIT.

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Received October 5, 1927.

ABSTRACT.

A calorimeter for measuring the heat evolved by apples during the senescence stage is described. A differential arrangement is used, one container being filled with apples and the other with dummy apples of the same thermal capacity, made of thin-walled glass spheres filled with moistened glass wool. These containers are enclosed in a metal box thermally insulated, and set up in a constant temperature room. Resistance thermometers composed of 80 yards of nickel wire in glass tubes are disposed among the apples and the dummies. They are connected differentially to a slide wire, on which a difference of one thousandth of a degree causes a movement of the contact maker of 3 millimetres. The air circulation is controlled so as to maintain at will atmospheres containing from 2 to 10 per cent. of CO_2 around the apples. The moisture content of the air before and after circulation is determined by a dew-point apparatus, and a correction applied for the heat absorbed in evaporating water from the apples.

The results show that at a temperature of 20°C. sound apples generate heat at the rate of about 0·012 calorie per second per kilogram of apples. Or for an apple of average size (2½ inches in diameter) the rate of heat generation per second is one and a half thousandth calories.

IT has long been known that fruit generates heat by some process analogous to "respiration," but the quantitative determination of this heat evolution is a problem of considerable experimental difficulty.

A knowledge of the rate of heat production by apples in various stages of ripeness would be of value to both the plant physiologist and the refrigerating engineer. The theoretical interest of the problem is that quantitative data might help to elucidate the nature of the complex reactions taking place in apples which involve the oxidation of sugar and malic acid, and result in the evolution of carbon dioxide and other products in minute quantities.

The practical interest of the problem is that the refrigerating engineer has to contend with the heat generated by quantities of the order of 2,000 tons of apples stowed in the insulated holds of a ship.

PRINCIPLE OF METHOD EMPLOYED.

In devising a method for the measurement of the heat evolution by fruit, the conditions of the experiment must be so arranged that it is possible to maintain any desired composition of the atmosphere around the fruit. Hence provision has to be made for ventilation of a controlled amount. In the experiments to be described, atmospheres containing from 2 to 10 per cent. of carbon dioxide have been maintained at will over periods of several days at each concentration of carbon dioxide.

The method employed was a differential one. Two identical containers were made—one to contain apples, and the other "dummy" apples, consisting of thin walled glass spheres filled with glass wool soaked in water, to give the "dummy" approximately the same heat capacity as an apple of the same size.

Differential resistance thermometers were constructed in the form of two sets of eight tubes projecting into the enclosures. Each of the tubes contained about 10 yards of nickel wire uniformly wound to occupy the full length of the tube. The object of this arrangement was to distribute the resistance thermometer coil over the space occupied by the apples and the dummies. All the coils in one enclosure were connected in series and constituted one thermometer, the corresponding coils in the other enclosure forming the other thermometer. These were connected to a slide wire bridge, on which one centimetre movement along the slide wire corresponded to a change of temperature of 0·003°C. between the thermometers.

The two enclosures were painted black, so as to ensure uniformity in the emissivities of their surfaces.* They were supported on wooden knife edges in a larger rectangular metal box well lagged on all sides with granulated cork. The large box was set up in the centre of a small room, whose temperature was maintained constant by a thermostat, heaters and a circulating fan. Rubber pipes were connected to the enclosure in which the apples were placed, so as to permit of the circulation at a steady rate of fresh air through the fruit.

Since the air which had been in contact with the apples was of a different humidity to the room air, and in the process of evaporation of moisture from the fruit heat is absorbed, arrangements were made to determine the humidities of the atmosphere in the large enclosure, and also of the stream of air drawn from the container in which the apples were situated.

The general arrangement of the apparatus is shown diagrammatically in Figs. 1 and 2, while Fig. 3 shows the thermostat arrangement for maintaining a constant temperature in the room in which the apparatus was situated.

The apparatus in the top left-hand corner, Fig. 1, is the aspirator for maintaining the circulation of air through the enclosure. By regulating the number of drops per second of the water from the constant head tank, a range of air speeds could be secured. The relation between the number of drops per second and the time for the passage of 1/1000 cu. ft. of gas through the apparatus is given in Table I below.

TABLE I.

Time in minutes for $\frac{1}{1000}$ cu. ft. of gas to circulate through apparatus.	Drops of water per minute.
0·5	284
1·0	129
1·5	86
2·0	66
2·5	56
3·0	49

The procedure in an experiment was as follows: The "zero" of the bridge was determined by packing both enclosures with "dummy" apples, and observing the bridge wire setting after the lapse of 24, 48 and 60 hours respectively. If these readings were in close agreement, it was presumed that the interior of the box was at a constant temperature, and that the two thermometers were in equilibrium. The bridge wire reading constituted the datum mark from which all subsequent

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* In some of the preliminary experiments the space between the calorimeters and the container was packed with thin cork shavings; this reduced the thermal transfer to about half its value.

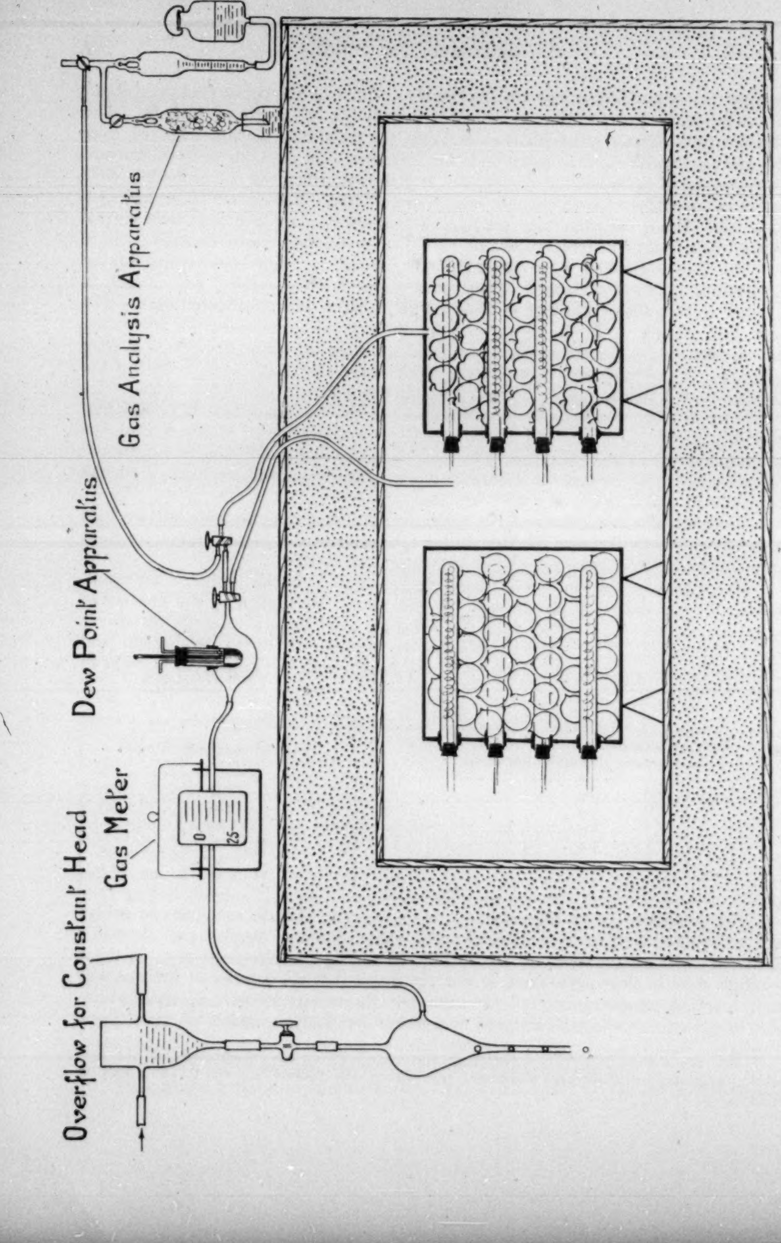
Overflow for Constant Head
Gas Meter
0
25
Dew Point Apparatus
Gas Analysis Apparatus

Fig. 1.

readings were measured. The " zero " was always checked at the completion of a series.

Having obtained the equilibrium point on the bridge wire, the " dummy " apples in the one enclosure were replaced by sound apples. The rate of gas flow through the apparatus was adjusted so as to give the desired percentage of carbon dioxide in the atmosphere around the apples. Usually percentage of carbon dioxide from 2 to 10 were employed. The apples generated heat, and consequently the bridge reading changed in the direction indicating that the thermometer in the enclosure containing apples was at a higher temperature than the one among the dummy apples. The order of magnitude of the temperature rise was 0·15°C. Periodically bridge wire readings, humidities of the atmosphere in the box and in the enclosure containing the apples were taken, and also an analysis made of the gas drawn off from the apparatus.

In order to convert the readings obtained on the bridge wire into rate of heat evolution by the apples in the container the apparatus was calibrated as follows: The apples were replaced by dummy apples, and a heating coil composed of 80 yards of cotton-covered copper wire was interspersed around the " dummy " apples. This coil was connected to a battery with a sensitive ammeter in series, a voltmeter was connected across its terminals so that the watts dissipated in the coil could be determined. For a steady rate of heat evolution the temperature rise was determined in a precisely similar manner to that adopted in an experiment with apples in the enclosure. The relation between energy generated in the coil and bridge reading was a linear one: a temperature rise of 0·1°C. or 0·02 ohms corresponding to an energy input of 0·037 calories per second.

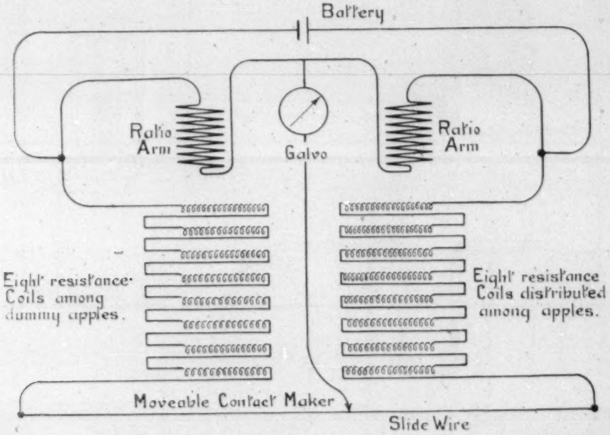
Battery
Ratio Arm
Galvo
Ratio Arm
Eight resistance Coils among dummy apples.
Eight resistance Coils distributed among apples.
Moveable Contact Maker
Slide Wire

FIG. 2.

In order to ascertain whether the distribution of the coil in the enclosure had any influence on the reading, a check experiment was made, in which the heating coil was pressed up into a compact ball the size of an apple, and placed in the centre plane of the enclosure one layer up from the bottom. The rise in bridge reading now produced by a given current through the coil differed from the observed when the coil was distributed by 4 per cent. This is an extreme test to apply to the apparatus, and it is unlikely that an uncertainty of anything like this magnitude

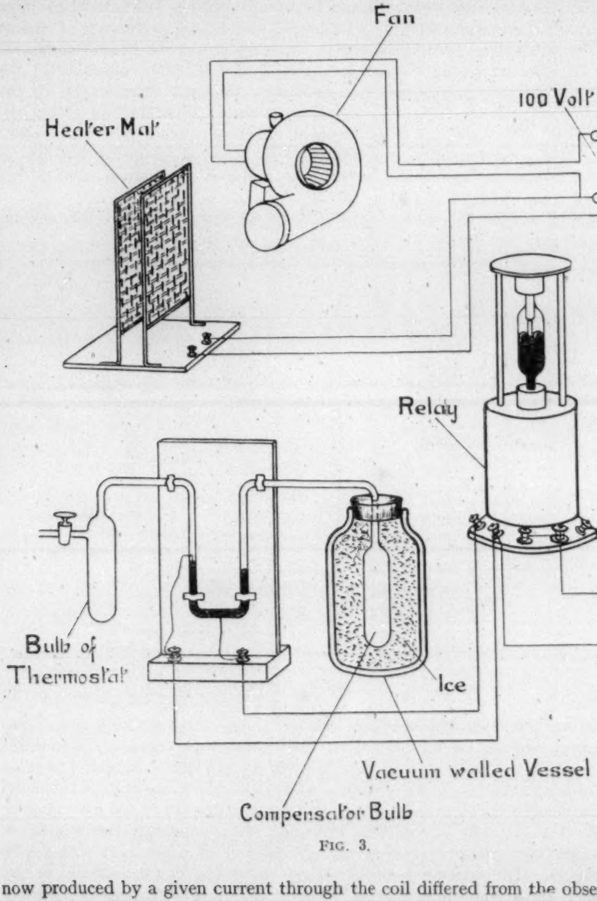
Fan
100 Volt Supply
Heater Mat
Relay
Bulb of Thermostat
Ice
Vacuum walled Vessel
Compensator Bulb

FIG. 3.

exists in the calibration results, since the distributed heating coil generated heat throughout the volume in a manner simulating the apples.

Another check experiment carried out was to ascertain whether the air circulation through the apparatus removed a measurable amount of heat due to the warming up of the air. This was, of course, for the case when no change in moisture content of the air was produced during its passage through the enclosure. That this effect was negligible was proved in two ways :—

(1) When a steady temperature in the enclosure had been obtained for a certain quantity of energy expended in the heating coil, the circulating pump was started and air drawn through at a slightly faster rate than the maximum employed in an actual experiment. No measurable effect on the bridge reading could be observed.

(2) Calculations were made assuming the volume of air circulated per minute, its temperature rise and the specific heat of air ; these calculations showed that the maximum amount of heat carried off by the air stream was only 0·3 per cent. of that normally generated by the apples.

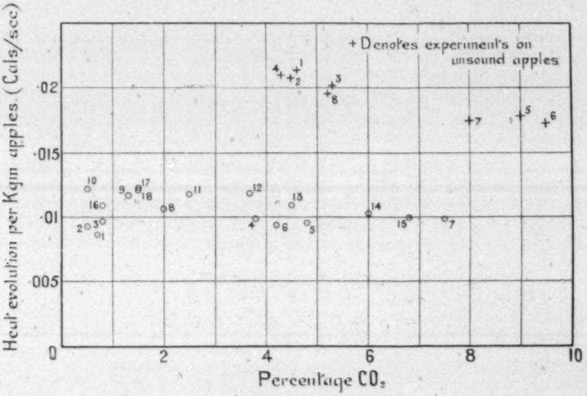

FIG. 4.

The heat absorbed in evaporation of moisture from the apples was of a very different magnitude, and, in extreme cases of rapid circulation, amounted to 8 per cent. of the heat evolution of the apples.

Numerous series of experiments have been made, and a typical set is reproduced in Table II. The results of another two series on different batches of apples is shown in Fig. 4.

In one series, Fig. 4, the apples studied were unsound, most of them containing codling moth. The heat evolution for this consignment was considerably greater than in the case of the others tested.

In studying Fig. 4 it should be remembered that abscissæ give the percentage of carbon dioxide in the atmosphere around the apples, and not the rate at which carbon dioxide is generated.

TABLE II.

Bridge zero, 0·0470. Experiments at 20°C. Weight of apples = 11 lbs. 4½ ozs. = 5·12 kgm.

Date (time).	% CO_2.	Bridge	Pump Drops min.	Time for $\frac{1}{100}$ c. ft. by gas-meter.	Humidity of air from enclosure containing the apples. %	Humidity of air from large enclosure, %	Heat evolution from calibration		Heat abstracted by air current.		Total.	Cals./sec. per kgm.
							Bridge rise.	Cals./sec.	Hum. diff.	Cals./sec. taken from box.		
Jan. 31 (10.30)	2·2	0·0831	350	43 sec.	92·3	61·6	0·0361	0·0652	30·7	0·0025	0·0677	0·0132
Feb. 1 (3.0)	1·5	0·0824	330	35 ,,	94·2	58·2	0·0354	0·0648	36·0	0·0037	0·0685	0·0134
,, 2 (10.0)	2·0	0·0818	320	35 ,,	93·2	67·8	0·0348	0·0636	25·4	0·0026	0·0662	0·0129
,, 3 (10.0)	2·0	0·0812	290	36 ,,	93·7	62·2	0·0342	0·0625	31·5	0·0031	0·0656	0·0128
,, 4 (9.45)	2·3	0·0812	145	61 ,,	94·2	60·4	0·0342	0·0625	33·8	0·0020	0·0645	0·0126
,, 5 (9.30)	2·6	0·0806	145	61 ,,	92·5	63·0	0·0336	0·0614	29·5	0·0017	0·0631	0·0123
,, 7 (9.30)	2·5	0·0796	140	75 ,,	93·0	47·0	0·0326	0·0596	46·0	0·0022	0·0618	0·0121
,, 8 (9.30)	3·5	0·0794	10	900 ,,	89·0	46·0	0·0324	0·0592	43·0	0·0002	0·0594	0·0116
,, 9 (9.30)	5·0	0·0788	26	360 ,,	87·2	47·0	0·0318	0·0580	40·2	0·0004	0·0584	0·0114
,, 10 (10.0)	5·8	0·0782	33	270 ,,	84·6	47·7	0·0312	0·0571	36·9	0·0005	0·0576	0·0112
,, 11 (9.30)	8·5	0·0770	24	460 ,,	83·6	55·0	0·0300	0·0548	28·6	0·0002	0·0550	0·0107
,, 12 (9.30)	8·0	0·0762	48	185 ,,	89·0	50·0	0·0292	0·0533	39·0	0·0007	0·0540	0·0105
,, 14 (9.30)	6·5	0·0744	84	110 ,,	84·0	47·0	0·0274	0·0500	37·0	0·0012	0·0512	0·0100
,, 14 (3.15)	8·0	0·0743	80	140 ,,	80·4	45·4	0·0273	0·0498	35·0	0·0009	0·0507	0·0099
,, 15 (10.30)	4·5	0·0764	320	33 ,,	90·6	46·0	0·0294	0·0537	44·6	0·0048	0·0585	0·0114
,, 16 (10.0)	1·6	0·0790	320	33 ,,	91·0	42·0	0·0320	0·0586	49·0	0·0053	0·0639	0·0125
,, 17 (10.30)	4·0	0·0795	105	87 ,,	89·5	46·7	0·0325	0·0595	42·8	0·0018	0·0613	0·0120

It should be noted that the primary object of this investigation was to develop the apparatus for investigations of heat evolution by fruit, and, consequently, in these preliminary runs involving the use of apples, no attempt has been made to observe the various " controls " customary in physiological experiments.

It may be of interest to compute the gross magnitude of the quantities of heat involved in the case of a ship load of fruit. From the preliminary observations recorded in this Paper, it may be assumed that a cubic foot container of apples at a temperature of 20°C. generates heat at the rate of 0·14 calorie per second.

The refrigerating plant on a ship capable of carrying 100,000 bushel cases (=290,000 cu. ft. net insulated capacity) of apples would be from 100 to 150 B.H.P. The heat absorption capacity or coefficient of performance of such a plant is, according to tests, from 3 to 10, and theoretically could be as high as 50.* So, taking it as 10, it could remove an amount of heat from the hold which is the thermal equivalent of 1,500 horse-power.

From the data obtained in the present investigation, the heat evolution at a temperature of 20°C. from the 100,000 bushel cases of apples would be at a rate equivalent to 132 H.P., so that the heat produced by the process of respiration of the fruit would account for not less than 1/10th the duty on the machine.

Comparing the heat evolution by the apples with that conducted in through the insulated sides of the ship, it may be stated that the ratio of the heat evolution by the apples to the heat conducted in through the sides is as 1 is to 3.

This investigation was carried out on behalf of the Engineering Committee of the Food Investigation Board, and the author desires to record his thanks to Sir William Hardy, Director of the Board, for permission to publish this account. His thanks are also due to Mr. F. W. Morris, Junior Observer in the Physics Department, for his able assistance.

DISCUSSION.

Mr. J. H. **Brinkworth** suggested that the influence of humidity on rise of temperature would be affected in practice by the boxes and paper wrappings in which apples are stored.

Dr. A. H. **Davis** remarked that the author had painted both boxes dead black to ensure uniform emissivity. Did he suggest that the kind of paint made any difference at such temperatures?

Dr. C. **Chree** asked whether any substantial heating effects occur in turnip and beetroot pits.

Dr. E. H. **Rayner** pointed out that if research on fruit storage had not been set on foot, as a direct result of the war, importers would still go on losing occasional cargoes worth £100,000. He noticed that the temperature of the experiments was 20 degrees C. How did that compare with actual transport temperatures ?

The Author, in reply, said that he had only considered the effect of humidity on the estimate of the heat produced : he had not investigated the effect which wrappings would have on temperature in actual transport. He admitted that black paint had no special virtue in the connection described, except that it provided an easily reproducible surface. He did not know much about the conditions in beetroot pits : his aim had been to provide an apparatus which biologists could use for inquiries into matters of that kind. The transport temperature was important, particularly in the tropics : he had chosen 20 degrees as the maximum which was likely to occur in practice.

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* Lower temperature limit 5°C. ; upper 10°C.

IX.—THE SCATTERING OF LIGHT BY INDIVIDUAL PARTICLES IN SMOKES.

By H. P. Walmsley.

Received April 21, 1927.

(*Communicated by* Dr. A. B. Wood.)

Abstract.

Applying the expression given by Maxwell for the number of collisions that occur between two sets of spherical uncharged molecules in a gas to the case of a smoke whose particles cover a wide range of sizes, it appears that particles of a given set collide less frequently with themselves than with those in sets of much larger or much smaller size. If the particles unite on contact, one would expect therefore that the units of the resulting aggregates would differ greatly in size. As this result is at variance with the deductions of Patterson and Whytlaw Gray from experimental data, the validity of their argument is discussed.

I.

IN a recent Paper with the above title,* Patterson and Whytlaw Gray describe experiments from which they find that if the relation between the intensity of the light scattered by a particle and its radius be given by the empirical relation

$$I \propto r^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the experimental values of n for complex particles (aggregates) dispersed in air from mercury, silver and magnesium vary from 4·07 to 3·32. In discussing these results, it is assumed that if the packing of the units composing the aggregates is very loose, each unit will scatter light individually, and if the light scattered by any given unit is not screened by other units, the intensity of the scattered light I from the aggregate will be proportional to x, the number of units in the complex, and thus to the cube of the radius r, i.e.,

$$I \propto x \propto r^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

With close packing and consequent screening, the incident light tends to be reflected from the surface of the complex, in which case the intensity becomes proportional to the surface, i.e.,

$$I \propto r^2 \propto x^{2/3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These are taken to be the two extreme cases, and they lead to values of n of 3 and 2 respectively.

Finding that the average scattering is more than proportional to the number of units in the complex—that is, the average scattering for each unit increases with the size of the aggregate—it is concluded that as the units themselves show a considerable range of sizes, the ones composing the larger aggregates consist of particles of larger average size than those constituting a smaller aggregate, " since the chance of large units combining to form a complex is the same as for small." The increased scattering by the larger units is called upon to explain the effect.

The conclusion seems unwarranted. If the disturbance at the point of observation due to a unit in the complex be represented by $\cos(pt-\epsilon)$, that due to x units loosely packed is $\Sigma_x \cos(pt-\epsilon)$. The intensity is therefore given by

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* Proc. Roy. Soc., A. 113, p. 312 (1926).

$$\begin{aligned} I &= (\Sigma \cos \epsilon)^2 + (\Sigma \sin \epsilon)^2 \\ &= x + 2\Sigma \cos(\epsilon-\epsilon') \qquad (4) \end{aligned}$$

where the summation is taken over the $\frac{1}{2}x(x-1)$ phase differences between the units. This leads to equation (2) only when the distribution of the units makes the second term vanish. If the dimensions of the complex are small compared with the wavelength of the incident light, the second term clearly cannot be zero, for, as the size of the complex diminishes, $\cos(\epsilon-\epsilon')$ tends to the value unity for each of the $\frac{1}{2}x(x-1)$ phase differences, so I tends to the value x^2. Thus the scattering becomes in the limit proportional to r^6, as in Rayleigh's law. As the size of the aggregate increases from this limit, the general effect of the second term of equation (4) will make the average scattering for each unit decrease. Even where the units can perform independent internal random movements—a case where the statistical average value of the second term is a minimum—it would be necessary for the dimensions of the complex to be of the order of the wavelength of the incident light before I becomes proportional to x. Hence, as r increases from zero, the index n of the empirical equation (1) should diminish from the value six to the value three.

With close packing and consequent screening, the light ultimately tends to be scattered from the surface units, and in the limiting case, where the second term of equation (4) vanishes, it becomes proportional to the number of particles on the surface—i.e., to r^2. Thus excluding special distributions of the units, we should in general expect n to range between six and two for these ultramicroscopic particles.

The experimental values for particles from the different dispersoids examined decreased in the order—mercury ($n=4{\cdot}07$), silver ($n=3{\cdot}97$), magnesium oxide ($n=3{\cdot}32$). As the average sizes of the particles examined in each case increased in this order, the results apparently conform with theory. The values for silver ($n=3{\cdot}97$) and mercuric chloride ($n=2{\cdot}10$), where the average range of sizes was about the same, show the influence of the packing on the value of n. As Patterson and Whytlaw Gray point out, this seems confirmed by their density measurements, the silver aggregates having about one-tenth the normal density, whereas the mercuric chloride aggregates have about one-quarter the normal density.

This result, of course, makes it difficult to apportion the relative effects of size and packing on the values of n in the above series. However, as a relation between n and r was actually looked for in each experiment and, "except in one or two doubtful cases, no evidence whatever was obtained that the value of n varied appreciably over the range of size considered within the limits of experimental error," it would appear that the relative effect of packing was the dominant factor influencing the values of n in the series. Two factors may contribute to the negative experimental result. Equations (2) and (3) pre-suppose a uniform distribution of units throughout the aggregate. If, as is highly probable, the aggregates are skeleton crystal growths about a nucleus, the density of the units will diminish radially from the nucleus. As the outer units are the more effective in producing interference, the value of I (equation 4) will diminish less rapidly with the radius than it would if the distribution were more uniform. The radius in this case we will take as equal to that of a circumscribing sphere. Since the experimental values of r are deduced from the Stokes-Cunningham law, they can hardly be expected to agree with the radius of the circumscribing sphere, and it may happen that the values so obtained accentuate the former effect and produce the observed negative result. A radial diminution in the density of the units should cause the mean density of the aggregate to diminish as the size increases. The measured sizes and densities of individual aggregates* seem to show this tendency in the cases of silver and magnesium oxide, but the number of observations quoted is too small to establish the point.

Taking into consideration the sizes of the particles examined (their radii ranged from $130\mu\mu$ to $1600\mu\mu$), the fact that the average scattering for each unit of a complex particle may increase with the number of units (i.e., when $n > 3$) is not, of itself, sufficient to prove that the average size of the units is greater in large than in small complexes. It can be due to vibrations from the individual units arriving at the point of observation in phase.

II.

The distribution of size amongst the units of an aggregate must, to a large extent, be governed by their collision frequencies before aggregation. Maxwell† has given an expression for the number of collisions that occur in unit time between two sets of spherical particles, from which it appears that the number of occasions per unit time in which two spherical units of radius r and r_1 come into contact is $\pi(r+r_1)^2\sqrt{u^2+u_1^2}\,.\,\rho\,\rho_1$, where u and u_1 are their velocities and ρ and ρ_1 the numbers per unit volume. Assuming the densities Δ are equal, and that the kinetic energies of the particles are the same as the mean kinetic energy E of a gas molecule of the surrounding medium, we have

$$\tfrac{1}{2}\times 4/3\times\pi r^3\Delta\times u^2=E$$

Eliminating the velocities of translation, the number of collisions becomes proportional to

$$(1+p)^2\,(1+p^{-3})^{\frac{1}{2}}\,.\,\sqrt{r} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

where $p=r_1/r$. For a sphere of given size this quantity becomes very great for both relatively small and relatively large values of p, and has a minimum value close to $p=0{\cdot}8555$. From this it is evident that, where an extensive size range exists, a given particle is much more likely to collide with a particle of larger or much smaller size than with one of its own size, and, owing to the presence of the factor $\sqrt{r}$, this tendency increases as the size of the particle increases. If the particles unite at contact, one would not therefore expect the units of the aggregates to consist of particles of fairly uniform size, but rather the reverse, and this is in effect the impression one obtains when they are examined under the microscope. The string-like aggregates produced by CdO and ZnO consists of units of various sizes linked together. When seen by oblique illumination, the increased scattering due to the larger units causes the chains of particles to appear like beads on a thread; often as if they were separated from each other by a small space. The spaces occur where the units are of too small size to scatter an appreciable amount of light. The combination of large and considerably smaller particles in many aggregates is also evident in the photomicrograph of a deposit from a magnesium oxide smoke published with the Paper of Patterson and Whytlaw Gray. It has been adapted in the construction of the models of the previous Paper.

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[*] Loc. cit., p. 307.

[†] J. C. Maxwell, Phil. Mag. (4), 19, p. 28 (1860); Collected Papers, Vol. 1, p. 387.

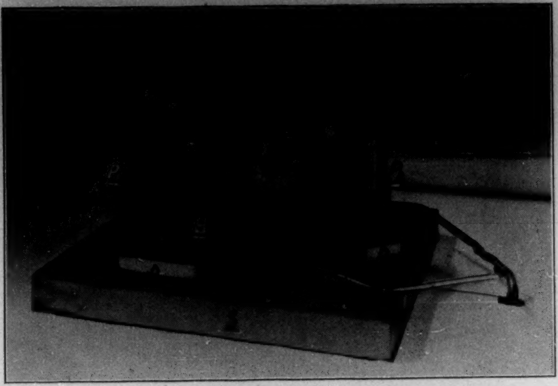

FIG. 1.

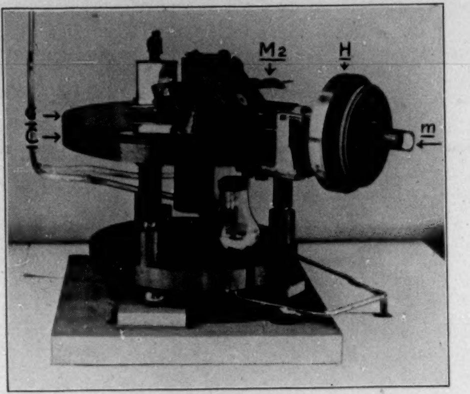

FIG. 2.

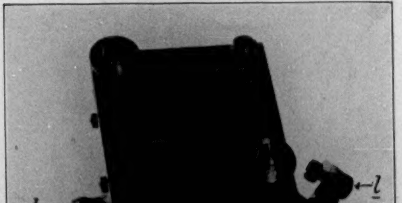

FIG. 3.

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X.—ON THE CONSTRUCTION AND STANDARDIZATION OF AN INTERFEROMETER PRESSURE GAUGE.

By J. J. Manley, *M.A.*, Research Fellow, Magdalen College, Oxford.

Abstract.

This Paper deals with the application of a Michelson interferometer to the determination of gas pressures ranging from 0·0001 to 20 mm. of mercury. The construction of the instrument is fully described as are also the methods used for standardising the registering micrometer screw and its graduated head. The gauge can be instantaneously set and the pressure which obtained at the moment of setting measured at leisure.

The gauge dealt with in this Paper, although applicable for the measurement of small pressures in general, was primarily designed for determining the rate of the electric eruption of any gas-grown skin found upon the interior surfaces of highly evacuated vessels. A chief merit of the instrument lies in the fact that, unlike some gauges, it can be instantly set at any given moment and then read at leisure. Briefly, the gauge is a Michelson interferometer, in which the usual vertical mirrors of silver-upon-glass have been replaced by two horizontal surfaces of mercury. These mercurial surfaces are the summits of two similar barometric columns, one of which is used as a standard of reference for the other. The gauge, which was carried by a large and massive slate bench cemented to concrete pillars, was constructed as follows.

Upon a marble slab *S* (Fig. 1), supported by three lead pellets for ensuring stability, were placed three marble plates, each having a cavity for the reception of a 1″ steel ball. Two of these plates, *p*, *p*, with their balls, are shown in Fig. 1. The plates were so disposed that the balls coincided with the apices of an equilateral triangle, and respectively engaged the hole, slot and plane of the under surface of a heavy brass disc *B*. The legs of a second massive disc *C* (Fig. 2), terminated respectively in a hole, a slot and a plane; and these, with *C in situ*, rested upon corresponding steel balls carried by the pillars P_1, P_2, P_3, shown in Fig. 1. The disc *C* (Fig. 2) was covered with a thick glass plate *G*, upon which were built the *ways* required for two radial slides, each carrying a plane mirror. Each way was formed by cementing to *G* two plane glass plates, the one being vertical and the other set at an angle approximating 45°. All four plates were buttressed. The brass slides running in the ways were each fitted with 2 steel balls *b*, *b* (Fig. 3), and an adjustable support *l*, the arrangement being such that the balls made contact not only with *G*, but also with the vertical plate of the way. The support *l* had contact with the sloping plate only; the contact system was therefore one of five points. Hence, given accurately prepared ways or guides, the slides with their mirrors run true. The plane mirrors carried by the slides were mounted and adjusted after Michelson's plan, but instead of being vertical they were inclined downwards through an angle of 45°. The two beams of light emanating from the half-silvered plate of the interferometer were by these inclined mirrors reflected downwards through radial slots cut in the discs *C* and *G*. Thus, one beam impinged upon the surface of the standard mercury column (*vide infra*), and the other upon that of the column responding to an introduced gas. The slides were controlled by elastic bands, and actuated by micrometer screws having a pitch of 0·5 mm. The screws terminated in fine and extremely hard points, and these were made to bear upon glass plates cemented to the slides. One screw M_1, termed the working micrometer, was fitted with a large head H (Fig. 2), having a scale of 600 divisions which were viewed with a micrometer microscope. Any traverse of the screw could be read to within 1/10th of a division—that is to say, the limit of accuracy attainable in reading was approximately $\frac{0{\cdot}5}{600 \times 10} = 0{\cdot}00008$ mm. The second micrometer screw M_2, termed the compensator, was left unmodified and used as received from the makers, Messrs. Pye & Co., of Cambridge. The smallest division upon its head $= 0{\cdot}005$ mm. The half-silvered parallel-plane glass plate used for dividing the beam of light, and also the compensating plate of the interferometer, were mounted in the usual way, and definitely located upon the base-plate G.* For maintaining uniformity of temperature within the instrument the upper part was enclosed by an aluminium wall and cover, and the lower portion surrounded by asbestos millboard, as seen in Fig. 4. The incident and reflected beams of light were transmitted through glazed windows in the aluminium wall. A brief description may now be given of the twin barometric columns and of their disposition.

In Fig. 1 are shown two flasks f, g,† which rest in leather-lined conical holes made in B. The mouth of each was turned in and given the form of a flat sloping ring with an aperture of 12 mm. The rings were ground true and then closed by parallel-plane glass plates attached with Canada balsam. Each flask had two tubes, the one leading out from the neck and the other from the bottom. One of the two side-tubes was joined to a Sprengel pump and the other to a second pump, and also to any apparatus to be evacuated. The second or vertical tube constituted in each instance the upper portion of a barometric column, terminating in the flask with its wide surface of mercury. These twin barometric tubes were bent and led side by side through a hole in the slate bench, and finally fused, the one to E and the other to F, as shown in Fig. 5. The lower ends of the cisterns E and F were placed in cavities in a block W. By means of the stop-cocks t_1 and t_2 communication between the two barometric columns and a main wooden cistern T could be established or broken at will. An auxiliary cistern N was added, so that the mercury in T might be varied with ease. Any desired level of the adjusted mercury was maintained by closing the tap t_3. Equality of temperature within and about the barometric tubes and their cisterns was secured by completely surrounding them with a case of wood and asbestos. The short exposed portions of the two columns seen close to the plate B in Fig. 1 were finally wrapped in asbestos fibre.

Before using the gauge, the scale upon the head H of the working micrometer, and also that portion of the screw traversing the nut when measuring the major pressure of 20 mm., were calibrated according to the methods detailed below.

(*a*) *Calibration of the Micrometer Head H.*—To aid in determining the errors of the scale upon H, the spindle protruding from the centre was fitted with a small wooden cylinder, which could be rotated and clamped as required. The anterior

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* A plan view of the gauge is given in Fig. 6.

† The diameter of these was 5 cm.

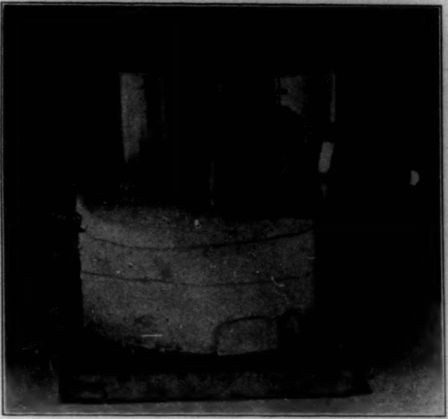

FIG. 4.

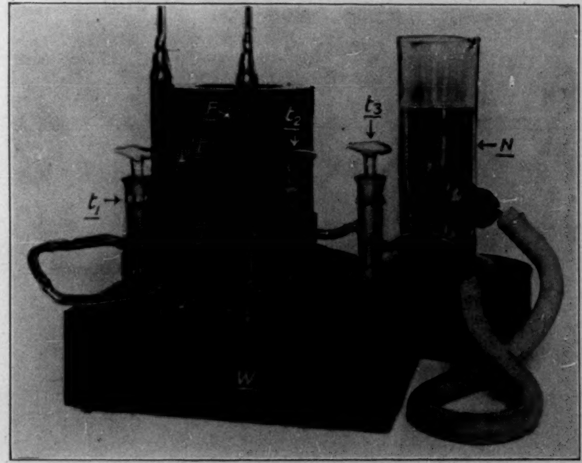

FIG. 5.

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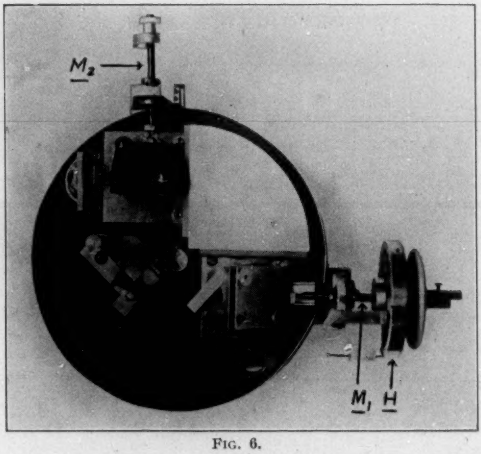

FIG. 6.

To face page 58]

portion of this cylinder was cut along its axis and one half removed; and to the central flat surface thus provided was secured a concave mirror *m* (Fig. 2). Opposite the mirror, and at a distance of 2 m. from it, was placed a vertical mm. scale; also a lamp having a rectilinear filament. These preliminaries ended, the errors of *H* were determined as follows.

First, the zero of the scale was set to coincide with a cross-thread of the reading microscope. Next, *m* was adjusted and clamped so that the image of the filament was near the top of the vertical scale and a first reading r_1 taken. *H* was then turned until the sixth instead of the zero division of its scale coincided with the cross-thread, and a second reading r_2 taken. A measure r_2-r_1 was thus obtained for the first portion of the scale having the indicated value of 0·005 mm.

For the next step, the micrometer was left undisturbed, whilst the mirror was returned to its original position, or approximately so, and a new deflectional reading obtained. Then, following the plan just described, the next 0·005 mm. was calibrated. Proceeding thus for 100 steps data were obtained for standardizing the whole scale. The calibrated steps were found to vary within the limits of ±1·6 per cent.

As illustrating the degree of accuracy here attained, it may be stated that, in a series of determinations for one and the same step giving a mean deflection of 257 mm., the greatest variation was ±1, or ±0·39 per cent. The errors of the micrometer head having been ascertained, it was then possible to standardize the screw itself. This was done as follows.

(*b*) *Calibration of the Screw.*—For calibrating the working screw M_1 the interferometer was first adjusted for the fringes obtained with a sodium flame, and the reading of M_1 made equal to zero. Then the compensator M_2 was manipulated so that one fringe was accurately sighted. Next M_1 was slowly turned until *x* fringes had crossed the fiducial line, and then by means of its micrometer microscope read a second time. Knowing *x*, the *apparent* distance traversed by the screw was therefore $0{\cdot}000589 \,.\, x$ mm. It will, however, be noted that with the advance of the screw the paths of the incident and reflected beams were simultaneously and equally shortened. Hence the *real* traverse was $0{\cdot}000589 \,.\, x/2$ mm. This known, the error in the first region is determinable.*

For the actual calibration, the plan adopted was that in which a portion of the screw corresponding to 0·005 mm. was tested in the neighbourhood of each successive 0·25 mm. division throughout the whole required range of 10 mm.; the number of calibrated regions thus being 41. By taking $x=17$, the standard length was $0{\cdot}000589\, x \times 17/2$ mm., a quantity differing from a compared length of 0·005 mm. by +0·1 per cent. only.

The error of the first region having been determined, M_1 was advanced 0·25 mm. and an accurate reading taken. Then, as already described, the coincidence of a fringe with the cross-threads of the telescope was effected by means of M_2. Lastly, M_1 was again turned until the standard number of fringes had crossed the threads of the telescope, and a further reading taken. From the data thus acquired the error for region 2 of the screw was calculated. Proceeding thus, the standardisation was completed, and a table of corrections prepared for future use. It was found that the mean errors of $M_1 = \pm 3{\cdot}2$ per cent. For the sake of clarity, an example of the method of using the gauge may now be given.

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* **When standardizing M_1, plane silver mirrors may be substituted for the mercurial surfaces of the barometric columns. If this be done, the calibration is not interrupted by passing tremors.**

Determination of the Pressure of a Gas.—First, communication between the two barometric tubes is established by opening an intervening mercury trap not shown in the figures. Both pumps are then brought into use and the system evacuated to the highest attainable degree. This done, the mercury in the cistern T is varied by operating the auxiliary cistern N until the upper and common level of the mercurial columns coincides approximately with the centres of the flasks f, g. The stopcock t_3 is then closed, and subsequently, at one and the same moment, t_1 and t_2 are also closed. Thus, for the time being, the two barometric columns, having one and the same length, are rendered invariable. Next, for the sodium flame used in standardising M_1 is substituted a 40 W. "Fullolite" lamp, and M_1 set at or just within the zero of its scale and its position a_1 read. Then M_2 is adjusted so that the central band of the coloured interference fringes appears upon the fiducial line of the telescope. Finally, the trap between the barometric columns is closed, t_1 and t_2 opened, and the gauge thus made ready for use.*

If now any small quantity of gas be generated within an apparatus attached to the barometric column of M_1, the mercury will be correspondingly depressed and the optical path associated with it lengthened; the fringes will therefore move across the field and probably disappear. The micrometer M_1 is then slowly turned and the central fringe restored to its original position. Thus the optical path is shortened by precisely the same amount as by the pressure of the gas it was lengthened. Let the new micrometer reading be a_2. Then, since the incident and reflected rays are equally shortened, the actual pressure of the gas is $2(a_2 - a_1)$ mm. With a little experience M_1 can be adjusted to within 2/10th of a fringe; hence the pressure values are determinable to within 0·0001 mm.†

If it be necessary to apply a correction for the temperature of the mercurial columns, the required data may be obtained in the usual way by thermometers appropriately placed. Since, however, the *difference* in the height of the intimately associated twin columns is never greater than 20 mm., the correction will in general be small, and in most cases may be altogether neglected.

It will be obvious that a series of sufficiently small gas pressures may be measured either individually or differentially, the latter being the more expeditious.

In conclusion, it must be said that the gauge can be used only in the complete absence of vibration. Any small tremor gives rise to ripples upon the mercurial surfaces, and a consequent distortion of the fringes. But given tranquil conditions, pressure measurements may be carried out quickly and with ease.

DISCUSSION.

The **AUTHOR**, in presenting his Paper, added that he hoped to overcome the difficulty arising from ripples on the surface of the mercury by floating on it thin slips of glass, which, unlike thicker strips, keep to the middle of the mercury surface. He also explained two simple methods of making optically perfect mirrors (Phil. Mag., January, 1928).

Prof. **O. W. RICHARDSON** asked what degree of sensibility could be obtained with the apparatus, and whether the temperature errors were serious.

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* The standard barometric tube working in conjunction with M_2, having been trapped off, is kept highly evacuated by ceaselessly working the pump with which it communicates.

† The two barometric columns, although continuously responding to atmospheric pressure, are *simultaneously and equally* affected. Hence the differential readings are not invalidated.

The Author replied that readings could be taken to 0.1μ, and that temperature errors were compensated by the differential use of two similar mercury columns.

Mr. R. S. Whipple congratulated the author on having worked out a very elegant piece of apparatus. He suggested that instead of glass slips small mirrors made of rustless steel might be floated on the mercury. This plan would be free from difficulties due to bubbles. He had found that results reproducible to 1μ could be obtained by the simple method of measuring displacements of a mercury or oil surface which consists in bringing down a point till it touches the surface. How long did the observations take?

The Author replied that the time taken by the observations was merely that needed to set the micrometer screw.

Mr. T. Smith asked whether the author employed fringes running in any special direction (for instance, vertically). Had he investigated whether one direction was theoretically preferable to the others?

The Author replied that he preferred vertical fringes which move laterally, as a matter of convenience. He thought that errors of the kind which might make one direction preferable to another would be balanced out by the method of calibration. Also, the observed fringes being arcs of equally spaced concentric circles, it is possible to select and use either horizontal or vertical portions. Theoretically, the value of a measurement is the same for both horizontal and vertical fringes.

Dr. D. Owen expressed great interest in the apparatus. Owing to its somewhat elaborate construction it was hardly suitable for universal use, but could not the author compare its readings with those of a McLeod gauge? One of the light pencils in the interferometer has to pass through the film of air adsorbed on the surface of the mercury, while the other does not. Would it not be possible, by a comparison of the kind suggested, to find evidence as to the nature of this film?

The Author replied that if, as stated by Langmuir, the gas skin was only 1 molecule thick, it could hardly be detected. If, as he believed, the skin was about 20 molecules thick, he did not know whether its detection would be possible by the method suggested by Dr. Owen. His gauge measured the actual pressure, whereas in the McLeod gauge the changes of pressure would presumably cause changes in the amount of gas adsorbed on the containing walls and so would give rise to errors.

Mr. D. H. Black asked why the author took the displacement of the mercury to be double the compensating displacement of the micrometer mirror, since the light pencil has to traverse twice the increment of path to which the displacement of the mercury gives rise? He suggested that, instead of using two manometers both open to the air, the author might use one manometer with a closed end containing a vacuum, and replace the other mercury surface by a fixed mirror.

The Author replied that the object of using two manometers differentially was to compensate for variations in temperature and atmospheric pressure. If the level of the comparison surface be fixed by closing the appropriate tap a constant movement of the fringes can be observed.

XI.—THE DIELECTRIC CONSTANT OF LIQUID BROMINE.

By ANNE I. ANDERSON, *B.Sc.*

Received November 22, 1927.

ABSTRACT.

An account is given of a re-determination of the dielectric constant of liquid bromine, the value found being 3·12 at 15°C., and at a frequency of 187,000 per sec., with a temperature coefficient of $-0{\cdot}00191$ referred to 0°C. Applying the dipole theory of Debye and Gans to the results, a value of $0{\cdot}40 \times 10^{-18}$ is deduced for the electric moment of the bromine molecule Br_2. The electrical conductivity was also measured.

INTRODUCTORY.

DETERMINATIONS of the dielectric properties of a limited number of liquids have recently been made by various experimenters with a view to testing Debye's well-known theory, elaborated by Gans and Isnardi to include liquid dielectrics. The theoretical side has been well summarised by Prof. Compton in his address on " Dielectric Constants and Molecular Structure."*

Liquid bromine, being an element, should be helpful in this respect: but it has recently received little attention, doubtless owing to the experimental difficulties of dealing with it.

Previous published determinations of the dielectric constant of liquid bromine are as follows:—

Author.	Date.	ε	t	λ
Schlundt	1901	3·18	23°C.	84 cm.
Schmidt	1903	3·10	...	...
Walden	1910	4·6	1°C.	" ∞ " (i.e., > 10^4cm.)

Drude's " second " resonance method was used. The first mentioned determination alone seems to merit serious consideration.

Schlundt and Eggers estimate the limit of error for this method at as much as 2 or 3 per cent., while none of the above workers record observations either upon the temperature coefficient of the dielectric constant or upon the electrical conductivities of the samples used.

Hence the opportunity of making a fuller and more accurate re-determination of the dielectric constant was taken when a sample of specially purified and dried bromine was kindly placed at my disposal by Prof. H. B. Baker. It is not claimed, however, that the liquid upon which the tests were carried out was of the extraordinary degree of dryness of the sample as first supplied. Exposure to the air for a brief period, in the course of transference to the testing vessel, was inevitable. The bromine was introduced into a glass bulb *A* (Fig. 1), containing specially purified phosphorus pentoxide. After sealing off, the bromine was allowed to stand for three weeks. It was then distilled over into the test condenser *B*, which was immersed in a freezing mixture. A mercury seal was used above the glass stopper of the condenser vessel.

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* **Science, 63, pp. 53-58, January 15 (1926).**

Determinations of the dielectric constant and its temperature coefficient were

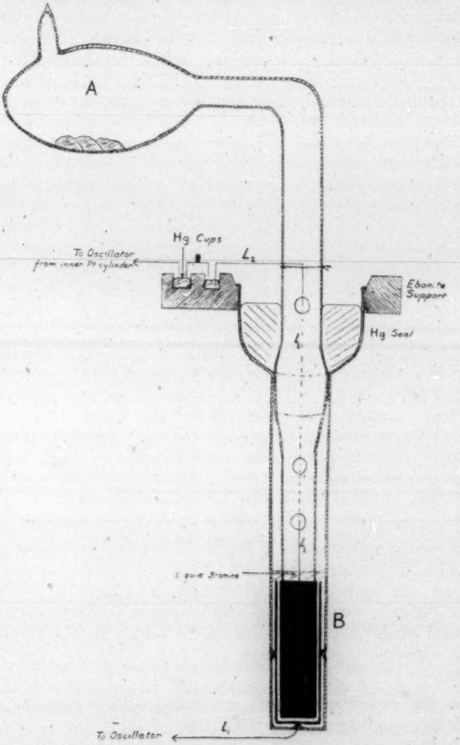
A
Hg Cups
To Oscillator
from inner Pt cylinder
L_2
Ebonite
Support
Hg Seal
l_1
l_2
Liquid Bromine
B
To Oscillator
L_1

FIG. 1.—BROMINE CONDENSER.

then carried out upon this sample; measurements were also made of the electrical conductivity at different temperatures (with due regard to polarisation effects exhibited), and the density and its temperature coefficient were observed. The effect of frequency on the dielectric constant was also examined.

THE DETERMINATION OF THE DIELECTRIC CONSTANT.

The method adopted has been employed by other experimenters, and is shown in Fig. 2. Two generators of H.F. oscillations produced a heterodyne note in a third detector circuit operating a loud-speaker. The first generating circuit consisted of a Sullivan wave-meter, oscillating at a set frequency. The second circuit included two variable air condensers, C_1 and C_2 (0·001 μF. max. and 100 $\mu\mu$F. max.), and the experimental condenser, C_3, containing the bromine; these three condensers being connected in parallel across the grid circuit. The generating circuits were lightly coupled to the detector circuit. Fig. 1 is a drawing of the vessel used as a condenser. The vessel consists of a piece of glass tubing, ground, at a distance of 13·5 cm. from its lower end, to form a hollow stopper to an outer glass tube closed at its lower end. Fitting tightly over the inner glass tube is a platinum sleeve, 5 cm. long, with platinum base. This sleeve is surrounded by a hollow concentric platinum cylinder—also closed at its lower end—which is fixed rigidly to the outer glass tube. These two cylinders, separated by a space about 1 mm. wide, constitute the plates of the condenser C_3 of Fig. 2. Platinum wire electrodes, L_1 and L_2, are led off from each cylinder as indicated. The method of introducing and removing the bromine from the condenser has already been described.

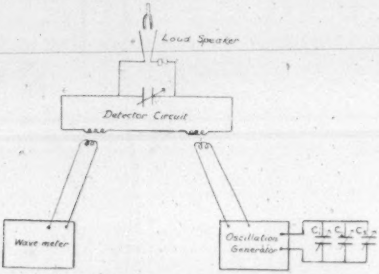

FIG. 2.—DIAGRAM OF ELECTRIC CIRCUITS.

After the 100 $\mu\mu$F. variable condenser had been adjusted so that the note in the loud-speaker made very slow beats with a tuning fork of frequency 768, the corresponding condenser scale readings, *a*, *b*, *c* and *d* for the following conditions, were observed :—

(*a*) When the experimental condenser, filled with bromine at a known temperature, was connected in the circuit;

(*b*) When the experimental condenser was disconnected by means of a small key;

(*c*) When the experimental condenser, with the bromine removed and air as dielectric, was re-connected in the circuit;

(*d*) When the experimental condenser was again disconnected.

When these readings have been corrected by the condenser calibration curve, and allowance has been made for the capacity of the fine wire lead L_2, the dielectric constant is given by

$$\varepsilon=\frac{b'-a'}{d'-c''}$$

where a', b', c', d' are the corrected values of the readings a, b, c, d.

This is a ratio and is independent of the actual value of the units in which the capacities were measured.

To minimise stray capacities, the observer and loud-speaker were situated at a distance of about 2 metres from the apparatus, and the 100$\mu\mu$F. condenser was operated from this point by means of a horizontal dial rotating a belt of fine silk cord. All observations were made on the same side of the wavemeter frequency. Care was taken to ascertain that the drift of frequency of the generators was negligible over the duration of a measurement. The filaments in the two generating circuits were connected in series, and, with a common high-tension battery of about 60 volts, the frequencies of oscillation became steady some 20 minutes after closing the circuits. The heterodyne note, when adjusted to the frequency of the standard fork, then remained constant to within two beats per second for at least a couple of minutes. In taking readings, the rate of beating was adjusted so as to be well within two beats per second. A change of this amount corresponds to an alteration in capacity of at most 1 in 44,000, hence the errors of adjustment of frequency were quite negligibly small.

The limit of accuracy in the measurement is essentially determined by the degree of constancy, at the various points of its scale, of the 100$\mu\mu$F. condenser which was substituted for the test condenser when the latter was cut out of circuit. This condenser could be read to 0·015 $\mu\mu$F. by the aid of a long ebonite pointer moving over a scale mounted on a marble base. It was calibrated by the aid of a small fixed parallel plate air condenser (5 $\mu\mu$F. approx.), which could be either connected in parallel or disconnected by a small double-pole key. Readings were taken at short intervals along the scale, and a calibration curve was drawn to a large scale, whence differences of readings were tabulated in arbitrary units of capacity. This laborious operation of calibration and reduction of results to the form of a curve was repeated several times in the course of the measurements. The air capacity of the vessel was about 20$\mu\mu$F., and the pointer readings should, with perfect reproducibility, have given this value correct to 1 part in 1440. Actually, however, the lack of exact reproducibility of the 100 $\mu\mu$F. condenser was quite appreciable, the effect being to reduce the accuracy of the final determination of the dielectric constant to $\frac{1}{4}$ per cent.

When the readings with bromine had been taken, the bromine was redistilled back into A, and allowed to cool. The capacity of the condenser, with air as dielectric, was then observed. No appreciable alteration in the air capacity was detected after the vessel had been opened and cleaned and finally re-assembled. A slight chemical action appeared to have occurred at the electrodes, forming a very fine film on the platinum—presumably of platinum bromide, insoluble in bromine. The thickness of the film was quite insufficient to affect the thickness of the dielectric sensibly.

The main measurements having been completed, the capacity of the platinum lead, L_2, was determined directly in the same way, the lead being cut, for this purpose, immediately above its connection to the inner platinum cylinder. This capacity amounted to 1·84 arbitrary units, as compared with 39·50 units for the air capacity—that is, to about 5 per cent. of the air capacity of the test condenser. This has to be subtracted from each of the capacities $b-a$ and $d-c$.

Theoretically the effective capacity of the bromine condenser is affected by the electrical conductivity of the bromine. Calculation shows that the effect of this shunt is to cause a virtual drop in capacity of about one part in a million; the effect is thus negligible.

The above experiments gave the value **$3{\cdot}11_9$ for the dielectric constant of liquid bromine at 15°C. and at a frequency of 187,000 per sec.**

At the same temperature, but at a frequency of over a million per sec., no variation exceeding 1 in 350 could be detected in the dielectric constant. The dimensions of the vessel did not permit of an accurate determination at audio-frequency.

TEMPERATURE COEFFICIENT OF THE DIELECTRIC CONSTANT.

A determination of the temperature coefficient of the dielectric constant was made at a frequency of 187,000 per sec. Readings were taken at a temperature slightly above 0°C. (by placing the condenser in a container filled up with melting ice) and also at the practically steady temperature of the laboratory. The values obtained were

$$3{\cdot}20_6 \text{ at } 1{\cdot}02^\circ;\quad 3{\cdot}13_1 \text{ at } 13{\cdot}4^\circ;\quad 3{\cdot}12_3 \text{ at } 14{\cdot}4^\circ;\text{ and } 3{\cdot}10_3 \text{ at } 17{\cdot}65^\circ\text{C}.$$

When these observations are plotted it is found that, for this small range, ε_t (the D.C. at t°C.) may be expressed by the linear relation $\varepsilon_t=\varepsilon_0(1+at)$ where ε_0 (the D.C. at 0°C.)$=3{\cdot}212$, and $a=-0{\cdot}00191$ per degree Centigrade. This value of the temperature-coefficient is in very close agreement with the results of measurements made six months earlier on the same sample of bromine.

ELECTRICAL CONDUCTIVITY.

The resistance of the bromine between the condenser electrodes was observed at various temperatures by measuring the current when a steady voltage was applied to the condenser in series with a high resistance galvanometer. Polarisation effects were observed, and readings were accordingly taken of the growth and decay of the galvanometer deflections with time after each reversal of the sign of the applied electromotive force. The maximum deflection observed was adopted as representing the true conductivity. The variation of readings with time is illustrated in Fig. 3. The vessel was calibrated by substituting a 1/100 N solution of KCl and determining its resistance by an A.C. bridge at audio frequency. The resulting value of the electrical conductivity was $9{\cdot}6\times10^{-10}$ ohm^{-1} cm.$^{-1}$ at 0°C., with a temperature coefficient of $-0{\cdot}0188$ per degree Centigrade. The actual resistance of the bromine was $5{\cdot}46\times10^{6}$ ohms at 0°C.*

ELECTRIC MOMENT OF THE BROMINE MOLECULE.

A careful examination of the results of the above experiments shows that the Clausius-Mosotti relation $(\varepsilon-1)/(\varepsilon+2)D=$constant (where $D=$ density) is not satisfied, this function falling unmistakably with rise of temperature. On the theory of Debye, and its extension by Gans, this decrease in $(\varepsilon-1)/(\varepsilon+2)D$ with rise of temperature indicates the presence of dipoles or electric doublets. It is, therefore, of interest to deduce the value of the electric moment of the Br_2 molecule.

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* **The conductivity of liquid bromine has recently been investigated by Rabinowitsch (Zeit. Phys. Chem., 119, February, 1926), who records observations on samples under different conditions.**

Using the formula

$$(\varepsilon-1)/(\varepsilon+2)D=A+B/T$$

(where A and B are constants and T denotes the absolute temperature), the electric moment is obtained by the relation $B=4\pi N\mu^2/9kD$, where N=number of molecules per c.c., μ=the electric moment and k=Boltzmann's constant ($1{\cdot}372\times10^{-16}$).

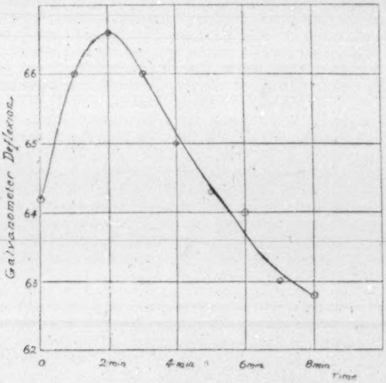

FIG. 3.—POLARISATION EFFECTS IN LIQUID BROMINE RESISTANCE.

On substituting experimental results from this Paper, the value $B=6{\cdot}3$ ($\pm0{\cdot}7$) is deduced.

The determination of B involves the determination of the density, D, and its variation with temperature. Since no modern data appeared to be in existence as to the density of liquid bromine at 0°C., this determination was made, and a mean temperature-coefficient of $-0{\cdot}00104$ was found over the range 0°C. – 20°C.,* this value being used in the calculation of B. It should be noted, however, that the percentage accuracy of the deduced value of B is lower than that of the determination of the dielectric constant, or rather of $(\varepsilon-1)/(\varepsilon+2)$.

Taking $1{\cdot}64\times10^{-24}$gm. for the mass of the hydrogen atom, the electric moment of the bromine molecule Br_2 is found to be

$$\mu=0{\cdot}40(\pm0{\cdot}04)\times10^{-18}.$$

Cordial acknowledgment is made to Mr. G. L. Addenbrooke, at whose suggestion the work was undertaken and who kindly provided the condenser vessel already constructed to his design. The experiments were carried out at the Sir John Cass Technical Institute, London, under the direction of Dr. D. Owen, who kindly filled the vessel with bromine and for whose valuable help I am greatly indebted.

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* Cf. with the value $-0{\cdot}0011$ given in the Int. Critical Tables over the range 0°C. – 30°C.

DISCUSSION.

Prof. O. W. RICHARDSON said that he was surprised to find such a high estimate of the electric moment of the bromine molecule. That given by the author was of the order of the product of the electronic charge and the diameter of an atom.

Mr. L. HARTSHORN congratulated the author on having obtained a reliable value of the dielectric constant in the case of such a difficult substance as bromine. Some time ago he had had to examine the published values of some 200 dielectric constants, and had found reason to believe that many of them were affected by very large sources of error, most of which the author had avoided. He thought, however, that she was optimistic in taking her result to the third decimal place. The main error would arise from the difficulty in measuring a small capacity accurately: the accuracy would be that of the smallest (20 $\mu\mu$F.) condenser, and although the author had used a single disconnection in the leads of the experimental condenser (thereby avoiding the errors introduced by a double disconnection), still, errors would arise from cross capacities between the leads and also from the capacity between the separated contacts of the disconnecting switch; thus between mercury cups in ebonite a capacity of 1 $\mu\mu$F. is not uncommon. These residual capacities would not be the same when the dielectric in the experimental condenser was bromine as when it was air. He did not think an accuracy of more than 1 per cent. was attainable in such measurements without the use of the elaborate bridge he had described to the Society three years ago. As regards the electric moment of the molecules, that obtained for bromine was about half as great as that of ethyl ether, and so seemed improbably high. The small temperature coefficient from which it had been deduced was partly due to change in density, and the small residuum ascribable to the presence of dipoles was not ascertained with sufficient accuracy or based on sufficiently numerous observations to justify the assumption of a linear relation between temperature and dielectric constant. He thought that the author's value of the electric moment could not, therefore, be relied on.

Mr. D. H. BLACK: Over 18 months ago, in connection with some research on the D.C. properties of insulating oils upon which I have been engaged for some considerable time, I observed that an oil gives a similar current-time curve to that shown in Fig. 3, the time to rise to a maximum varying according to the nature and the condition of the oil. I fail to see how such a curve can be explained as being due to polarisation as mentioned in the Paper, for on reversing the applied E.M.F. the polarisation E.M.F., which had previously been in opposition, would now be assisting it, thus causing a larger current to flow than that which would be due to the applied E.M.F. alone. This latter current can be observed on the initial application of the voltage after the film of oil has been short-circuited for some hours, and it is found to decrease from the moment of application. My own opinion is that the observed facts are due to the formation of a contact resistance on one or both of the electrodes similar to that observed when dealing with ordinary electrolytes. Such an assumption gives a very simple explanation of the observed facts, for on reversing the applied E.M.F. the contact resistance which had been formed would take time to disappear, and at the same time a similar resistance would be built up on the opposite electrode, giving the rising and the falling portion necessary to give such a curve as Fig. 3. Polarisation electromotive forces may be present, but in the case of oils, at any rate, their effect is small in comparison with the effects of the contact resistances. This theory is strongly supported by other experiments which I have carried out on various oils, and it gives a simple explanation of many of the apparent anomalies met with when making electrical measurements on liquid dielectrics.

Mr. G. L. ADDENBROOKE: Miss Anderson mentions that the condenser vessel for these experiments was provided by me and that the work was undertaken at my suggestion. It may, therefore, add something to the value of the Paper if I mention something of my preliminary work on the subject. In a discussion at the last meeting of the Society I sketched a diagram dealing with the electrical properties of celluloid, since reproduced in the Society's Bulletin. The experiments on celluloid were made in 1916. To pursue the matter further, I began to look out for some less complicated substance. With this view, I consulted the late Sir James Dewar, when he expressed the view that for scientific purposes experiments on substances of definite chemical composition were the only ones of value, and suggested work on crystals. There are a good many difficulties, however, in working with crystals, and after consideration of these and the cost, I was deterred from pursuing the subject in this form, because such crystals would be formed of at least two elements with their water of crystallisation. It seemed to me, therefore, that it would be more fundamental, if possible, to begin by a study of the electrical properties of the dielectric elements themselves as far as they could be obtained, free from impurities, and especially moisture. This would thus form a basis for the study of composite substances later. With this view, I looked up the available data regarding the non-metallic elements, and ultimately found the capacity of 10 elements, including liquified gases, were quoted in recent tables of physical constants, and that they were in general agreement with the squares of these refractive indexes. Amongst these elements were chlorine, bromine and iodine. I was particularly desirous of knowing, also, the electrical resistance of all the elements and their losses in alternating fields as showing how far absorption might be present in the capacity measurements. With iodine I was able to make some approximate measurements by a fall-of-potential method, which convinced me that the ordinary resistance of dry iodine is very high. As regards chlorine, I found that when prepared for experimental purposes the gas is always passed over a drying agent before liquefaction: an inspection of the capacity results, of which there are two or three separate determinations, led me to believe that the dielectric constants quoted are probably correct within a few per cent. This led me to bromine. I could, like Miss Anderson, at the time find no data as to its electrical resistance. I therefore procured the purest specimen I could and placed some in a U-tube with carbon electrodes. The resistance worked out at only 40 thousand ohms per cm. cube. On placing a few fragments of calcium chloride in the bromine the resistance soon increased to one megohm. As it was obvious there was still much moisture present, an attempt was made to distil the bromine, but without a well-organized chemical laboratory this was found both troublesome and dangerous. A friend then drew my attention to Professor H. B. Baker's work on highly-dried bromine, and I called on him and stated my difficulties and the object in view. He then kindly offered to place some of his highly-dried bromine at my disposal. The first specimen was in a rather narrow U-tube with platinum electrodes. I found it impossible to separate the resistance of this bromine from that of the glass tube in which it was contained. It was therefore clear that the resistance was far higher than the figures given above. The bromine remained in the tube for several weeks, and no action was observable on the platinum electrodes. This gave confidence to go further, and Professor Baker put up a further specimen of bromine in a glass bulb with platinum electrodes having about 5 cms.^{2} area each, spaced about a millimetre apart. Through the good offices of Mr. Dye, I was able to take this as soon as it was put up to the National Physical Laboratory; here tests were made of the resistance, the figure found was 140 megohms. A subsequent test by myself some three weeks later, at a rather lower temperature, gave 160 megohms. These tests correspond with a specific resistance of about 10^{11} megohms per cubic centimeter, and accord satisfactorily with the figures found under better conditions by Miss Anderson. Tests were also made by Mr. Dye of the capacity over a range of frequencies from about 400 up to one million. The change of capacity with changing frequencies was only nominal over this great range. It was impossible to measure the dimensions and spacing of the electrodes, and the outside capacity, with a sufficient accuracy to obtain the dielectric constant of such a small specimen. I was, however, able to measure the dielectric losses over a range of frequencies from 4 per second to 1,250. The curve is similar to curves obtained with other liquid dielectrics of about the same resistance. Taking the loss at equal voltages in steady and alternating fields, and assuming the d.c. loss at unity, the losses in alternating fields were 192 at 17∼ and 4·5 at 1,240∼. The results established clearly that Professor Baker's highly-dried bromine was a dielectric of a fairly high class, behaving in an ordinary manner for a liquid, and also that it could be kept in contact with platinum surfaces for some time without sufficient action to ionise it appreciably. In consultation with Professor Baker, the apparatus was then designed which Miss Anderson has used. It was still too small to enable good capacity measurements to be made by the method I was using; but just at this time Dr. D. Owen drew my attention to the heterodyne method of measuring very small capacities, and mentioned that the Cass Institute was setting up the necessary apparatus. After inspection of the platinum condenser I already had made, he agreed that the method should be sufficiently sensitive to obtain the dielectric constant of bromine with it; and on his telling me that Miss Anderson would be willing to undertake the work, it was placed at her disposal. That the work has been well done I am satisfied from my previous experience, and Miss Anderson is to be congratulated on the ability with which it has been carried out and for her patience and perseverance in dealing with a troublesome substance. Miss Anderson's work confirms within 2 or 3 per cent. the previous work of Schlundt and Schmidt, whose work I had previously relied on, but was inclined to doubt, because of the difference between their figures. The square of the refractive index for bromine is larger than that in the case of the other nine elements mentioned above, and it seemed to me that probably the larger relative capacity was due to absorption. I think it is now fairly shown that this is not so. It was my intention with the above apparatus to have made more complete measurements of the losses in alternating fields over a wide range of frequencies than the approximate results given above, but I have been waiting to do this until Miss Anderson's work was completed. The ten elements I have mentioned above include all those occurring in organic matter ; they also form a large number of compounds amongst themselves. Since matter itself, it is now agreed, is of electrical origin, it has for some years appeared to me of the first importance that the electrical properties of the non-metallic elements themselves in a pure state should be known as completely as possible. These elements form many compounds amongst themselves, of which many are dielectrics. Knowing the electrical properties of the constituents individually, we should then be in a position to study their properties when joined in compounds, and thus to trace differences brought about in their combination. There are other important questions turning upon the properties of this highly-dried bromine, but it would take too long to enter into them now ; but I would like to end by paying a tribute to Professor H. B. Baker, whose beautiful work has enabled this further advance to be commenced on a solid foundation. There seem strong reasons for anticipating that these non-metallic elements are all in a pure and dry state high-class dielectrics and practically free from serious absorption. Absorption may, therefore, come in from combination or from the presence of some extraneous substance.

Dr. D. OWEN said that, as regards the value found for the electric moment, it must be remembered that the bromine molecule is a heavy one, with some 40 electrons to the atom. The point raised by Mr. Hartshorn as to the effect of density variations on the temperature coefficient had been fully considered, and he thought the result must be taken as significant.

AUTHOR'S reply: I quite agree that the bridge described by Mr. Hartshorn in his Paper to this Society is capable of a higher degree of accuracy than can be obtained by the method used in the present Paper owing to the inevitable break of circuit in the latter method, but it is unfortunate that bridge methods are inapplicable except at lower frequencies. The mercury cups were small and shallow, hence $\frac{1}{3}\mu\mu$F. would probably more nearly represent the value of the capacity across the disconnected switch than $1\mu\mu$F. I admit, however, the force of Mr. Hartshorn's criticism and that the effect of this error slightly reduces the accuracy originally claimed in the Paper for the value of the dielectric constant. Surprise was expressed by Prof. Richardson at the apparently large value deduced for the electric moment μ, but, on making the following calculation, I find that this value is considerably less than Prof. Richardson made it appear. Taking the electronic charge e as $4{\cdot}77 \times 10^{-10}$E.S., and, for the diameter of the bromine atom the value $2{\cdot}5 \times 10^{-8}$cm. used by Prof. W. L. Bragg and derived from X-ray grating measurements, we obtain the figure $11{\cdot}9 \times 10^{-18}$ for the product of electronic charge and diameter. This product is nearly 30 times the value $0{\cdot}40 \times 10^{-18}$ obtained for μ in the present Paper and is hence of a distinctly higher order than this value of μ. Similarly Gans and Isnardi's value of μ for liquid ethyl ether is given by Smyth (Phil. Mag. 47, p. 541, 1924) as $1{\cdot}435 \times 10^{-18}$. This is much higher than Mr. Hartshorn's estimate of $2 \times 0{\cdot}40 \times 10^{-18}$. I also feel that Mr. Hartshorn's line of criticism on the value found for μ, based apparently on chemical considerations, is one which must of necessity remain in abeyance until the present dearth of accurate data from actual observations by numerous observers has been removed. Since reading the present Paper before this Society, I have discovered that Dr. Smyth (Amer. Chem. Soc. J., 46, Oct., 1924) assigns the value $0{\cdot}58 \times 10^{-18}$ to the electric moment of the bromine molecule. I must, however, emphasize the fact that, since the final form of the dipole theory is hardly yet agreed upon, any value of μ here offered depends upon the application of experimental observations in the particular formula chosen and is therefore liable to revision. I fully agree with Mr. Black's observations. The term "polarization" used in the Paper was intended to include not only the back E.M.F. due to electrolytic action, but also the effect of a film of high resistance. This latter overwhelms the effect of the polarization E.M.F. when the current is reversed, and causes the rise shown in Fig. 3 of the Paper.

XII.—THE SWAN BAND SPECTRUM OF CARBON.

By W. E. Pretty, *A.R.C.S.*, *B.Sc.*, Demonstrator in Physics, Imperial College of Science, South Kensington.

Received December 24, 1927.

(*Communicated by* Prof. A. Fowler, F.R.S.)

Abstract.

The Swan spectrum has been obtained in carbon monoxide and in the carbon spark in the absence of hydrogen. A discussion of the results of experiment and theory leads to the conclusion that the emitter of the Swan band system is the molecule of carbon C_2.

Introduction.

This well-known spectrum, which predominates in the blue base of a bunsen flame, was first discovered by Wollaston in 1802, and independently in 1857 by Swan, from whom it receives its name. Since then the origin of the spectrum has been the subject of numerous investigations and speculations. These have been reviewed by Watts,* and only a brief summary need be given here.

The spectrum has been variously attributed to (i) a hydrocarbon, (ii) an oxide of carbon, (iii) carbon. Swan himself suggested acetylene as the emitter, and this view was adopted by Ångström and Thalén (1875). For the carbon-oxygen origin carbon monoxide has usually been specified, and has found supporters in Baly† and Smithells.‡ Among those who have favoured a purely carbon origin mention should be made of Attfield (1862-75), Dibbits (1864), Wullner (1872), Secchi (1873), Deslandres (1888), Eder (1890), La Rosa (1911), and Kayser (Handbuch der Spectroscopie, Vol. V, p. 206, 1910). From their earlier experiments, Liveing and Dewar, in agreement with Ångström and Thalén, were led to attribute the spectrum to acetylene, though the origin of the Swan spectrum was not the primary object of their investigation. Later,§ however, when giving special attention to this spectrum, they abandoned the view that hydrogen was essential to the production of the spectrum.

In spite of the enormous amount of work done on the subject, there has hitherto been no general agreement as to the origin of the Swan spectrum, and the question has recently been reopened by the work of R. C. Johnson.‖ From an analysis of the bands, and from a critical discussion of his own and other experimental work, Johnson has concluded that the emitter of the Swan band spectrum is a hydrocarbon.

The present investigation was undertaken with a view to submitting this conclusion to further experimental tests.

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* Watts, Phil. Mag., Vol. 28, p. 117 (1914).
† Baly, "Spectroscopy." Vol. II, p. 220 (1927).
‡ Smithells, Phil. Mag. (i), p. 476 (1901).
§ Proc. Roy. Soc., 34, p. 418 (1882).
‖ Phil. Trans. Roy. Soc., A, Vol. 226, pp. 157-230 (1926).

EXPERIMENTAL.

Two methods of producing the spectrum were employed.

(i) Discharge through carbon monoxide at low pressure.

(ii) Carbon spark in various gases at atmospheric pressure.

It is well known that when an electric discharge is passed in carbon monoxide at a suitable low pressure the Swan spectrum is yielded brilliantly, and is often accompanied by traces of the hydrocarbon band at $\lambda 4315$, and by lines of hydrogen. The object in these experiments on carbon-monoxide was, therefore, to prepare the gas as free from hydrogen impurity as possible, and to examine the effect of such purification on the production of the Swan spectrum.

To this end the apparatus was made entirely of glass. The flask in which the gas was generated had a glass funnel fused into the neck, as shown in the diagram (Fig. 1). The electrodes were of thick platinum wire sealed into the limbs of the discharge tube. To facilitate the sealing in of the electrodes, the limbs of the discharge tube were made of lead glass. The discharge tube itself was of the ordinary H-pattern, having a capillary about 8 cms. long and 1 mm. in diameter. A palladium tube was sealed on to the apparatus for the purpose of introducing or removing hydrogen as should be desired. With the apparatus so constructed the only source of hydrocarbon impurity was the small amount of grease used to lubricate the taps; but as these were well removed from the discharge tube it is very improbable that any grease could have reached the latter. The carbon monoxide was prepared by the action of pure sulphuric acid on pure formic acid (A.R. standard). From the generator the gas passed into a reservoir, where it could be stored when necessary. It was then passed through a U-tube containing phosphorus pentoxide to the discharge tube, and was exhausted through another U-tube containing phosphorus pentoxide.

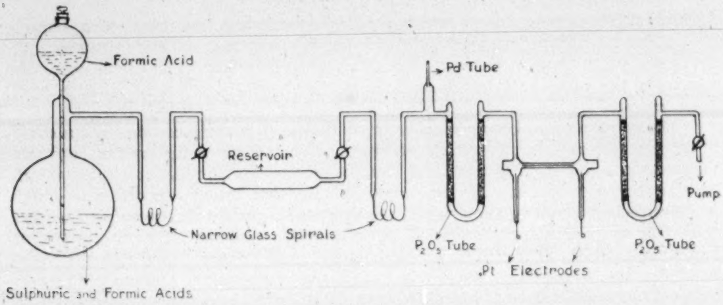

FIG. 1.

The apparatus was first filled with carbon monoxide, then exhausted to a very low pressure, and a discharge passed for some time. This process was repeated several times to remove any impurities in the electrodes or on the walls of the apparatus. It was then left for a considerable time in order that any water present might be absorbed by the phosphorus pentoxide. The presence of water in the carbon monoxide itself was not to be expected, as the gas was prepared from pure concentrated sulphuric and formic acids, and was therefore dried before leaving the generator. For the excitation of the tube an induction coil giving a 6″ spark in air was used; condensers, a variable inductance and spark gap being introduced into the circuit when required. The photographs were taken on a single prism instrument having a dispersion of about 5 *A* per mm. at λ4000 and 18 *A* per mm. at λ5700.

Various types of discharge were employed. With a spark gap and fairly large inductance in series, and condenser in parallel, the only spectrum seen was that consisting of the Swan bands. On reducing the amount of inductance the band system became relatively weaker, and lines due to carbon and oxygen made their appearance. Finally, when all the inductance was cut out, only the line spectrum remained. Some of these photographs are shown in Plate I. The pressure of the gas in these experiments was of the order of a few mms. of mercury. When the pressure of the gas was reduced the Ångstrom bands appeared, and eventually replaced the Swan bands, particularly with an uncondensed discharge.

The striking result from this part of the investigation was that not one of the very many photographs taken showed the least trace of the hydrogen line spectrum or of the hydrocarbon band at λ4315.

Hydrogen was next introduced into the apparatus by means of the palladium tube, and a discharge passed through the mixture of carbon monoxide and hydrogen. As was expected, the hydrogen lines Hα, Hβ appeared, together with the hydrocarbon band; the intensity of the Swan spectrum, however, was not appreciably changed.

A discharge was next passed continuously through the mixture of carbon monoxide and hydrogen, observations being made with a direct vision spectroscope. After a short time the hydrogen lines became fainter and the pressure decreased. Eventually the hydrogen lines disappeared, and a photograph taken with the tube in this condition showed no trace of the hydrogen lines or hydrocarbon band. By admitting further amounts of hydrogen the pressure was lowered sufficiently for the Ångstrom bands to replace the Swan system almost entirely. The explanation of this is, no doubt, that the hydrogen was oxidised to water by oxygen from the carbon monoxide, the water so formed being absorbed by the phosphorus pentoxide with considerable lowering of the pressure. The liberated carbon was deposited in a finely divided state on the walls of the discharge tube.

These experiments showed that the Swan spectrum could be obtained in carbon monoxide free from hydrogen to the extent that the latter could not be detected. Also, since hydrogen, which had been intentionally introduced, had disappeared during the passage of the discharge, it would appear highly improbable that traces of hydrogen impurity, if such had been present during the experiments on pure carbon monoxide, could have persisted for the many hours during which the different photographs of the Swan spectrum were taken.

EXPERIMENTS ON THE CARBON SPARK.

In order to vary the mode of production of the spectrum, a spark was passed between carbon poles in different gases, free, as far as could be obtained, from hydrogen impurity. In this case the surrounding medium was at atmospheric pressure.

The carbon poles were about 5 mms. long and 2-3 mms. in diameter, and were supported by platinum leads, the distance between the poles being about 2 mms. The object of making the poles as small as practicable was to minimise the amount of any gases which might be evolved during the passage of the spark. The tube containing the poles was made of glass about 15 cms. long and 3 cms. in diameter. The first gas used was argon. This was passed over heated calcium (to remove hydrogen and nitrogen), through a long tube containing phosphorus pentoxide, and thence to the sparking tube. With a condenser and inductance included in the circuit, the Swan spectrum was readily obtained, together with the brighter bands of cyanogen, indicating nitrogen impurity. There was, however, no visible trace of hydrogen. The inductance was then cut out, and a photograph taken under these conditions showed a faint Hα.

On repeating the experiment with nitrogen (the calcium tube having been removed) a similar result was obtained, except that the cyanogen bands were relatively stronger than the Swan bands. This is not surprising, as the excitation potential for the CN bands is probably lower than that for the Swan system.

In an endeavour to remove the hydrogen impurity the spark was passed in air, and in this case the phosphorus pentoxide was contained in the spark tube itself, instead of in a separate tube. At first the result was the same as for argon and nitrogen ; but after continued running of the spark the hydrogen disappeared, no trace of it being seen on the plates even when the inductance was cut out and the line spectrum thereby obtained (Plate I (*f*)). As in the case of carbon monoxide, continued passage of the spark had brought about the oxidation of the hydrogen with formation of water, which had then been absorbed by the phosphorus pentoxide. In all cases the Swan spectrum was yielded, and the removal of the last trace of hydrogen did not affect in any way the intensity or appearance of the bands.

The evidence obtained from these experiments on the carbon spark is in agreement with that obtained from the experiments on carbon monoxide—that the Swan spectrum can be obtained in the absence of hydrogen.

Discussion of Results.

Among the chief experimental investigations which appear to show that oxygen is not necessary to the production of the spectrum are those of Watts* on the passage of sparks under carefully purified liquid carbon compounds. In heptane, hexane, octane and carbon tetrachloride the Swan bands were consistently observed. Similar results were obtained by Konen. Prof. Fowler has also informed the author that he has several times made similar experiments with benzene which had been boiled for several hours with sodium in a reflux condenser, and that he always found the Swan bands to be very bright when the spark was passed.

The earlier experiments on the appearance of the Swan bands in burning hydrocarbons, and in the carbon arc burning in hydrogen, clearly do not afford conclusive evidence that both hydrogen and carbon are necessary for the production of the bands. R. C. Johnson,† however, has described a new series of experiments which have led him to the conclusion that the emitter is a hydrocarbon, and from additional theoretical considerations he specifies in particular a molecule of the form HC-CH. Briefly these results were as follows: A discharge was passed between carbon electrodes in argon at about 30 mms. pressure, the discharge tube being of the ordinary H-type having a capillary about 20 cms. long and 1 mm. in diameter. When hydrogen was admitted to the tube the Swan spectrum was yielded brilliantly. A palladium tube was then heated to remove the hydrogen, and the Swan system grew fainter and finally disappeared, leaving only the continuous spectrum of high pressure argon, together with a few lines of the line spectrum of red argon. As hydrogen was readmitted the Swan bands reappeared. From these results it was concluded that hydrogen is essential to the production of the Swan spectrum; and, further, since only hydrogen and carbon were present, apart from argon, the origin of the spectrum must be hydrocarbon.

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* Watts, loc. cit.

† R. C. Johnson, loc. cit.

From the results of the experiments carried out in the present investigation, however, it would appear that the emitter cannot be a hydrocarbon. Also, since the Swan spectrum can be obtained in hydrocarbons free from oxygen, the only conclusion consistent with the results of experiment is that the emitter is purely carbon. On this view, Johnson's experiments may possibly be interpreted as follows: In the case of a discharge between carbon electrodes in pure argon the production of the Swan spectrum will depend on the presence of some agent which will enable the carbon to be freed from the poles and then be excited in the tube. In the complete absence of such an agent the discharge becomes such as would result if any electrodes other than carbon were used in pure argon, yielding only the continuous spectrum of argon together with some line spectrum. When, however, hydrogen is admitted, the carbon first forms a hydrocarbon, which is then excited and decomposed in the tube, the Swan spectrum being yielded by the carbon during the time that it is in the free state. According to this suggestion, it follows that a similar result would have been obtained by the admission of any other active gas, the function of the gas introduced being merely to bring about the conditions necessary for the carbon to be appropriately excited. Johnson admitted oxygen from a side bulb containing $KMnO_4$, and the Swan system appeared; but as the oxygen admitted contained sufficient hydrogen for the triplet system* (attributed by Johnson to a hydrocarbon) also to appear, it affords no test of the suggestion made above.

If a C_2H_2 origin is assumed, it is to be expected that there would be dissociation, to a greater or lesser extent, to CH and H; for this is certainly true of the higher hydrocarbons (Plate I (*a*)). In the author's experiments on CO no trace of the CH band or of the hydrogen line spectrum was seen on any of the plates, although hydrogen was readily indicated when it was intentionally introduced. Further, in order to account for the observed structure of the bands, Johnson assumes that the C_2H_2 emitter is of the form HC-CH, the binding in each CH group being sufficient to permit of its being treated as a single nucleus. In view of the fact that the CH molecule is known to emit bands of two systems, $^2D \rightarrow {}^2P$ and $^2S \rightarrow {}^2P$ neat $\lambda 4300$ and $\lambda 3900$ respectively, it does not seem probable that in the case of a C_2H_2

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* **Merton and Johnson, Proc. Roy. Soc. A, Vol. 103, p. 390 1923).**

emitter the binding would be sufficiently close to permit of the CH group being treated as though there were no internal oscillation or rotation.

The Swan bands are found to consist of a triplet *P* branch and a triplet *R* branch, but no *Q* branch has yet been detected. The absence of a *Q* branch is taken by Johnson to indicate a high degree of symmetry in the emitting molecule. This assumption does not appear to be justified, for although *Q* branches have not been found for the symmetrical molecules N_2 and O_2, they are strongly developed in He_2. Also the *Q* branch is absent, or at any rate very weak, in systems due to CuH and CaH, and in the violet system of CN. These molecules are certainly not symmetrical. The apparent absence of or the presence of a very weak *Q* branch appears to be characteristic of systems which are due to transitions between like electronic states—e.g., CuH, $^1S \rightarrow {}^1S$; CN (violet) $^2S \rightarrow {}^2S$; NO β bands recently assigned to a $^2P \rightarrow {}^2P$ transition.*

The component lines of the triplets in the Swan bands show alternations of intensity which, up to the present, have been found only in systems known to be due to diatomic molecules composed of like atoms—e.g., N_2, $N_2{}^+$, O_2 and He_2. Hund† has recently interpreted the "Second Positive" band-system of N_2 as a $^3P \rightarrow {}^3P$ transition. Mulliken‡ has extended this to the Swan bands, and explains the apparent absence of a *Q* branch, and also the alternating intensities in the triplets, pointing out the strong analogy with the "Second Positive" system of N_2. This evidence does certainly point to a symmetrical diatomic molecule as the emitter of the Swan bands.

Johnson has analysed the structure of the bands, and finds that the moment of inertia of the emitter is of the same order as that obtained for the molecules CO, N_2 and O_2. Such a value would be given by a molecule of acetylene C_2H_2, assuming each CH group acted as a single nucleus, and also by a molecule composed of two carbon atoms C_2. The determination of the moment of inertia, therefore, while showing that both C_2H_2 and C_2 are of the right order, fails to decide between the two.

Experiment leads to the conclusion that the emitter is purely carbon. Theory indicates that the emitter is a symmetrical diatomic molecule. The molecule fulfilling these conditions, and also giving a value for the moment of inertia which is of the order obtained from an analysis of the structure of the bands, is the molecule of carbon C_2.

In conclusion, the author wishes to express his thanks to Prof. A. Fowler, at whose suggestion this investigation was undertaken, for his continued interest and advice during its progress.

NOTE ADDED JANUARY 25, 1928.

Since the completion of the above investigation a paper on the "Structure of the Swan Bands," by J. D. Shea,§ has appeared, in which the opinion is expressed that the carrier is a purely carbon molecule; either C – C or, if such a combination can exist, $C^- - C^-$.

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* **Jenkins, Barton and Mulliken, Phys. Rev., Vol. 30, p. 150 (1927).**

† **Hund, Zeits. f. Phys., 36, p. 657 (1926).**

‡ **Mulliken, Phys. Rev., Vol. 29, p. 637 (1927).**

§ **Shea, Phys., Rev., Vol. 30, p. 825 (1927).**

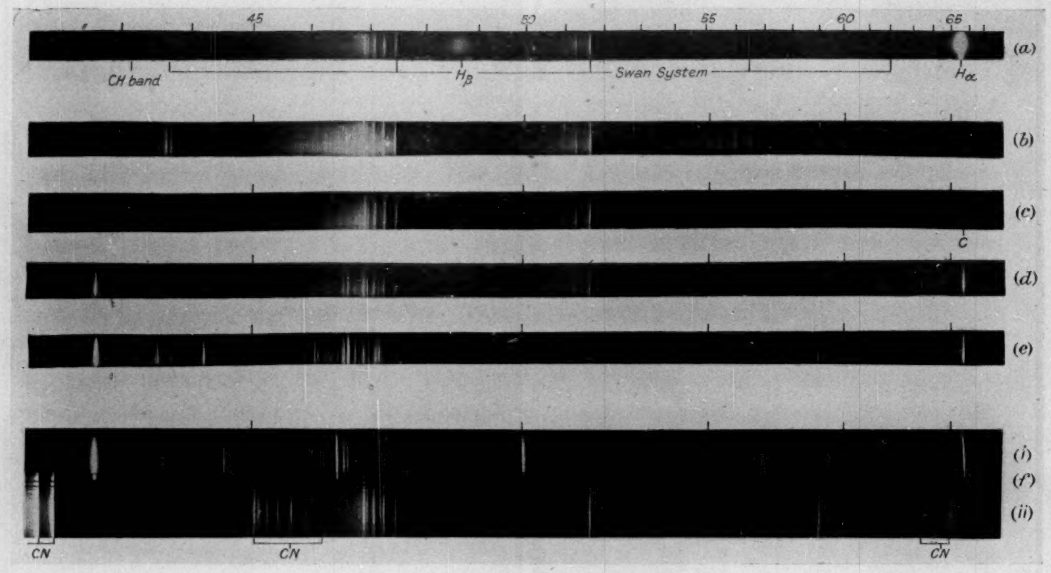
45
50
55
60
65
(a)
CH band
H_β
Swan System
H_α
(b)
(c)
C
(d)
(e)
(i)
(f)
(ii)
CN
CN
CN

PLATE I.—PHOTOGRAPHS OF THE SWAN SPECTRUM.

[*To face page* 77

Description of Plate I.

(*a*) Discharge through pentane (C_5H_{12}). Condenser and inductance, showing hydrogen lines and CH band in addition to Swan system.

(*b*), (*c*), (*d*), (*e*). Discharge through carbon monoxide. Condenser and diminishing inductance. (*b*) is a purely Swan spectrum; (*e*) shows lines of carbon and oxygen but no hydrogen; (*c*) and (*d*) are intermediate stages.

(*f*) Carbon spark in air.

(i) Condenser, no inductance; showing lines of oxygen, nitrogen and carbon, but no hydrogen.

(ii) Condenser and inductance; showing Swan system.

DISCUSSION.

Dr. R. C. **Johnson** said that, while the author's work was of great interest, he did not think that it justified the conclusion reached. The emission of the Swan band from carbon monoxide dried with P_2O_5 was a familiar occurrence, but he did not think that the absence of hydrogen could be assumed. The relative intensity of spectra depends more on the excitation potentials than on the percentage in which the components of a gas mixture are present. Thus a mere trace of a hydrocarbon in helium at a pressure of 40 mm. will drown the helium spectrum, owing to the relatively low excitation potential of the former. Mulliken, in the Physical Review, had recently pointed out that the molecules C_2, BN, BeO, LiF all have a similar outer structure; they are polar molecules of marked chemical stability, and are theoretically incapable of emitting band spectra, while extensive experimental research has hitherto failed to reveal band spectra in any of these cases, with the doubtful exception of BeO. Analogy, therefore, makes the C_2 origin of the Swan bands improbable, whereas the C_2H_2 molecule is similar in its external structure to N_2, which has two valency electrons and exhibits a band spectrum similar to the Swan spectrum. Again, if a gas jet be plentifully supplied with air the flame is blue, owing to the presence of the 4315 band of CH, whereas when the air supply is reduced the Swan spectrum appears and turns the flame green; this result may be attributed to the association of 2CH into C_2H_2, which may be regarded as a rotating dipole HC–CH. Shea had suggested $(C^-)_2$ as the Swan emitter, but there appeared to be no justification for the revolutionary assumptions involved in this suggestion, and such a structure, if it could exist, would not resemble the N_2 molecule. It was desirable to investigate the Swan absorption spectrum. Hori had obtained it by exploding by sudden heat a thread of mercury in a vacuum chamber which happened to contain hydrocarbon gases from the oil of a pump. It would be interesting to fuse the filament of a carbon-filament lamp, after exhausting the latter if necessary, and to see whether the absorption spectrum was obtained.

Prof. E. N. **da C. Andrade** pointed out that hydrogen would be occluded in the carbon of the lamp, and the speaker agreed that the result would be inconclusive.

Dr. W. **Jevons**: An experimental investigation as to the emitter of the Swan band system was perhaps one of the most urgent in molecular spectroscopy, and I tender my hearty congratulations to the author on the very thorough manner in which he has carried it out. His result will, I am sure, win immediate and general acceptance. It is unfortunate that Mr. Pretty's Paper did not appear, say, two months ago, for a very important Paper (Phys. Rev., 30, 825, December, 1927) has just been published by J. D. Shea, who has analysed the 0,0 (λ5165), 0,1 (λ5635), 1,0 (λ4737), 1,1 (λ5129), and 1,2 (λ5585) bands, and, like Mr. Pretty, has pronounced in favour of C_2 as the emitter, but has added a suggestion that it might be $(C^-)_2$. In a preliminary report of this work to the American Physical Society (Phys. Rev., 27, 245, abs. 14, 1926) Shea and Birge drew attention to the surprising similarity of the values obtained for the moments of inertia (I_0) and the electronic angular momentum (ε) of the emitters of the Swan system and the N_2 "Second Positive" system. And, since the acetylene molecule $(CH)_2$ has the same number of electrons and nearly the same mass as N_2, they thought that $(CH)_2$ *might* be the Swan emitter if it could vibrate and rotate as a dipole. The $(CH)_2$ suggestion, though supported in Dr. Johnson's Paper (which appeared shortly afterwards), was not acceptable to several practical spectroscopists who had very many years' experience as to the occurrence of the Swan bands, nor to the investigators who have done most to bring about the rapid development of the theory in the last twelve months. On the face of it, a tetra-atomic molecule like $(CH)_2$ was unlikely to emit a band-system of the same simplicity as one emitted by the di-atomic molecule N_2. Mulliken has, as Mr. Pretty states, attributed the Swan bands to the transition $^3P \rightarrow {}^3P$ (or, less probably, $^3D \rightarrow {}^3D$) in the C_2 molecule ; and I might add that Prof. Mulliken informed me last summer that Hund had deduced that the Swan emitter has, like He_2, etc., zero nuclear spin —a result compatible with a pole like C, but not with one like CH. Shea and the author of the present Paper are, then, in agreement with Hund and Mulliken. Like Dr. Johnson, I have a difficulty in understanding why the Ne-like (no-valence-electron) molecule C_2 should emit a band-system of the same type as one emitted by Mg-like (two-valence-electron) molecule N_2. Shea's $(C^-)_2$ would remove the difficulty, *if* it could reasonably be assumed to exist in an electronically excited ^3P state, and also to be capable of vibration and rotation as a dipole. I would like to add a word about two other twelve-electron molecules BN and BeO, which Dr. Johnson mentions with C_2 as being improbable emitters. BN, for which my early argument was mostly based on analogy with CN and SiN in active nitrogen, is of course entirely withdrawn, as Dr. Johnson says, in view of Mulliken's unassailable evidence for BO as the emitter of the bands in question. I have quite recently started an investigation of the beryllium band-system described by Glaser, but I have done too little yet to decide whether BeO is its emitter. I cannot altogether agree with Dr. Johnson's statement that CH is incapable of vibration and emits only non-vibrational (0,0) bands. For, in addition to the twelve branches of the 0,0 band of the $^2D \rightarrow {}^2P$ system near λ4300 Hulthén (Dissertation, Lund, 1923) detected eleven fainter branches which Birge (National Research Council Bulletin, No. 57, p. 193, footnote 132) regards as possibly belonging to the 1,1 band of the same system.

AUTHOR'S reply : I should like to point out to Dr. Johnson that the object in reducing the amount of inductance included in the circuit was to ensure that the discharge was sufficiently violent for hydrogen lines to appear if any of the gas were present. This is evidenced by the fact that the carbon doublet at λ6580 (excitation potential—16 volts) appeared very strongly. The calculated excitation potential for $H\alpha$ is about 12 volts. Why the molecules C_2, BN, BeO and LiF should not be expected to radiate is not altogether clear, for the corresponding atom Ne is easily excited. The phenomena accompanying the change in a gas flame when the air supply is varied can be explained on a carbon hypothesis. With an insufficiency of air there is a greater proportion of carbon available. As the air supply is increased the carbon is oxidised to a greater extent, and is then incapable of yielding the Swan spectrum. I might point out that, according to Smithells, the flame of pure dry cyanogen in pure dry oxygen yields a brilliant Swan spectrum. This is very difficult to reconcile with a hydrocarbon emitter. Dr. Johnson attempts to justify the close binding of the CH group, which yields the systems at λ4300 and λ3900, by pointing out that only the non-vibrational (0,0) bands have been observed. Kratzer has analysed the two systems, and has calculated the values of the internuclear distance for the different electronic states. The smallest of these values is approximately the same as the value obtained by Johnson for the separation of the nuclei in the emitter of the Swan system. It cannot therefore be held that the binding in the CH group is close compared with that in the emitter of the Swan bands.

XIII.—THE ELECTRODELESS DISCHARGE THROUGH GASES.

The Thirteenth Guthrie Lecture, Delivered March 9, 1928.

By Sir J. J. Thomson, *O.M., F.R.S.*, Master of Trinity College, Cambridge.

By " electrodeless discharge " I mean a discharge where the currents form closed circuits in the gas, and have not to pass from gas to metal, as in discharge tubes with metallic electrodes, or from gas to glass, as when metallic electrodes are placed outside the tube. This ring discharge can be produced by placing outside the bulb or tube containing the gas at a low pressure a solenoid placed in a circuit connecting the outsides of two Leyden jars. The insides of these jars are connected respectively with the two terminals of an induction coil ; when sparks pass between these terminals, rapidly alternating currents, whose frequency is inversely proportional to the square root of the product of the capacity of the jars and the self-induction of the circuit, pass through the solenoid. The electromagnetic induction due to these currents starts in the exhausted vessel currents which flow in rings whose axis coincides with that of the solenoid.

Theory of the Discharge.

When the solenoid is long and straight, the theory of the discharge is as follows :

The tangential electric force at a point distant r from the axis of the solenoid is equal to

$$\frac{1}{2\pi r}\int_0^r \frac{dH}{dt} 2\pi r \,.\, dr = \frac{1}{r}\int_0^r \frac{dH}{dt} r dr,$$

where H is the magnetic force at the distance r. The tangential force on a moving electron at r is therefore

$$e\left(\frac{1}{r}\int_0^r \frac{dH}{dt} r dr + H\frac{dr}{dt}\right)$$

$$= \frac{e}{r} \,.\, \frac{d}{dt}\int_0^r Hrdr.$$

Hence, if m is the mass of the electron

$$m\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = \frac{e}{r}\frac{d}{dt}\int_0^r Hrdr$$

or

$$mr^2\frac{d\theta}{dt} = e\int_0^r Hrdr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

if H and $d\theta/dt$ vanish simultaneously.

If there are no currents through the gas, or if these are very small compared with those through the solenoid, as they will be while the discharge through the gas is being initiated, H will be independent of r, so that equation (1) gives

$$mr^2\frac{d\theta}{dt}=e\frac{Hr^2}{2}$$

or

$$\frac{d\theta}{dt}=\frac{1}{2}H\frac{e}{m}$$

thus all the electrons have the same angular velocity round the axis of the solenoid.

In order that the discharge should take place, the energy given to an electron must be great enough to enable it to ionise the gas—i.e., it must not be less than We if W is the ionising potential. If $W=\frac{1}{2}mw^2$, then the electron must acquire a velocity at least as large as w to enable it to start the discharge.

if $H=H_0\sin pt$

$$\frac{d\theta}{dt}=\frac{1}{2}H_0\frac{e}{m}\sin pt$$

The maximum value of $d\theta/dt=H_0e/2m$.

If a is the radius of the discharge tube, whose axis is supposed to coincide with that of the solenoid, the maximum velocity of an electron is $aH_0e/2m$, and one condition for the discharge is that this velocity should not be less than w. The electron will not, however, acquire this velocity under the electric force if it makes a collision with the molecules of the gas before it attains it, for this would take away the energy it had already acquired. Thus the condition for the discharge is that the electron should acquire a velocity w before it has travelled over a distance λ, where λ is the free path of the electron through the molecules of the gas. The distance travelled by the electron is $a\theta$; since

$$\frac{d\theta}{dt}=\frac{1}{2}H_0\frac{e}{m}\sin pt$$

$$a\theta=\frac{1}{2}a\frac{H_0}{p}\frac{e}{m}(1-\cos pt)$$

Hence the condition for the discharge is that when

$$\frac{1}{2}aH_0\frac{e}{m}\sin pt=w$$

$$\frac{1}{2}a\frac{H_0e}{mp}(1-\cos pt)=\lambda$$

From these equations we get

$$a\frac{H_0e}{m}=\frac{w^2+(p\lambda)^2}{p\lambda} \qquad \text{. . . . . (2)}$$

Hence H_0 is infinite when $p\lambda$ is either zero or infinity—i.e., when the pressure is infinite or when it vanishes, and H_0 will have a minimum value when $p\lambda=w$, and the minimum value of H_0 is $\frac{2m}{ae}w$.

Thus for the electrodeless discharge, as well as for the spark discharge between electrodes, there is a critical pressure at which the spark passes most easily; the critical pressure for the electrodeless discharge depends on the frequency of the currents, and is directly proportional to this frequency; the critical pressure for the spark discharge depends upon, and is directly proportional to, the spark length.

The magnetic force required to produce the electrodeless discharge is a function of $p\lambda$, or p/P if P is the pressure, and does not alter as long as this is constant. We may compare this with Paschen's law for the spark discharge, according to which the potential required to produce the spark is a function of the product of the pressure and the spark length.

The electromotive force round a circuit of radius r in the discharge tube is

$$\pi r^2 \frac{dH}{dt}, \text{ i.e., } \pi r^2 p H_0 \cos pt$$

on substituting the value of H_0 from equation (2), we find that the E.M.F. round the circuit when the discharge is passing is

$$\pi r^2 p \, . \, \frac{m}{ea} \, \frac{w^2 + (p\lambda)^2}{p\lambda} \cos pt$$

Thus the peak value of this has a minimum value when $p\lambda = w$, and the E.M.F. round the circumference of the tube is $\frac{2\pi a \, . \, mw^2}{\lambda e}$.

In the non-striated uniform positive column in the discharge between electrode the electric force F at a point is given by the equation

$$Fe\lambda = \frac{mw^2}{2}$$

Hence the difference of potential in a length $2\pi a$ of the positive column—i.e., in a length equal to the circumference of the ring discharge—is $2\pi a \, . \, F$—i.e.,

$$\frac{2\pi a \, . \, mw^2}{2\lambda e},$$

This is one-half of the peak E.M.F. round the ring when the electrodeless discharge is passing most easily. A further analogy between the ring discharge and the positive column is that within certain limits of pressure and current density the electrodeless discharge, like the positive column, shows striations. These for the electrodeless discharge are much more conspicuous in hydrogen than in any other gas I have tried.

In hydrogen I find the electrodeless discharge sometimes has the appearance shown in Fig. 1. There are well marked striations, which, however, flicker about too much to allow of photographs being taken.

I have measured the E.M.F. round the bulb when the electrodeless discharge is passing at different pressures, and found that there is a minimum at a certain pressure; the minimum is, however, a very flat one, so that it is not possible to determine the critical pressure accurately. Experiments showed, however, that it was much higher for helium than for any other gas, and that it increased with the frequency of the alternating currents.

In the preceding investigation the currents through the gas have been assumed to be very small, as they will be when the discharge only just passes. When the spark-gap between the terminals of the induction coil is lengthened a very bright discharge passes through the gas, and the currents through it are comparable with those through the solenoid ; the magnetic force, instead of being uniform over the cross-section of the tube, is now very much greater near the walls of the tube than it is near to the axis. Experiments showing this effect are described in the Phil. Mag. iv., p. 1128, 1927. When these large discharges pass through the gas there are many free electrons and positive ions distributed through the gas, and the pressure is generally so low that the motion of the electrons may be regarded as free, and not influenced by collisions with the molecules of the gas. The properties of a medium of this kind are exceedingly interesting and important. For example, the "Heaviside layer," which plays such an important part in the transmission of electrical waves, has this constitution, and the electrodeless discharge is a convenient method for investigating experimentally its properties.

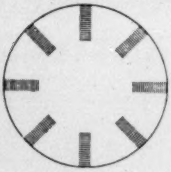
FIG. 1.

The theory of the electrical properties of such a medium is quite simple.

Let X, Y, Z be the components of the electric, α, β, γ those of the magnetic force ; u, v, w the components of the electric currents in the gas, x_r, y_r, z_r, the co-ordinates fixing one of the electrons ; c the velocity of light. Then the field equations are

$$u=\frac{1}{4\pi}c^2\frac{dX}{dt}+\Sigma e\,.\,\frac{dx_r}{dt}$$

With similar equations for v and w

$$m\frac{d^2x_r}{dt^2}=Xe$$

where m is the mass of an electron ; we suppose that the current through the gas carried by the electrons is large compared with that carried by the positive ions.

The other equations are

$$4\pi u=\frac{d\gamma}{dy}-\frac{d\beta}{dz}$$

with similar equations for v and w

$$\frac{d\alpha}{dt}=\frac{dy}{dz}-\frac{dz}{dy}$$

with similar equations for $d\beta/dt$ and $d\gamma/dt$.

From these equations it follows that α, β, γ, X, Y, Z satisfy differential equations of the type

$$\frac{d^2\psi}{dt^2}+4\pi c^2\frac{Ne^2}{m}\psi=c^2\left(\frac{d^2\psi}{dx^2}+\frac{d^2\psi}{dy^2}+\frac{d^2\psi}{dz^2}\right)\quad\ldots\ldots\ldots\quad(3)$$

when N is the number of electrons per unit volume. A vibration of frequency p and wave length λ will be represented by

$$\psi=\cos\left(pt-\frac{2\pi}{\lambda}(lx+my+nz)\right)$$

when l, m, n are the direction cosines of the normal to the wave front. Substituting this value of ψ in equation (3) we find

$$p^2=4\pi c^2\,.\,\frac{Ne^2}{m}+\frac{4\pi^2}{\lambda^2}c^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

If the gas is confined within a discharge tube, the possible values of λ will be determined by the boundary conditions, and will depend on the size and shape of the tube. The possible values of λ will be comparable in magnitude with the linear dimensions of the tube. To each value of λ will correspond a particular frequency p; but we see from equation (4) that whatever be the value of λ, p cannot be less than $c\sqrt{4\pi Ne^2/m}$. Thus the frequencies of the free vibrations of the gas will form a series with a definite lower limit. The ionised gas will thus have definite periods of vibration, and when the ionisation is very dense all these periods will cluster round the value $2\sqrt{4\pi Ne^2/m}$, and will be but little influenced by the size of the vessel. Thus we conclude that the ionised gas would emit vibrations—i.e., it would send out electrical waves—and that the greater the density of the ionisation the higher the frequency of the waves.

These results refer to the free vibrations excited in the gas. Let us now consider the forced vibrations due to the alternating current through the solenoid. If p is the frequency of these currents

$$\frac{d^2\psi}{dt^2}=-p^2$$

and for a plane wave travelling along the axis of x equation (3) becomes

$$\frac{d^2\psi}{dt^2}+4\pi\frac{c^2Ne^2}{m}\,\psi=c^2\frac{d^2\psi}{dx^2}$$

or

$$\left(4\pi c^2\,.\,\frac{Ne^2}{m}-p^2\right)\psi=c^2\frac{d^2\psi}{dx^2}$$

If p^2 is less than $4\pi c^2Ne^2/m$, the coefficient of ψ on the left-hand side is positive; hence ψ will not be represented by a harmonic, but by an exponential function of x, and the expressions for ψ will contain a term $\varepsilon^{-x/R}$

where

$$\frac{1}{R}=\sqrt{\left(4\pi\frac{Ne^2}{m}-\frac{p^2}{c^2}\right)}$$

So that a disturbance would die down to $1/t$ of its original value in a distance equal to $1/\sqrt{4\pi Ne^2/m-p^2/c^2}$. Thus no forced disturbance whose frequency is less than the critical frequency can travel as a wave through the gas; it only penetrates a distance equal to the value just found.

In our experiments the forced disturbances are magnetic forces spreading from the solenoid through the ionised gas. When the ionisation is strong, the frequency of the alternating currents is small compared with the critical frequency, so that the magnetic force will only travel a distance measured by $1/\sqrt{4\pi Ne^2/m}$ into the gas. The experiments show the existence of this effect very plainly, for when the ionisation is very intense the discharge consists of a thin, very brightly luminous ring close to the surface of the discharge tube. The thickness of this ring enables us to form some idea of the intensity of ionisation, for since the light is very bright at the outer edge of the ring, and very faint at the inner one, the magnetic force must have fallen to much less than $1/\varepsilon$ of its value in a space equal to the thickness of the ring. Thus in an experiment in hydrogen, at a pressure of 1/50 of a millimeter of mercury, I estimated the thickness of the bright ring as about a millimeter; hence we must have

$$\frac{1}{\sqrt{4\pi Ne^2/m}} < 0{\cdot}1$$

or

$$4\pi Ne^2/m > 10^2$$

Now $e^2/m = 3 \times 10^{-13}$, hence N must be greater than $10^{15}/12\pi$. At a pressure of 1/50 of a mm. the number of molecules per cubic centimetre is about $0{\cdot}7 \times 10^{15}$. So that in this experiment the number of electrons must have been comparable with the number of molecules.

EFFECT OF LIGHT ON THE ELECTRODELESS DISCHARGE.

The electrodeless discharge supplies a very convenient means for studying the effect of various physical agents on the passage of the discharge through gases. The first example I shall take is the effect of light on the discharge.

It has been known for many years that the passage of a spark between electrodes is facilitated by exposing the spark-gap to light; this, however, is due to the photoelectric effect at the cathode producing a discharge of electrons which helps to start the discharge. In the experiments with the electrodeless discharge there are, of course, no electrodes, and precautions were taken to prevent the light striking against the glass walls of the tube in the neighbourhood of the ring discharge. The arrangement adopted is shown in Fig. 2.

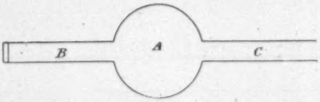

FIG. 2.

The ring discharge passes in the bulb A, two long tubes B and C connect with this bulb, and at the end of B there is a quartz window, and through this a parallel beam of light passes down the axis of the tube; the ends of the tubes B and C, which are far away from the bulb, are the only parts of the apparatus hit by the rays of light.

The length of the spark between the terminals of the induction coil is adjusted until, when the light is screened off, the ring discharge just ceases to pass through the gas. When the light is sent through the tube a ring discharge with considerable luminosity appears. The effect has been found in a great variety of gases, in air, hydrogen, helium, argon, oxygen, nitrogen, carbonic acid, iodine vapour and mercury vapour. It is not so apparent when the pressure is very low as at somewhat higher pressures. The pressure at which the effect is a maximum depends on the nature of the gas; it is higher for the lighter gases, like hydrogen and helium, than for the heavier ones. It is well developed in most gases at a pressure of about 0·1 mm. of Hg; the range of pressure over which it exists is very considerable—e.g., I have observed the effect in air over a range of pressures from 0·007 to 7 mm.

The effect is produced by light from very varied sources, by light from a mercury lamp, from an arc lamp, from sparks between metallic terminals, and from the brush discharge. What is perhaps rather surprising is that the light from an ordinary spirit lamp is very effective, as is also the light from a Bunsen burner, or from the oxyhydrogen flame, especially the light coming from the luminous cone. The light from the electric discharge through a gas at low pressure is effective if the tube containing the gas is made of quartz, but not if it is made of glass.

It is essential for the success of the experiment that the light should not pass through anything opaque to ultra-violet light before going through the gas; very thin films of glass or mica, or even of celluloid, are sufficient to destroy the effect, though it is not stopped when the light passes through plates of quartz or of rock salt several millimetres thick.

This effect does not show much evidence of selective absorption. We might, for example, have expected that when using the light from a mercury lamp the effect would be abnormally large when the ring discharge went through mercury vapour. To test this, I compared the relative effects produced by a mercury lamp and the arc light, (*a*) when the ring discharge passed through pure oxygen, and (*b*) when it passed through a mixture of oxygen and mercury vapour. The ratio of the effect in (*b*) to that in (*a*) seemed to be a little greater for the mercury lamp than for the arc, but there was not much difference.

In another experiment I used as the source of light a quartz tube filled with hydrogen; this did not seem to produce abnormally large effects when the ring discharge passed through hydrogen.

The light does not by itself ionise the gas or produce a supply of free electrons. This was proved by measuring the rate at which negative electricity escaped from a metal plate placed in the bulb *A* (Fig. 2) out of the way of the light and when no ring discharge was passing; the small leak due to residual ionisation was not increased by exposure to the light.

The effect of illumination on the ring discharge is evidently due to the absorption of ultra-violet light; such absorption seems from the experiments just described to be a frequent accompaniment of the passage of light through gases. There would be such an absorption of ultra-violet light from a variety of sources by many different gases either if these sources gave out a continuous spectrum in the ultra-violet, or if the gases had continuous absorption in that region. The energy absorbed, though not sufficient to eject electrons from the atoms or molecules of the gas, may be sufficient to put these for a time, which is not infinitesimal, into a state in which they are more easily ionised than when they are normal. Thus, as the work of ionisation is already half done, the external electric field will not be called upon to supply as much energy to the free electrons to make them ionise these excited molecules as would be required to make them ionise normal atoms or molecules; and thus the ring discharge will pass more easily when the gas is illuminated than when it is not.

EFFECT ON THE RING DISCHARGE OF IMPURITIES IN THE GAS.

I have observed some curious and suggestive effects produced by the presence of various substances in the gas. The kind of effect is illustrated by the following experiment. The discharge tube was a long cylindrical glass one, along which the solenoid carrying the alternating currents could be moved from end to end. A thin cylindrical glass rod *AC* ran along the axis of this tube, and on a portion *B* of this tube a thin layer of sulphur was deposited ; the length of this deposit was about the same as that of the solenoid. When the solenoid did not encircle the rod, or when it encircled the part of it not covered by the sulphur, the ring discharge was very bright, but it stopped when the solenoid came over the part covered by the sulphur, and started again when the solenoid was moved away from this part. When the gas in the tube was either hydrogen, air or nitrogen there was a rapid diminution in the pressure, showing that chemical combination was going on ; but the effect just described occurred also when the tube was filled with helium, when there was no diminution in the pressure.

With this rod in the tube it was found impossible, whatever might be the gas filling, to maintain the discharge for long, even though the solenoid did not encircle the sulphur. Though the discharge might be bright to begin with, it got fainter gradually, and finally disappeared. Sometimes it was found impossible to get the discharge to start again however much the spark-gap between the terminals of the induction coil was increased. If the tube was left for some hours, or if a current of gas was drawn through it for some minutes, it recovered its power of giving a luminous discharge. The effect seems to be connected with a deposit on the walls of the tube, for if the tube be heated for some minutes with a gas flame it recovers its power of giving a bright discharge. If one half the tube has been heated while the other half has been left cold, the discharge will often pass when the solenoid is over the part which has been heated, but will not do so when the solenoid is over the other portion.

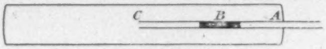

FIG. 3.

I have tested the effects produced by deposits of substances other than sulphur. Phosphorus produces a large effect, and appreciable effects are produced by deposits of silver, platinum, cadmium ; the effect, if any, produced by graphite was much smaller than with the other substances.

The effect is most marked when the pressure of the gas is only a few hundredths of a millimetre, and gets much less at considerably higher pressures ; it does not seem to depend on the nature of the gas, as it occurs in hydrogen, air, oxygen and helium. If after the discharge has been running for some time the tube is heated strongly in the neighbourhood of the place where the substance is deposited on the glass rod, it often happens that immediately after the heating the discharge will pass when the solenoid is over the patch of substance, the place where it was stopped before the heating, but that it is now stopped at some other part of the tube where it went before. This suggests that the effect is due to a deposit on the walls of the tube which is driven off when it is heated.

The following considerations help us to understand the retardation produced by substances like sulphur and phosphorus on the discharge. To start the discharge there must be some electrified bodies to begin with. In a gas in which there was nothing but neutral molecules it would be impossible to start the discharge unless the electric field were strong enough to tear the electrons out of the molecules; to do this would require a field of quite a different order of intensity from that in our discharge tubes.

It is to stray electrons that we must look for the initiation of the discharge. Under ordinary conditions there is always a certain amount of ionisation going on in the gas, but before an electron produced in this way can ionise the gas it has to get from the electric field energy greater than the ionising potential, and it has to convey this energy to an electron in a molecule before that electron can be liberated. When the electric field is not much more intense than the limit which will just produce discharge, it may be only one out of a very large number of collisions which will result in the transfer of a sufficient quantity of energy. Anything which reduces the life of the electron below the time necessary for it to make these collisions will reduce its efficiency for producing the discharge. The electron loses its power of ionisation when it unites with a molecule to form a negative ion. Loeb's experiments show that the number of collisions an electron makes with the molecules before it unites with one of them depends very greatly upon the nature of the gas. Thus, it is certainly many millions for pure nitrogen or helium, for oxygen in which the electron has a comparatively short life the number of collisions is about 30,000, while in chlorine it is less than 300. Thus, for the electronegative gases, the number of collisions is exceptionally small.

Now sulphur is a very strongly electronegative element, and presumably a sulphur molecule would be very likely to capture an electron striking against it. Thus a slight amount of sulphur vapour would result in a great diminution in the life of an ion, and therefore of its power, as an ioniser. That there is sulphur vapour produced when the discharge is going is proved by the fact that there is a visible deposit of sulphur on the walls of the tube after the discharge has been going for some time.

The effect produced by electronegative elements can be explained by the effect they produce on the life of the electron, but the electronegative elements are not the only ones which retard the discharge. Effects of the same type, though not so pronounced, are produced, for example, by certain metals, and the preceding explanation would not apply to them. It would seem as if these produced the effect, not by removing the electrons—the ionising agent—but by removing from the gas something which is more easily ionised than the normal molecule. The discharge, when it passes through the gas, besides producing chemical combinations and decompositions, may produce other modifications of the gas which only last for a short time, but which are more easily ionised than the normal molecules. In many gases—oxygen and nitrogen are conspicuous examples—the existence of such modifications is shown by bright after-glows which may persist for several seconds after the discharge has stopped. In other gases, where the duration of these modification is too short for them to show visible after-glows, their existence may be inferred from the changes which occur in the properties of the gas through which the discharge has passed. The discharge shows very marked hysteresis. If we begin with a very small spark-gap between the terminals of the induction coil and increase its length until the ring discharge begins to pass through the gas, this length will be longer than that at which the ring discharge will disappear if we start with a very long spark-gap and gradually diminish it until the discharge stops. It does not require so large an electromotive force round the circuit to maintain the discharge as it does to start it. The simplest explanation of this is that the discharge produces systems which are more easily ionised than the normal molecule. These systems may either be aggregates formed by the molecules, or they may be molecules in an "excited" state, like those by which we explained the action of light on the discharge. When the molecules are in the "excited" state they are more active chemically than the normal molecules, and they will thus tend to combine with substances present in the discharge tube. If, then, any substance which has a strong affinity for these easily ionised molecules is present in the discharge tube, it will remove them from the gas as fast as they are formed, and thus make the passage of the ring discharge more difficult. Evidence of the effect of certain metals on systems formed in the gas by the discharge is afforded by experiments on the after-glows to which we have already alluded; the presence of these metals in the discharge tube completely stops the after-glow in gases which in their absence give intense glows. The metals destroy the systems which produce the glows. These systems are probably analogous to the easily ionised ones which assist the discharge; if the metals destroy these, their presence will make the discharge more difficult.

CHEMICAL COMBINATION PRODUCED BY THE RING DISCHARGE.

A property of the electrodeless discharge which may, I think, have important applications to chemistry, is that of giving to the gas through which it passes very active chemical properties, and causing it to enter into chemical combinations and produce compounds, some of which are unexpected. The discharge puts some of the atoms and molecules of the gas into the "excited" state, and when in this state their chemical activity is enhanced. One reason for this is that their electrostatic moment is increased, and they are therefore more likely to combine with other systems than when they are in the normal state. I shall this afternoon confine myself to a few cases where the combination is between the gas through which the discharge is passing and solids placed in the tube. This combination may be detected by measuring the changes of pressure due to the passage of the discharge; the chemical combination will produce a diminution in pressure. In some cases this diminution is so rapid that the pressure falls below the value at which the discharge will pass, and to maintain the discharge a stream of gas must be kept flowing through the tube. It is necessary when investigating the combination between the gas and a solid to measure the changes of pressure both before and after the introduction of the solid into the tube, for in some cases—hydrogen especially—there is often a considerable diminution of pressure even without the introduction of other bodies into the tube. The first case I shall consider is that of the formation of oxides of magnesium higher than MgO. The ring discharge through oxygen at certain pressures produces a brilliant phosphorescence in any MgO inside the tube. When experimenting with this I found that the pressure of the oxygen fell so rapidly that the discharge stopped after a minute or two. On measuring the pressure it was found to have fallen to less than 0·01 mm. On introducing fresh oxygen the process was repeated, and a volume of gas which at atmospheric pressure would be many times the volume of the MgO in the tube was absorbed without any sign of the process coming to an end. The substance in the tube remained white, but it was evident that it was not the original substance, because it no longer showed the brilliant phosphorescence characteristic of MgO. The experiment was repeated many times with the same result, and no change of pressure in the oxygen occurred unless the MgO was in the tube. This I think showed that oxygen through which the ring discharge is passing combines with MgO to form a higher oxide. I understand that a higher oxide of magnesium is manufactured in considerable quantities, though it is not mentioned in the majority of text books on chemistry.

I found by the same method that oxygen through which the ring discharge is passing combines with CaO, and also with ZnO, to form higher oxides, though the rate of formation is not so rapid as for MgO.

I made an experiment to see if the thermionic emission from the higher oxide of calcium differed from that from CaO. Specks of the two oxides were placed close together on a platinum wire which was heated to incandescence by an electric current. The stream of electrons from the higher oxide was, I thought, visible at a slightly lower temperature than that from the CaO, but there was not a great deal of difference. Another reaction which goes on with great rapidity under the ring discharge is the combination of arsenic with hydrogen.

This chemical effect of the discharge is very intimately connected with the action of "getters" in producing high vacua. One "getter" which is much employed is phosphorus, and the rapidity with which the pressure falls when the ring discharge passes through nitrogen in the presence of phosphorus is very remarkable. It was so large that I thought it worth while to see if there would be any reduction of pressure if helium were substituted for nitrogen; the pressure of the helium showed, however, no alteration.

I think that the study of chemical reactions produced by the electrodeless discharge is a promising field of investigation, and may throw light on the mechanism of some reactions and the discovery of new compounds.

XIV.—SOME ACOUSTICAL PHENOMENA ILLUSTRATED BY RIPPLES—TRANSMISSION THROUGH QUINCKE FILTERS, CURVED CONDUITS AND VIBRATING PARTITIONS.

By A. H. DAVIS, *D.Sc.*, Physics Department, The National Physical Laboratory.

ABSTRACT.

The Paper employs ripple photographs to illustrate the action of the Quincke acoustical filter, and draws attention to the filtering action which is likely to occur at bends in curved sound conduits or in curved horns owing to resonant transverse vibration of the contained air. It also illustrates by the ripple analogy the transmission of sound through partitions, showing how the characteristics of the transmission may vary as the natural frequency of the partition is raised.

IN a previous Paper,* in which the analogy between ripples and acoustical phenomena was established, some examples were given of ripple analogues to acoustical conditions. The present Paper gives further illustrations of some interest.

I. CONDUIT WITH QUINCKE FILTER.

Photographs have been taken illustrating wave transmission through a straight conduit fitted with a model acoustical filter of the type due to Quincke. A Quincke filter consists of a resonator or a stopped pipe of adjustable length fixed to the side of a sound conduit. It prevents waves of certain definite frequencies from passing along the conduit. The frequencies suppressed are those natural to the resonator, so that a stopped pipe of (corrected) length l acts as a filter to waves of length λ if $l=\lambda/4$, $3\lambda/4$, $5\lambda/4$, etc.

In photographing ripple transmission through narrow conduits by the method of the earlier Paper, any water menisous at the boundary of the conduit gives optical distortion of the wave image on the screen. On this account a special technique was developed for the work. The boundaries of the conduit and of the filter were constructed of teak of 2 cm. thickness, and were firmly cemented in proper relative position on a sheet of plate glass. The upper surface was planed carefully to ensure that an accurately level upper surface was obtained. The model thus assembled was then laid on levelling wedges in the ripple tank, and was adjusted to be level, and thus parallel with the surface of the water. The depth of water in the tank was adjusted so that the surface of the wood was barely emergent. With this adjustment, the meniscus effect was eliminated when the water was undisturbed, but occurred to some extent as the trough of the wave passed along the conduit.

Photographs of wave transmission through the conduit were then taken for different adjustments of the length of the Quincke filter tube at the side. The frequency of the ripples, about 8 cycles per second, was the lowest at which good waves were obtained with the apparatus used. For lower frequencies the main waves were accompanied by small subsidiary wavelets, which apparently travelled

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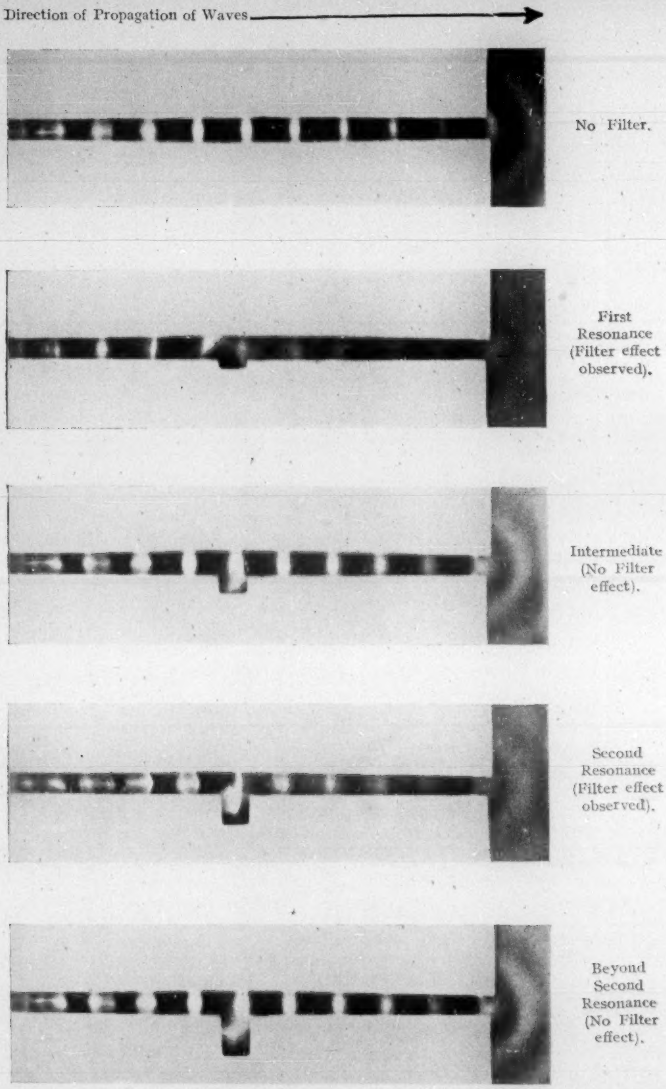
* Proc. Phys. Soc., Vol. 38, p. 234 (1926).


Direction of Propagation of Waves

No Filter.

First Resonance (Filter effect observed).

Intermediate (No Filter effect).

Second Resonance (Filter effect observed).

Beyond Second Resonance (No Filter effect).

FIG. 1.—QUINCKE FILTER EFFECTS.

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slowly and dropped back into the main waves after about 4 wavelengths had been traversed. To obtain a clear outline of the conduit, a second photograph of the boundary was taken in each case by stopping the wave generator and increasing the depth of water in the tank slightly, so that the model was completely immersed. For printing purposes a negative of the wave photograph was superimposed upon a glass positive of the outline, and prints were obtained by printing in parallel light through the two plates when accurately registered.

The results are shown in Fig. 1. The various photographs show wave transmission through a conduit in the absence of the filter, the success of a Quincke filter when tuned to resonance directly or harmonically, and the absence of filtering when the filter is mistuned.

II. Curved Conduit.

During some experiments with a curved conduit representing a section of a loud-speaker horn, it was noticed that pronounced stationary waves occurred at a certain bend, the direction of vibration being transverse to the axis of the horn. For frequencies at which such transverse resonance occurred, waves were largely suppressed in transmission through the conduit, little energy emerging from the flare. The bend was, in fact, acting as a wave filter, owing to the dissipation of wave energy in transverse vibration. The filter effect was observable over a moderate band of frequency, but when the frequency of the waves was markedly altered the transverse stationary undulation broke down, and waves emerged freely from the flare of the horn.

The scale of the experiment was such that the resonant frequency observed would correspond to a frequency of 5400 cycles per second if the horn were a moderately large one with a flare of 2½ feet wide. The suppression of this particular frequency would not be of great importance as far as the reproduction of speech or music were concerned. It was not practicable at the time to try lower frequencies ; but, on the above scale, the conduit at the bend had a width corresponding to 7 inches, for which lowest mode of transverse stationary undulation should occur with a frequency of about 900 cycles per second, well within the important range of hearing.

In connection, therefore, with curved or folded horns, the tendency for transverse vibrations of the air to be set up at bends must not be overlooked. The effects in the acoustical case, however, would not necessarily be so pronounced as in the ripple experiment, unless there were the possibility of corresponding dissipation of energy at the bend.

As regards other forms of conduit, it is interesting in this connection that, in testing voice tubes in 90° bends of 10-foot length, Eckhardt, Chrisler and Quayle* found their largest tube, which was 4 inches in diameter, transmitted a lower percentage of sound for frequencies in the region of 2000 cycles per second than for those in the regions of 1000 and of 3000 cycles. They comment on this, but are unable to offer an explanation. Now, according to the formula in Lamb's "Sound," 2nd ed., p. 263, 1926, the lowest frequency of radial vibrations of the air contained in a 4″ cylindrical tube is about 1950 cycles per second, and it seems possible that the low transmission they found in this region may have been due to the filtering effect arising from radial resonance. Consistently with this view transmission through the 2-inch tube fell off at higher frequencies, and it may be that it was the 3-inch tube, which showed no falling off, which was exceptional.

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* Bur. Stds. Tech. Papers 21, No. 333, p. 162 (1926).

III. WAVE TRANSMISSION THROUGH A PARTITION.

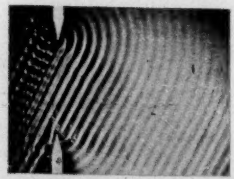
In a certain method of measuring transmission of sound through building partitions, a test partition covers an aperture in a sound-proof wall between two adjacent rooms. On one side a more or less parallel beam of sound is incident obliquely upon the partition, and on the other measurements of sound intensity are made along the continuation of the axis of the incident beam.

Now it may be shown by calculation that sound is transmitted by partitions mainly through vibration of the partition, and not by wave transmission from particle to particle in the wall. It might then be expected that vibration of the partition would result in sound being sent out at right angles to the partition rather than in the oblique direction of the incident beam, particularly if the stiffness of the partition were a predominant factor. But it must be remembered that the fundamental natural frequency of building partitions is usually very low compared with the frequency of ordinary sounds, and it is mass, and not stiffness, which is the dominant factor. In actual practice the author has frequently observed by ear that a beam of sound incident obliquely upon a partition emerges as an oblique beam from the other side, and measurements with light partitions, in collaboration with Mr. T. S. Littler, M.Sc., contribute substantially to this general observation.

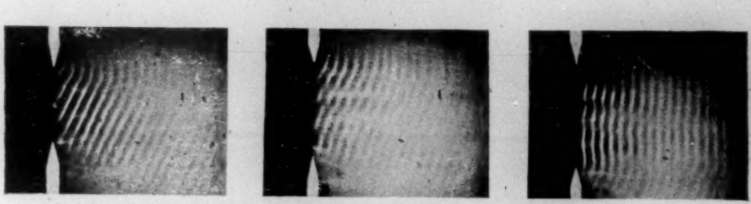
In this connection, therefore, the ripple photographs of Fig. 2 were taken some time ago as a preliminary test of the validity of a beam method for transmission tests of partitions. The first photograph shows a parallel beam of ripples incident upon the left-hand side of an aperture in a solid wall dividing the ripple tank into two parts; transmission of the oblique beam through the aperture is observed. The distance of the observing screen from the tank was chosen to give good definition for the transmitted waves, and the blurring of the incident waves arises from this choice rather than from any imperfections of the waves themselves.

The position of the aperture has been clearly marked on the photographs in white, and the pointed ends shown are in reality the plans of two small vertical vices, in which a rubber membrane could be so clamped that the lower half was in water and the upper half in air. In the second photograph a thin rubber membrane covers this aperture, but is quite slack. Transmission is now less pronounced than when the aperture is uncovered, but the oblique beam is preserved. In the next photograph the membrane has been appreciably stretched—evidenced by an increased gap width—and is approaching the condition of resonant vibration of the membrane. There is now evidence of transmission normal to the membrane as well as in the direction of the incident beam. Slightly greater stretching resulted in the fourth photograph of the series, in which the emergent beam is mainly normal to the membrane, presumably owing to resonant vibration. Stretching beyond this point resulted, as in the fifth photograph, in a return to more complicated conditions. Stretching the membrane so tightly that the aperture points lay outside the limits of the photograph resulted again in the emergence of waves normal to the membrane. This completes the series of photographs obtained.

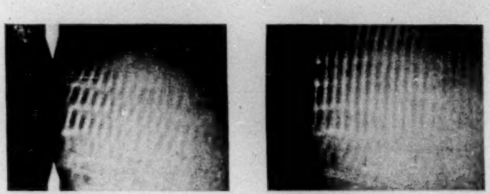
It should be stated that the photograph with the slack membrane was easily



Membrane Absent.



Membrane Slack. Approaching Resonance. Resonance.



Beyond Resonance. Membrane Tightly Stretched.

FIG. 2.—PHOTOGRAPHS SHOWING VARIATION IN WAVE TRANSMISSION THROUGH A MEMBRANOUS PARTITION AS THE NATURAL FREQUENCY OF THE MEMBRANE IS RAISED BY INCREASING ITS TENSION.

To face page 92]

obtained in the first instance, but was more difficult to repeat. The rubber membrane was very thin indeed, and, after it had been in position and thus partially immersed in the water for some time, the upper part of the membrane which emerged from the water tended to be drawn down upon the surface by meniscus forces. With the membrane slack this was troublesome, and the membrane had to be supported above the water at a number of points by means of a fine thread. Presumably, this " leaning over " of the upper part of a slack membrane stiffened its resistance to the progress of waves.

I am indebted to Mr. R. Berry, Observer in the Physics Department, for assistance with the experiments and photographs.

DISCUSSION.

Major W. S. TUCKER said that the author had shown a mastery of the difficult technique of the ripple tank. He was particularly interested in Section II of the Paper, which dealt with transverse resonance occurring at an elbow bend in a loud-speaker horn. In listening trumpets much distortion is experienced owing to the fact that the polar curve of reception of such a trumpet is a function of the frequency. Even when the source of sound is on the axis of the trumpet some distortion occurs, and this rapidly increases as the direction of the sound becomes more oblique. It is on this account desirable to turn the horn towards the source; but, apparently, when this is done another cause of distortion is introduced in the transverse resonance described by the author. The author's work on partitions seemed to indicate that resonant walls may become secondary radiators of sound; this fact must be taken into account in designing the walls of an aeroplane cabin.

Dr. A. B. WOOD referred to the author's remark that the natural frequency of building partitions is usually low. If the frequency were 100, for instance, the corresponding wave-length would be 10 ft., which was large compared with the size of a door. The author's diaphragms (Fig. 2) appeared, on the other hand, to be large compared with the wave-length. Were his inferences valid in view of this disparity?

Dr. E. T. PARIS (communicated): I would like to express my appreciation of the very interesting photographs taken by Dr. Davis which appear to me to illustrate in a very clear manner the different acoustical phenomena mentioned in his Paper. In connection with the photographs illustrating the action of a Quincke filter, I would like to ask whether the width of the resonator has any influence on the amplitude of the transmitted ripple. The reason for asking this question is that the photographs seem to show that, even when there is no filter, there is a perceptible dying away of the ripples as they pass along the main conduit. In the corresponding acoustical case it is generally assumed that there is no dissipation of acoustical energy either in the main conduit or in the resonator, and in these circumstances there should, theoretically, be complete reflection of the incident sound-waves at the resonator when the length of the latter is one-quarter of a wave-length. Also this effect is independent of the relative diameters of the main conduit and the resonator. If, however, there is attenuation of the sound-waves it is not obvious that the same result will hold good. In fact, from a rough calculation, it appears likely that the filtering effect depends to some extent on the diameter as well as the length of the resonator. I would therefore like to know whether, in Dr. Davis's experiments, anything has come out that would indicate that the width of the resonator has any influence on the filtering effect. I would also like to suggest to Dr. Davis an experiment with the ripple-tank which I think would be of interest. The experiment I have in mind is one to illustrate the reflection of sound from a porous surface, and in particular the conditions under which the reflected wave may vanish. If plane waves are incident on a plane reflecting surface in which there is a number of long narrow pores—so long and so narrow that any sound which enters them is stifled (owing to viscosity) before it reaches the ends of the pores—then there is always some angle of incidence for which the reflected wave should vanish. That is, there is complete absorption of the sound for that angle of incidence. According to the theory given by Rayleigh, the cosine of the angle of incidence for total absorption is equal to the area occupied by the pore-openings per unit area of reflecting surface. This phenomenon would be a difficult one to show with sound-waves, but it seems that it could be readily illustrated with the ripple-tank apparatus which Dr. Davis has at his disposal. All that would be required is a straight barrier to reflect the ripples, with a number of canals leading off at right angles. If different angles of incidence were tried, one should be found at which there is no reflected wave. I think the experiment, if it can be performed, would be of considerable interest.

AUTHOR'S reply: In reply to Dr. Wood, the scale of the ripples in Fig. 2 is suitable for showing the phenomena which may occur in transmission of sound through building partitions. The wave-length of the ripples (1·4 cm.) relative to the size of the aperture (about 14 cm.) corresponds to a sound of frequency of 450 cycles per second incident upon a 25 ft. partition, or to one of about 2000 cycles incident upon a 6 ft. partition. In the earlier photographs the natural frequency of the partition is much lower than the ripple frequency; a condition which corresponds, within the limits of the analogy, to the case in actual practice where the gravest mode of building partitions (say 100 per second or much lower) lies well below the frequency of the majority of incident sounds. The beam of ripples in these conditions is seen to pass straight through—a confirmation of my experience that an obliquely incident beam of sound passes through a building partition without change of direction. The later photographs show how different things would be if the partition had an important natural frequency in the region of the frequency of the incident sound. In reply to Dr. Paris, there is relatively greater dissipation of energy in a ripple conduit than in an acoustical one, and he is correct in his deduction that the width of the Quincke filter did influence the amplitude of the transmitted ripples. The width of resonator (2 cm.) shown in the photographs was the minimum with which a reasonable demonstration of the filter effect could be obtained with the conduit used. It will be a pleasure to try, on some convenient occasion, the experiment on wave reflection from an absorbent wall that Dr. Paris suggests. In response to suggestions by Mr. Albert Campbell and Capt. C. W. Hume, a short definition of the Quincke filter has been included in the Paper.

XV.—A SIMPLE GRAPHIC METHOD FOR THE DETERMINATION OF GALVANOMETER AND FLUXMETER CONSTANTS, WITH A NOTE ON THE MEASUREMENT OF INTENSE MAGNETIC FIELDS.

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ABSTRACT.

This Paper deals with methods for evaluating the principal construction constants of moving coil instruments by the graphical treatment of observations of logarithmic decrement and its variation with circuit resistance. It also discusses the application of these methods to the measurement of intense magnetic fields.

GALVANOMETER.—GENERAL THEORY.

THE behaviour of a galvanometer may be defined in terms of its operation constants*—that is, in terms of its period and of the half-dozen sensitivities which it possesses. These operation constants in turn depend on the intrinsic or construction constants of the instrument, and it is the principal object of this note to describe a simple graphic method for their determination. A secondary object is pædagogic in character. From the teacher's point of view a galvanometer illustrates admirably the dynamics of a vibrating system, and a little drill of the type to be described greatly assists students to comprehend the capabilities and limitations of the instruments which they may be called on to use. The galvanometer designer is interested in the magnitude of the construction constants and in their relation to the operation constants. The construction constants with which we are concerned are the moment of inertia (K) of the deflected system, the displacing couple per unit current† (Q), the open circuit damping coefficient (p_0), the restoring couple per unit displacement (μ), and the resistance (G) and self-inductance (L) of the galvanometer. The effects of self-induction are commonly omitted, and the method we are about to describe has the advantage of showing whether or not the omission is justifiable.

Writing the equation of motion of the coil in the form

$$Q\gamma - p_0\dot{\theta} - \mu\theta = K\ddot{\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where θ is measured in the usual way and γ is the current at any instant, we have, where N is the flux through the coil in its displaced position, and e is the external electromotive force acting on the circuit, whose *total* resistance is R.

$$N = Q \sin\theta - L\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1{\cdot}1)$$

$$\gamma = (e - \dot{N})/R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1{\cdot}2)$$

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* Wenner, Bull. Bureau Stand. 13, No. 2 (1916).

† We restrict the discussion to underdamped moving-coil instruments with critical damping as a limit. Q is thus the " nAH " of elementary theory.

from (1), (1·1) and (1·2) we have, if the self-inductance is small.*

$$(K+LQ^2/R^2)\ddot{\theta}+(p_0+Q^2/R)\dot{\theta}+\mu\theta=Qe/R \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

If no external electromotive force is acting, (2) may be written

$$\ddot{\theta}+p\dot{\theta}+q\theta=0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

where

$$p=(p_0+Q^2/R)\div(K+LQ^2/R^2),\ q=\mu\div(K+LQ^2/R^2) \; . \; . \; . \; . \; . \quad (4)$$

GALVANOMETER.—SELF-INDUCTANCE NEGLIGIBLE.

If we suppose that the self-inductance may be neglected we have at once for an underdamped system, remembering that $4n\lambda=p$,

$$4R(n\lambda-n_0\lambda_0)=Q^2/K, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

where λ is the logarithmic decrement corresponding to a frequency n, and λ_0 and n_0 are the corresponding quantities on open circuit. It is not uncommon to assume that n is independent of λ, and keeps its open circuit value n_0, in which circumstances (5) becomes

$$Q^2=4KRn_0(\lambda-\lambda_0), \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

an equation which gives the justification of the well-known rule that λ is a linear function of the reciprocal of R.

Keeping to the general case and remembering that $q=4n^2(\pi^2+\lambda^2)$ we have, substituting for n in (5),

$$2q^{\frac{1}{2}}\times R\lambda/\sqrt{(\pi^2+\lambda^2)}=4n_0\lambda_0R+Q^2/K, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

and we see that a plot of $R\lambda/\sqrt{(\pi^2+\lambda^2)}$ as y against R as x gives a straight line whose slope and intercept determine Q^2/K and q(or μ/K). Whether L is negligible or not, we have on open circuit

$$4n_0\lambda_0=p_0/K\ ;\ 4n_0^2(\pi^2+\lambda_0^2)=\mu/K \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

which determines p_0/K and gives a check determination of μ/K. It remains to determine K, and this is done by taking periods and logarithmic decrements for the coil on open circuit, first unloaded (n_0, λ_0), second, when loaded (N_0, Λ_0) with a body of known moment of inertia K_1.† We then have, taking account of the fact that the loading may alter the damping coefficient p_0,

$$(K+K_1)/K=n_0^2(\pi^2+\lambda_0^2)/N_0^2(\pi^2+\Lambda_0^2) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

K being thus determined, the other intrinsic constants p_0, μ and Q follow at once from a knowledge of the variation of logarithmic decrement with total circuit resistance. It is of course assumed that G, the galvanometer resistance, has been independently determined.

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* If this restriction be removed, the equation of motion becomes a linear third-order differential equation, the development of which, in a form not so convenient for our purpose, follows the lines laid down in Maxwell, "Elec. and Mag.," II, 405 seqq.

† A glass (not metal) rod is suitable.

An important operation constant is R_1, the total circuit resistance which just produces critical damping. This may be determined directly by noting the resistance for which the motion just ceases to be oscillatory, or by making use of the property that the time of return to the zero is a minimum when the damping is critical. Whichever of these methods is adopted, the value obtained for R_1 depends considerably on the judgment of the individual observer, and a more reliable value is that deduced from the experimentally determined intrinsic constants. This may be done by means of equation (3), which for critical damping gives

$$R_1=Q^2/(2\sqrt{\mu K}-p_0)=Q^2/K\div(2\sqrt{\mu}/\sqrt{K}-p_0/K) \quad . \quad . \quad . \quad . \quad (10)$$

all the quantities on the right-hand side of equation (10) being known. More neatly, we may read it off at once from the straight-line graph of equation (7). For when $R=R_1$ the logarithmic decrement becomes infinite, and $\lambda/\sqrt{(\pi^2+\lambda^2)}$ becomes unity. Hence the critical value of R is the abscissa of that point for which y is equal to x, or in terms of the slope (m) and the intercept (c) of the straight line

$$R_1=c/(1-m) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

It may be noted in passing that one of the disadvantages of the approximate equation (6) is that it cannot be used in this way to determine the critical resistance.

GALVANOMETER.—SELF-INDUCTANCE APPRECIABLE.

Should the graph of equation (7) show any pronounced curvature, it may be due to the fact that the self-inductance of the galvanometer coil is not negligible.

Proceeding as before, we still have

$$4n\lambda=p,\ 4n^2(\pi^2+\lambda^2)=q,$$

where p and q now have the values given in (4).

Eliminating n we easily obtain

$$\frac{2\lambda}{\sqrt{(\pi^2+\lambda^2)}}\cdot\sqrt{\left(\frac{\mu}{K}\right)}\cdot\left(1+\frac{1}{2}\frac{LQ^2}{KR^2}\right)=4n_0\lambda_0+\frac{Q^2}{RK} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Treating μ/K as known from equation (8) we may put

$$\frac{2\lambda}{\sqrt{(\pi^2+\lambda^2)}}\cdot\sqrt{\left(\frac{\mu}{K}\right)}-4n_0\lambda_0=Y,\ \text{and}\ \frac{2\lambda}{\sqrt{(\pi^2+\lambda^2)}}\sqrt{\left(\frac{\mu}{K}\right)}=X,$$

and we have

$$RY=-\frac{1}{2}\frac{LQ^2}{K}\cdot\frac{X}{R}+\frac{Q^2}{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The test of the negligibility of L is therefore the constancy of the product RY. If this product shows signs of systematic variation, a plot of RY against X/R will determine a straight line from the slope and intercept of which both L and Q^2/K may be deduced.

FLUXMETER.—SELF-INDUCTANCE AND TORSIONAL CONTROL NEGLIGIBLE.

The simple theory of the fluxmeter gives us, for an impulsive couple,

$$\frac{Q}{R}\int e\,.\,dt=\frac{QF}{R}=K\omega_0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

where ω_0 is the initial angular velocity and F the flux cut. The equation of motion of the coil being

$$\ddot{\theta}+p\,\dot{\theta}+q\theta=0$$

we obtain at once, if q be negligible and α be the amplitude of the throw,

$$p\alpha=\omega_0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

or

$$(p_0+Q^2/R)\alpha=K\omega_0=QF/R \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

Writing this in the form

$$R\alpha=-Q^2\alpha/p_0+QF/p_0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

we see that if we vary the total circuit resistance and employ a constant known flux F, a plot of $R\alpha$ as y against α as x determines the constants Q and p_0.

FLUXMETER.—SELF-INDUCTANCE APPRECIABLE, TORSIONAL CONTROL NEGLIGIBLE.

The equation of motion is still

$$\ddot{\theta}+p\dot{\theta}=0, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

where p is now given by

$$p=(p_0+Q^2/R)\div(K+LQ^2/R^2).$$

Substituting in (17) we obtain on reduction

$$p_0R\alpha+Q^2\alpha=QF+LQ^3F/KR^2.$$

This is a four-term equation, but we can reduce it to a three-term form by taking the origin at some *known* point $(R_0,\ \alpha_0)$ on the curve between R and α considered as variables, F being constant. This gives us

$$p_0(R\alpha-R_0\alpha_0)=Q^2(\alpha_0-\alpha)+LFQ^3/K\,.\left(\frac{1}{R^2}-\frac{1}{R_0^{\,2}}\right) \quad . \; . \; . \; . \; . \; . \; . \quad (21)$$

which, divided throughout by $(\alpha_0-\alpha)$, obviously gives a rectilinear graph.

FLUXMETER.—TORSIONAL CONTROL APPRECIABLE, SELF-INDUCTANCE NEGLIGIBLE.

If we take the equation of motion as

$$\theta+p\dot{\theta}+q\theta=0$$

retaining the q term, which is *ex hypothesi* small, so that the system is heavily overdamped, we may write the solution as

$$\theta=exp.\ (-pt/2)[A\ exp.\ \sqrt{(p^2/4-q)}\,.\,t+B\ exp.\ -\sqrt{(p^2/4-q)}\,.\,t]$$

Introducing the conditions that when $t=0$, $\theta=0$ and $\dot{\theta}=\omega_0$, and that when $\theta=\alpha$, $\dot{\theta}=0$, we readily find that

$$q\alpha^2=\omega_0^{\,2}\left[\frac{p/2-\sqrt{(p^2/4-q)}}{p/2+\sqrt{(p^2/4-q)}}\right]^{p/2\sqrt{(p^2/4-q)}} \quad . \; . \; . \; . \; . \; . \quad (23)$$

which, remembering that q is small in comparison with $p^2/4$, may be written

$$(p^2-q)a^2=\omega_0^2\left(\frac{q}{p^2-q}\right)^{2q/p^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

This reduces, as it should, to (17) when q is negligible.

We see that for a constant circuit resistance ω_0 and therefore F remains proportional to a, but the constants are considerably more difficult to determine.

MEASUREMENT OF INTENSE MAGNETIC FIELDS.

A knowledge of Q ($\equiv nAH$) obviously affords a means for determining H, the field in which the coil swings, if the effective area of the coil be known. This method has been used by Klopsteg,[*] who, however, was content to work with the approximate equation (6). This restriction seems to be quite unnecessary, as the more exact equation (7) is just as easy to handle, and involves no restriction of the range over which logarithmic decrements may be taken. Klopsteg determines his moment of inertia directly. This is fairly easily done with a special coil, but in the handling of galvanometer coils occasions arise when a direct determination of K is not easy. In these circumstances it is much more convenient to determine Q by making a direct determination of the current sensitivity (S_e) or the voltage sensitivity (S_e) in a circuit of total resistance R. By putting $\ddot{\theta}=\dot{\theta}=0$ in equation (2) we see that

$$Se=Q/\mu R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Let us now write this in terms of the slope (m) and of the intercept (c) of the rectilinear graph of equation (7). We then have

$$Q=\frac{mc}{Rn_0\lambda_0S_e}=\frac{mc}{n_0\lambda_0S_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

This method of determining Q provides a very simple means for measuring strong magnetic fields, given a coil of known effective area. The necessary operations are quite straightforward, comprising as they do a measurement of sensitivity and of logarithmic decrement of the system for various external resistances.

MEASUREMENTS.

We proceed now to give the details of measurements made on certain moving-coil instruments.

Galvanometer A.—(Maker's stamp absent. Ordinary rectangular coil galvanometer.)

$R\times10^{-2}$:	5	7	10	20	30	40	50	60	80	100
$\lambda\times10^2$:	53·02	38·22	27·83	15·77	11·59	9·61	8·37	7·65	6·60	6·01
RY[†] :	112·0	110·6	111·0	112·0	110·4	110·9	110·1	112·1	111·0	111·8

Resistance (G) of galvanometer = 29·4 ohms.

$$T_0=8''\cdot70\,; \quad \lambda_0=0\cdot0358$$

Moment of inertia of vibrator employed $(K_1)=1\cdot874$ gm.cm.^{2}

$$N_0=1/14\cdot37\,; \quad \Lambda_0=0\cdot09613.$$

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[*] Physical Review, 2, 390 (1913).

[†] Vide equation (15).

From equation (8)

$$\frac{K+K_1}{K}=\frac{n_0^2}{N_0^2}\cdot\frac{\pi^2+\lambda_0^2}{\pi^2+\Lambda_0^2}=\left(\frac{14\cdot37}{8\cdot70}\right)^2\left(\frac{9\cdot871}{9\cdot966}\right)=2\cdot701$$

Hence $K=1\cdot102$ gm.cm.^{2}

$$\mu/K=4n_0^2(\pi^2+\lambda_0^2)=0\cdot5217 \text{ sec.}^{-2}$$

From graph (*see* Fig. 1)

$$\text{Intercept} \quad =76\cdot9=\frac{Q^2}{K\,.\,2\sqrt{q}}$$

$$\text{Slope} \quad =114\times10^{-4}=2n_0\lambda_0/\sqrt{q}$$

Hence $q=\mu/K=0\cdot5214$ (alternative value),

and* $Q^2/K \quad =76\cdot9\times2\times\sqrt{(0\cdot5214)}=111\cdot0.$

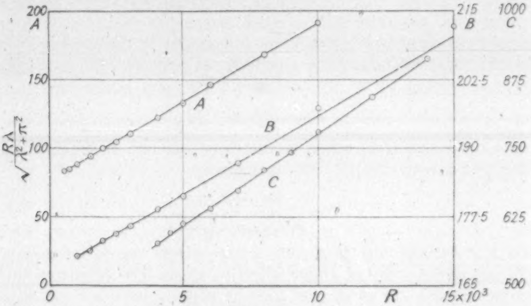

FIG. 1.

Hence for the constants of galvanometer A,

$$Q=\sqrt{(111\cdot0\times1\cdot102)}=11\cdot06 \text{ (arbitrary units)}$$

$$\mu=0\cdot5216\times1\cdot102=0\cdot5748 \text{ gm.cm.}^2\text{ sec.}^{-2}$$

$$p_0=4n_0\lambda_0K=0\cdot01814 \text{ gm.cm.}^2\text{ sec.}^{-1}$$

$$K=1\cdot102 \text{ gm.cm.}^2$$

$$R_1=c/(1-m)=76\cdot9(1+0\cdot01)=77\cdot7 \text{ ohms.}$$

Galvanometer C, by Nalder Bros. & Co., Westminster, No. 26806. (The coil is wound in two parts which for our purposes were connected in series.)

$R\times10^{-2}$	:	40	45	50	60	70	80	90	100	120	140
$\lambda\times10^2$	:	45·75	41·95	38·78	33·71	30·35	28·00	25·97	24·58	22·15	20·54
RY	:	1132	1138	1139	1122	1119	1129	1121	1134	1124	1128

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* Q^2/K is expressed in arbitrary units. The resistances concerned are measured in ohms.

Resistance (G) of galvanometer $=217$ ohms.

$$T_0=4''{\cdot}90\,;\quad \lambda_0=0{\cdot}10637$$

Moment of inertia of vibrator employed $(K_1)=2{\cdot}086$ gm.cm.^{2}

$$N_0=1/5''{\cdot}88\,;\quad \Lambda_0=0{\cdot}0999$$

From equation (8)

$$\frac{K+K_1}{K}=\frac{n_0{}^2}{N_0{}^2}\cdot\frac{\pi^2+\lambda_0{}^2}{\pi^2+\Lambda_0{}^2}=\left(\frac{5{\cdot}88}{4{\cdot}90}\right)^2\left(\frac{9{\cdot}881}{9{\cdot}880}\right)=1{\cdot}440$$

Hence $K=4{\cdot}742$ gm.cm.^{2}

$$\mu/K=4n_0{}^2(\pi^2+\lambda_0{}^2)=1{\cdot}646 \text{ sec.}^{-2}$$

From graph (*see* Fig. 1)

$$\text{Intercept}=440{\cdot}6=\frac{Q^2}{K\,.\,2\sqrt{q}}$$

$$\text{Slope}\quad=3375\times10^{-5}=2n_0\lambda_0/\sqrt{q}$$

Hence $q=\mu/K=1{\cdot}656$ (alternative value),

and $Q^2/K\quad=440{\cdot}6\times2\times\sqrt{(1{\cdot}656)}=1134$

Hence for the constants of galvanometer B,

$$Q=\sqrt{(1134\times4{\cdot}742)}=73{\cdot}33 \text{ (arbitrary units)}$$

$$\mu=1{\cdot}656\times4{\cdot}742=7{\cdot}830 \text{ gm.cm.}^2\text{ sec.}^{-2}$$

$$p_0=4n_0\lambda_0K=0{\cdot}4116 \text{ gm.cm.}^2\text{ sec.}^{-1}$$

$$K=4{\cdot}742 \text{ gm.cm.}^2$$

$$R_1=c/(1-m)=440{\cdot}6(1+0{\cdot}03)=454 \text{ ohms.}$$

Galvanometer B, by Crompton & Co. (This galvanometer has a circular coil and is fitted with a gold bifilar suspension.)

$R\times10^{-2}$	:	10	15	20	25	30	40	50	70	100	150
$\lambda\times10^2$	:	54·30	36·14	27·29	21·98	18·44	14·07	11·40	8·405	6·206	4·444
RY	:	397·0	396·2	396·6	396·5	396·3	396·4	395·6	396·0	399·6	400·4

Resistance (G) of galvanometer $=92$ ohms.

$$T_0=5''{\cdot}30\,;\quad \lambda_0=0{\cdot}009095$$

Moment of inertia of circular coil $=1{\cdot}851$ gm.cm.^{2} (calculated from its mass and dimensions)

$$\mu/K=4n_0{}^2(\pi^2+\lambda_0{}^2)=1{\cdot}406 \text{ sec.}^{-2}$$

From graph (*see* Fig. 1)

$$\text{Intercept} = \quad 167{\cdot}3 = \frac{Q^2}{K\,.\,2\sqrt{q}}$$

$$\text{Slope} \quad = 0{\cdot}00288 = 2n_0\lambda_0/\sqrt{q}$$

Hence $\quad q = \mu/K = 1{\cdot}420$ (alternative value),

and $\quad Q^2/K \quad = 167{\cdot}3 \times 2 \times \sqrt{(1{\cdot}420)} = 398{\cdot}7$

Hence for the constants of galvanometer C,

$$Q = \sqrt{(398{\cdot}7 \times 1{\cdot}851)} = 27{\cdot}17 \text{ (arbitrary units)}$$

$$\mu = 1{\cdot}413 \times 1{\cdot}851 = 2{\cdot}615 \text{ gm.cm.}^2 \text{ sec.}^2$$

$$p_0 = 4n_0\lambda_0 K = 0{\cdot}01271 \text{ gm.cm.}^2 \text{ sec.}^{-1}$$

$$K = 1{\cdot}851 \text{ gm.cm.}^2$$

$$R_1 = c/(1-m) = 167{\cdot}3(1 + {\cdot}003) = 167{\cdot}8 \text{ ohms.}$$

FLUXMETERS.

Fluxmeters are rather more difficult to handle. The absence, or almost complete absence, of any torsional control brings forward much more prominently any small variation in other parts of the system. In particular, the experiments which we have made emphasise the importance of ensuring that the deflections of the instrument in opposite directions shall be equal, and that, in measuring the flux due to a change in a secondary circuit, deflections at make and break shall be sensibly equal—always assuming that the time constant of the primary has a reasonable value.

We feel that recent minor changes in fluxmeter design are retrograde in character. We refer specially to those instruments in which, in order to obtain a more open scale, the zero has been moved from the middle to one side of the scale. We would further emphasise the necessity for the careful examination of the regularity of the permanent field in which the coil swings.

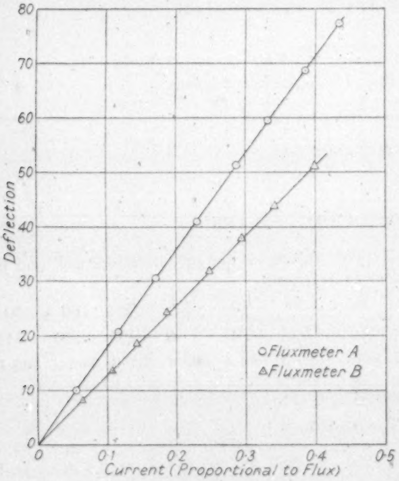

FIG. 2.

In the experiments made two fluxmeters of different make were employed.

The fluxes used were not measured absolutely, as the flux was obtained from a short coil of inductance about 1 henry and resistance 16 ohms, on the outside of which was wound a secondary. The current in the primary was determined by measuring, on a Tinsley potentiometer, the P.D. at the ends of a standard resistance included in the circuit.

The extent of the throw in any one instance was measured from the position of rest taken up by the coil just before the secondary was made.

Two types of experiment were carried out—one in which the flux was varied and was taken as proportional to the current in the primary, the other in which the flux was kept constant, and the resistance in the fluxmeter circuit varied.

The graphs obtained are approximately in agreement with the formulæ given, but slight curvatures are in some instances shown. With one instrument, left and right and make and break swings are in fairly good agreement; with the other the differences are quite distinct.

Fluxmeter A (1 division = 10,000 maxwells).

This instrument showed distinct differences between the readings to left and right, and smaller but appreciable differences between the readings at make and break. The mean readings, however, lay on a satisfactory straight line (*see* Fig. 2 *A*).

	Deflection.					Current.
	Left.		Right.		Mean.	(Proportional to flux.)
Make	9·9		10·4			
		9·9		10·4	10·1	0·056
Break	9·8		10·3			
Make	20·5		21·1			
		20·5		21·0	20·8	0·116
Break	20·5		20·9			
Make	30·3		31·1			
		30·3		31·0	30·6	0·171
Break	30·2		30·9			
Make	40·0		41·7			
		40·5		41·6	41·0	0·231
Break	40·9		41·5			
Make	50·3		52·3			
		50·3		52·4	51·3	0·288
Break	50·2		52·4			
Make	58·2		60·9			
		58·4		60·5	59·5	0·333
Break	58·6		60·1			
Make	67·0		70·3			
		67·3		70·2	68·7	0·386
Break	67·5		70·1			
Make	75·3		79·1			
		75·6		79·0	77·3	0·435
Break	75·9		78·9			

Fluxmeter B (1 division $=14{,}000$ maxwells).

With this instrument the readings left and right and at make and break were much more consistent. The plot of flux against deflection showed a slight but appreciable curvature (*see* Fig. 2 *B*).

	Deflection.					Current.
	Left.		Right.		Mean.	(Proportional to flux.)
Make	8·2		8·3			
		8·3		8·3	8·3	0·065
Break	8·3		8·3			
Make	13·6		13·6			
		13·6		13·7	13·6	0·107
Break	13·5		13·7			
Make	18·4		18·5			
		18·5		18·5	18·5	0·144
Break	18·5		18·5			
Make	24·3		24·5			
		24·3		24·5	24·4	0·189
Break	24·3		24·5			
Make	32·1		31·9			
		32·0		31·8	31·9	0·248
Break	31·8		31·7			
Make	38·0		38·0			
		38·0		37·9	37·9	0·295
Break	37·9		37·7			
Make	44·2		43·6			
		44·1		43·7	43·9	0·342
Break	44·0		43·7			
Make	51·2		51·0			
		51·3		51·1	51·2	0·400
Break	51·3		51·1			

This lack of linearity between flux and deflection was found to hold for the much smaller fluxes necessitating deflection measurements made by means of a lamp and scale.

With fluxmeter *B* the total resistance (R) in the fluxmeter circuit was varied and a plot of $1/a$ against R was made for a constant flux. Here, again, anomalies were apparent, for while reciprocal deflections to the left or the mean reciprocal deflections plotted against R showed distinct curvature, reciprocal deflections to the right when plotted against R lay on a good straight line (Fig. 3).

Elementary fluxmeter theory assumes the *constancy of the field in which the coil swings* and the *absence of torsional control.*

If slight torsional control is present then, as we have seen, deflections remain proportional to flux in the absence of *any initial torsion.* The importance of these conditions in an instrument where controls are small, but not zero, has induced us to criticise the recent step taken in fluxmeter design of moving the zero to one end of the scale, thus preventing the user of the instrument from testing the equality of right and left deflections. Where discrepancies of four or five per cent. may be introduced, it is very advisable that means for checking these errors or for eliminating them may be provided.

Our thanks are due and are tendered to Prof. C. H. Lees for the interest he has taken in this work, and for the facilities which he has given for the carrying out of the experiments; and to Prof. J. T. MacGregor-Morris our thanks are also offered for the loan of some of the instruments used.

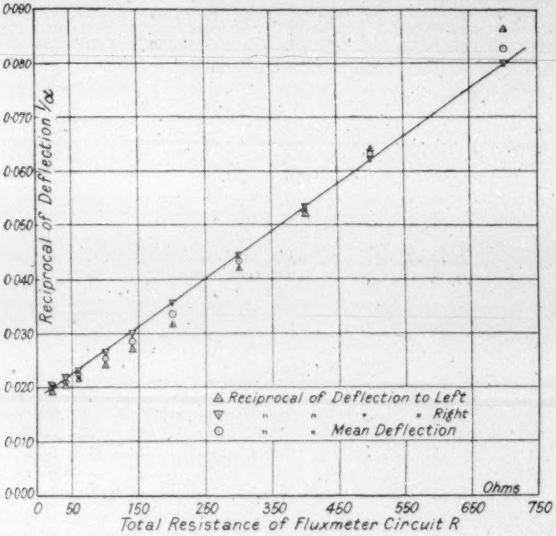

FIG. 3.

DISCUSSION.

In presenting the Paper, Dr. FERGUSON suggested that instrument makers should revert to the practice of locating the zero at the centre of the fluxmeter scale, as the sensitivity in one direction often differs slightly from that in the other; and also that they should mark on instruments the values of constructional constants such as the number of turns in a coil and its area.

Dr. D. OWEN said that the Paper would be much appreciated by those making a study of the principles underlying the use of that important instrument, the moving-coil galvanometer. The assumption, however, of constancy of field for all positions of the coil was scarcely justifiable in an accurate treatment. It would thus be of interest to learn how far the values of constants as deduced by the methods of the Paper agreed with direct calculation, wherever this was possible. In regard to the fluxmeter, there could be no doubt of appreciable departure from constancy of field, and this fact involved a calibration of the scale for accurate work. It was much to be desired, on this and other grounds, that the fluxmeter should be provided with a means of zero adjustment. He expressed the hope that the makers of these instruments would turn their attention to the remedy of what is a serious defect.

Dr. Ferguson, in reply, said that if necessary it would be easy to insert in equation (1) a term in $dQ/d\theta$ to account for non-uniformity of the field, but the linearity of the graphs relating to galvanometers showed that this was unnecessary. In the case of fluxmeters the curvature of the graphs obtained indicated the presence of anomalies, but the curvature was definite and reproducible. The advantage of being able to read the throw in both directions was that this procedure indicated whether anomalies were present or not.

XVI.—THE APPLICATION OF ELECTRICAL RESISTANCE MEASUREMENTS TO THE STUDY OF THE ATMOSPHERIC CORROSION OF METALS.

By JOHN CHRISTOPHER HUDSON.

Received October 20, 1927.

(*Communicated by* W. G. BICKLEY, M.Sc.)

ABSTRACT.

This Paper describes in detail an experimental procedure for quantitative field tests on the atmospheric corrosion of metals, which is based on the determination of the change in the electrical resistance of wire specimens produced on exposure. The method is capable of great accuracy, and the resistance changes produced in duplicate specimens agree to within 5 per cent. The validity of the results has been established, in the case of the metals investigated, by correlation with the corresponding weight changes of the specimens. In the case of copper, it has been found that the percentage resistance change due to corrosion is inversely proportional to the diameter of the wire ; it is thus possible to use the method as an " acceleration test " by conducting experiments on relatively thin wires, which are appreciably affected by corrosion within a few weeks.

CONTENTS.

I. Introduction.
II. Experimental Procedure :
  Brief description of the experimental methods.
  Method of fixing the terminals.
  Method of measurement.
  The experimental bridge.
  Methods of calculating and expressing the results.
III. Experimental Results :
  Short period corrosion tests on typical non-ferrous materials (Winter 1926-27).
  The effect of the length of the coil and of the gauge of the wire on the resistance changes produced by corrosion.
  Correlation of weight increment and resistance change (Stevenson screen exposure).
  Correlation of change in weight and resistance change for fully exposed specimens.
IV. Conclusion.
  Acknowledgments.
V. Appendix.
  Design and calibration of the stands used for outdoor exposure.

I. INTRODUCTION.

THE quantitative study of the corrosion of metals—by no means an easy task in the laboratory—becomes especially difficult when tests are to be made under actual service conditions. The difficulty is increased by the limited accuracy of the usual methods of measurement when applied to field tests, particularly in an investigation on atmospheric corrosion, where the rate of attack is, as a rule, much less than in other cases of practical importance. Thus, a direct determination of the loss in weight of a specimen may be sufficiently accurate for practical purposes when a marked change occurs, as in the corrosion of iron by sea water. On the other hand, the losses in weight of non-ferrous materials on exposure to the atmosphere for a reasonable period are probably too small to render the results of very great value.* The first necessity, therefore, when the author was appointed in April, 1925, by the Atmospheric Corrosion Committee of the British Non-Ferrous Metals Research Association, to undertake systematic field tests on corrosion, was to devise much more accurate methods of measurement.

A method has been developed in which the increase in the resistance of a metallic conductor is taken as an index of its corrosion. The first observations of this type were made by Wilson,† who has followed the changes in the electrical and the mechanical properties of a number of aluminium alloys on exposure to the London atmosphere for a period of over twenty years. Although Wilson was not concerned with the use of the resistance determinations as a standard corrosion test, his results showed that measurable resistance changes were produced, and after two years' exposure the resistance of copper, aluminium and duralumin wires, $\frac{1}{8}''$ in diameter, had increased by 2·0, 4·4 and 8·2 per cent. respectively.‡

The resistance method for studying corrosion is based on the fact that, as a result of corrosion, metal is removed from the specimen or replaced *in situ* by a non-conducting corrosion product.§ Consequently the effective conducting cross-section is diminished. Since the resulting percentage increase in the resistance of the specimen is equal to the percentage diminution in the cross-section, the method has the advantage of giving results that are immediately comparable for different materials. (The usual loss in weight test does not, unless the density of the metal is taken into account.) Apart from the accuracy with which the measurements can be made, the method has the further advantages that the tests can be conducted upon specimens of a convenient size, and that continuous observations may be made on the same specimen. These two assets are of especial value in the case of field tests.

It has been found that, if the tests are made on wires of a suitable diameter, measurable changes in resistance are produced after a few weeks' exposure, which are accurately reproducible from one specimen to another. Moreover, the results obtained are in reasonably good agreement with the changes observed in other properties of the specimens, more particularly with their change in weight. The relation between these two quantities may be deduced as follows :—

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* The whole technique of gravimetric tests on atmospheric corrosion has been considerably improved by Vernon (Trans. Faraday Soc., 1924, 19, 839; 1927, 23, 113). In his weight increment method, the progress of corrosion is followed by determining the increase in weight of the specimens at convenient intervals. This procedure is particularly suitable for the study of the tarnish films developed in indoor atmospheres, and the accuracy obtainable is well demonstrated by the fact that he was able to follow the development of tarnish films on lead and aluminium in cases where the final weight increment, after 25 days' exposure, was as low as 0·28 milligrammes. The method can be used outdoors if the specimens are exposed in a Stevenson screen, but when the specimens are fully exposed the results are complicated owing to the removal of the corrosion product by the rain. Vernon (l.c.) overcame this difficulty by making simultaneous determinations of the metal removed in the rain water. Unfortunately, this procedure is too laborious to be applied on a large scale.

† E. Wilson, Proc. Phys. Soc., 39, 15 (1926); J. Inst. El. Eng., 63, 1, 108 (1925), and numerous other Papers.

‡ E. Wilson, B.A. Reports, p. 606 (1913).

§ The specific resistance of the corrosion product of the more common metals was determined by Bruhl (J. Inst. Met., 1911, 6, 279). The minimum value observed was about 1500 ohms/cm.^{3} for moist cuprous oxide; the figures for the dry products were much greater and ranged from 0·1 to 10 megohms/cm^3.

The resistance of a wire of radius r is given by

$$R=k/r^2, \text{ where } k \text{ is a constant.}$$

By differentiation, $$100\, dR/R=-200\, dr/r.$$

The percentage increase in resistance ΔR, due to the corrosion of a thin uniform film of thickness dt on the surface of the specimen is thus

$$\Delta R=200\, dt/r \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

The weight of metal corroded $$W=Adt\,.\,D, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where A is the total area of the specimen and D the density of the metal.

From (1) and (2), $$dt=r\,.\,\Delta r/200=W/AD$$

Hence $$\Delta r=200\, W/A\,.\,D\,.\,r \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

or $$W=r\,.\,\Delta r\,.\,AD/200 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

By means of the last two equations the resistance change can be calculated from the loss in weight, and *vice versa*. The values so obtained are found to agree fairly well with the observed results; this fact supports the view that the resistance change is quantitatively proportional to the corrosion.

The experimental procedure for making corrosion tests by the electrical resistance method will now be described. It has already been successfully applied to most of the common non-ferrous metals, including copper, brass, bronze, nickel, nickel-silver, the copper-nickel alloys, nickel-chromium and aluminium.

II. Experimental Procedure.

Brief Description of the Experimental Method.

The tests are made on short lengths of wire wound into a coil of convenient size. In the earliest experiments the wires were 0·020 in. in diameter (24 S.W.G.); but, although the sensitiveness of the measurements is reduced by using thicker wires,* any irregularities due to pitting are correspondingly diminished, and it is therefore desirable to use the thickest wire that will give a measurable change. The gauge of the specimens has consequently been increased as this was warranted by the experimental results, and 18 S.W.G. wires (0·048 in.) have been used for the most recent work. This is approximately the gauge of a 40 lbs. to the mile telegraph wire.

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* With the apparatus available, the sensitiveness is limited by the errors in the contact resistances, which are independent of the gauge of the wire. Since, however, the resistance of the specimen is inversely proportional to the cross-section (for a constant length of wire), and, as will be seen later, the change produced by corrosion varies inversely as the diameter, the experimental error is inversely proportional to the cube of the diameter, and the limiting thickness of wire is soon reached.

The wire is supplied by the manufacturers in the annealed condition, and as free as possible from surface defects. The specimens are made by winding the wire into coils ¾ in. in diameter, and cutting these up into lengths of 36 turns, or about 200 cms. of wire. Terminals of the same material are then attached to the ends of the wire by the method described later. The tests are usually made on sets of four duplicate specimens.

After a rinsing in acetone to remove any traces of grease the specimens are annealed.[*] The ends of the terminals are then amalgamated,[†] and the resistance of each of the four specimens is measured. After the resistance measurements, the amalgam is usually removed from the terminals by filing, although the presence of a little mercury at the end of the terminals does not seem to affect the corrosion of the coil itself.

Three of the specimens are then exposed in the atmosphere under investigation,[‡] whilst the fourth is stored in the laboratory. The " blank " not only acts as a check on the constancy of the original resistances, but also compensates automatically for a number of possible experimental errors. At the end of the test, the resistance of the specimens is measured again, and the mean percentage change in the resistance of the exposed specimens, less any change that may occur in the " blank," is taken as proportional to the corrosion.

After this brief survey of the experimental procedure, it may be advisable to consider certain points in rather greater detail.

Method of Fixing the Terminals.

The resistance of the specimens is measured in an ordinary Wheatstone bridge circuit, and it is therefore necessary to minimise the contact resistances by fixing thick terminals to the end of the wire.[§] Solder cannot be used for this purpose, because the juxtaposition of metal and solder might constitute a corrosion couple and cause abnormal corrosion by electrochemical action.

It was therefore necessary to devise a purely mechanical joint which could have no effect on the corrosion of the specimens. This difficult problem was finally solved by Mr. R. May, A.R.S.M., who suggested the method shown in Fig. 1. Blocks of the same material as the wire were milled out as shown in *A*, and the wire was inserted into a hole drilled in one end, which would just take it at a " push " fit. On hammering the block flat, an excellent joint was obtained (*B*).

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[*] Usually in hydrogen at 250° for one hour, not including the time taken in heating up and cooling the furnace. This annealing is intended to remove any internal strain in the metal, and thus eliminate the possibility of " season cracking." This point has already been discussed elsewhere (Proc. Phys. Soc., 1926, 39, 24). The efficacy of low temperature annealing in preventing season cracking has been demonstrated by the work of Moore and his collaborators (J. Inst. Met., 1920, 23, 225 ; 1921, 25, 35 ; 1922, 27, 149 ; 1923, 29, 285) ; whilst Vernon (Trans. Far. Soc., 1927, 23, 169) has shown that differences in the physical condition of the specimens have no appreciable effect on their (atmospheric) corrosion.

[†] During all operations of this kind, the specimens are surrounded by paper covers to protect them from contamination.

[‡] The specimens are stretched to a span of about 4 in. during exposure, so as to separate the adjacent loops. Details of the method of exposure are given in the Appendix.

[§] The lowest resistance measured is 0·03 ohm ; for an accuracy of 1 part in 10,000 the contact resistance must therefore be reproducible to 3 microhms.

The method was not, however, suitable for the preparation of a large number of specimens, owing to the high cost of production. The manufacture of specimens on a large scale was rendered practicable by a slight modification, suggested by Mr. L. H. Wadsworth (Messrs. L. H. Wadsworth, Pimlico, S.W.1). In this case the terminals are made from tape of the necessary cross-section, and the wire is fixed by punching the terminal with a blunt tool, as shown in *C*.

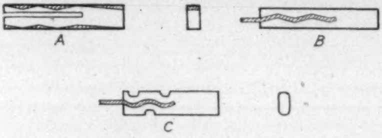

Fig. 1.—Methods of Fixing the Terminals.

Method of Measurement.

The resistance of the specimens varies from about 0·03 to 2 ohms, according to the material. After careful consideration, the method shown diagrammatically in Fig. 2 was adopted for the resistance measurements. This is an ordinary Wheatstone bridge, which is balanced by shunting one or other of the ratio arms. The standard resistances *P*, *Q*, *R* were made to a special design, illustrated in Fig. 3, and are carefully calibrated from time to time.* *Q* and *R* have a range from 1 to 15 ohms, in steps of 1 ohm, and *P* from 0·1 to 1·5 ohms, in steps of 0·1 ohm. The resistance of the specimen *X* may be approximately followed by varying the standard resistances. For instance, if the bridge is balanced when $R=12$ and $Q=13$, $X=12P/13=0{\cdot}923P$. If, now, *R* and *Q* are changed to 13 and 14 respectively, the new equilibrium value of *X* is $13P/14=0{\cdot}929P$. The small gap between these two values can be covered by applying the shunt *S*, which is a four-dial switch resistance box, variable by steps of 0·1 ohm up to a total resistance of 1111 ohms. The galvanometer *G* was obtained from Messrs. H. Tinsley, and has a resistance of 10·7 ohms, with a sensitivity of 51 mms. per microampere at one metre.

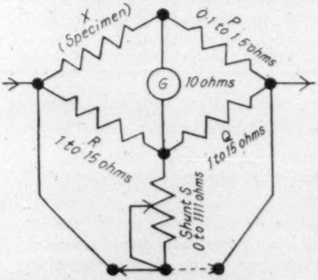

Fig. 2.—The Theoretical Circuit.

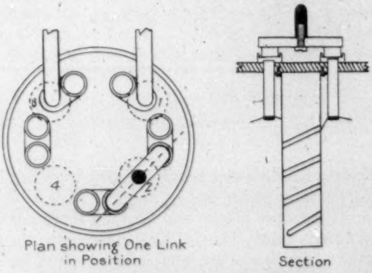

Fig. 3.—The Standard Resistances.

With this apparatus the resistance of the specimens over the range indicated can be measured with an accuracy of approximately 0·1 per cent.

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* Each resistance comprises four coils of one, two, four and eight units, connected in series, any or all of which may be short-circuited by means of copper links dipping into mercury cups, as shown in the diagram.

The Experimental Bridge.

The apparatus used for the resistance measurements is illustrated in Figs. 4 and 5. The specimens are stretched between the mercury cups *m*, milled out to a depth of 10 mms. in the heavy copper blocks *B*, which are supported on a fibre base *F*. The cups, which are 9 cms. apart, are just a little larger than the terminals of the specimens, so that, when the latter are pushed down to the bottom, very good contacts are obtained. The copper blocks *B* are connected to the upper part of the bridge by five copper rods *C*, $\frac{1}{4}$ in. in diameter, which are riveted to the floors of the mercury cups *M*. The lower ends of the rods are amalgamated and dip into the mercury cups *n*. The two parts of the apparatus are thus detachable, but perfect electrical connection between them is secured when they are clamped together by the distance pieces *D*.

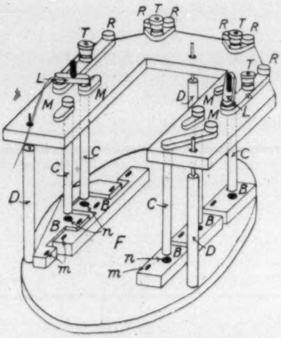

FIG. 4.—THE EXPERIMENTAL BRIDGE

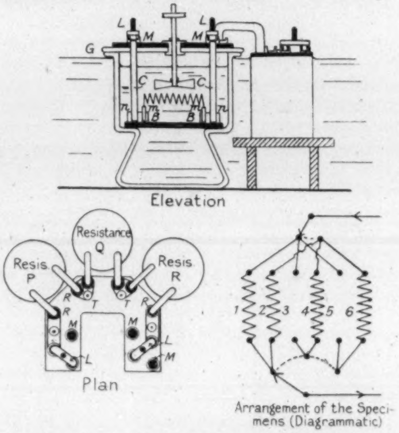

FIG. 5.—THE EXPERIMENTAL APPARATUS.

The upper blocks and mercury cups are mounted on an ebonite panel, resting on a thick glass plate *G*. This plate forms the lid of a large desiccator full of acetone, and the specimens are completely covered by the liquid when the plate is placed in position. The desiccator itself is immersed to within an inch of the top in a large oil thermostat, fitted with the usual electrical heating and stirring devices. The terminals of the standard resistances fit into the mercury cups *R*, and the screw terminals *T* are used for the necessary connections to the batteries, galvanometer and the variable shunt.[*]

---

[*] The insulation of all the materials used (acetone, fibre, glass and ebonite) has been tested and found satisfactory. In the case of acetone, the least perfect insulator, the resistance across a 7 cm. gap between two thick leads to a "megger" was found to be 2·5 to 3 megohms at 500 volts.

When the bridge has been assembled, the specimens are left in the acetone bath, which is stirred by a small auxiliary stirrer, until the temperature is within a tenth of a degree or so of 25°C., and the resistance measurements are then made. (The nominal temperature for the resistance determinations is 25·00°C., but it has been found expedient to work at a temperature reasonably near this, and to apply a small temperature correction, since there is an appreciable lag between the temperature of the oil bath and that of the acetone.)

The measurements are made by adjusting the resistances P, Q, R, so that PR/Q is slightly greater than the resistance of the specimens, and reading off the values of the shunts, applied across R, that are necessary to balance the bridge when each specimen is introduced into the circuit in turn by means of the links L. The temperature of the acetone bath is also recorded, and the resistance of the specimens is calculated as described in the next section.

Methods of Calculating and Expressing the Results.

Since the effects of corrosion are deduced from the *percentage* change in the resistance of the specimens, it is not necessary to calculate their absolute resistance. The results may therefore be expressed in terms of an arbitrary unit R_0, which, for convenience, may be taken as PR/Q. The absolute resistance of the specimens at 25·00°C. is given by

$$X = R_0(1 - a \times 10^{-4}),$$

where $a \times 10^{-4}$ is a small factor. The value of R_0, of course, varies for each set of specimens.

As an illustration of the methods of calculating and expressing the results, the data for the initial measurements of a set of 19 S.W.G. copper coils are given in Table I. In this particular case P, Q, R were 0·1, 8 and 3 ohms respectively, so that the value of the arbitrary unit, R_0, was $0{\cdot}1 \times \frac{3}{8}$, or 0·0375 ohm. The values of the shunts applied across the 3-ohm coil necessary to balance the bridge are given in the second column. In the case of the first determination of the resistance of coil 1, the shunt was 52·80 ohms. The resistance of this coil is therefore

$$X = 0{\cdot}1 \times \frac{3}{8} \times \frac{52{\cdot}8}{52{\cdot}8 + 2{\cdot}99_0{}^*}$$

$$= R_0\left(1 - \frac{2{\cdot}99_0}{55{\cdot}8}\right)$$

$$= R_0(1 - 536 \times 10^{-4}).$$

The temperature of the bath was 25·21°, and the corresponding temperature correction (to 25·00°)

$$-39 \times 10^{-4} \times 0{\cdot}21 = -8 \times 10^{-4}.$$

The corrected value of the resistance is thus

$$X = R_0(1 - 536 \times 10^{-4})(1 - 8 \times 10^{-4}),$$

$$= R_0(1 - 544 \times 10^{-4}).$$

---

[*] **The corrected value of the 3-ohm resistance.**

In this case $a=544$; the corresponding values of a for the other specimens are given in the last column.

TABLE I.—*Determination of the Initial Resistance of* 19 *S.W.G.* (0·040″) *Copper Specimens.*
Unit $R_0=0{\cdot}1\times\frac{3}{8}$. Temperature coefficient for Copper 0·0039.
Initial Resistances.

Coil.		Shunt across 3 ohms for balance.	Temperature.	a (uncorrected).	Temperature correction.	a
I	1.[*]	$52{\cdot}8_0$	25·21°	536	−8	544
	3.	$52{\cdot}0_0$	25·21°	543	−8	551
	4.	93·3	25·21°	310	−8	318
	5.	74·9	25·21°	384	−8	392
II	1.	52·0	25·01°	544	Nil.	544
	3.	51·0	25·01°	554	Nil.	554
	4.	91·7	25·01°	316	Nil.	316
	5.	73·35	25·01°	391	Nil.	391
III	1.	52·1	25·02°	542	−1	543
	3.	51·05	25·03°	553	−1	554
	4.	91·4	25·04°	316	−2	318
	5.	73·45	25·05°	391	−2	393

The three series of determinations given in Table I were made independently on different days. The results are, therefore, a fair indication of the accuracy obtainable. The average deviation of the individual values from the mean is 1 part in 10,000, and, as a rule, the resistance measurements are reproducible with this accuracy.

TABLE II.—*Change in Resistance of* 19 *S.W.G. Copper Specimens after Seven Months' Exposure.*

Coil.	α (initial) mean of I, II, III.	a (final).	$\Delta\alpha$.	ΔR p.c.
1.	544	278	266	2·82
3.	553	286	267	2·83
4.	317	38	279	2·88
5.	392	380	12	0·12

Table II illustrates the effect of seven months' exposure on the resistance of the specimens. If a_I and a_F are the initial and final values of a, the percentage increase in resistance

$$(a_I-a_F)10^{-4}\times100/(1-a_I\times10^{-4})$$
$$=(a_I-a_F)/(1-a_I\times10^{-4})100.$$

The values of ΔR calculated in this way are given in the last column. The increase in resistance due to corrosion is equal to 2·84 (mean of the three exposed specimens) −0·12[†] (change in the " blank "), or 2·72 per cent. From this value the equivalent loss in weight of metal could be calculated as on page 109. Since, however, for a number of different materials the changes in resistance per cent. are directly comparable, being equal to the percentage reduction in cross-section,* whereas the losses in weight are not (because they involve the density of the material, which may differ from one metal to another), there is usually no advantage in doing this, and the corrosion may be expressed directly in terms of the resistance change.

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[*] The numbers refer to the position of the specimen in the bridge.

[†] In this particular case the increase in the " blank " was largely accounted for by slight changes in the standard resistances.

III. EXPERIMENTAL RESULTS.

The experimental work that will now be described, was undertaken as a preliminary to a much more extensive series of field tests on the atmospheric corrosion of non-ferrous metals and alloys. The chief object of the experiments was to test the validity of the method rather than to obtain comparable data for different materials. The first necessity was to show that measurable resistance changes were produced in the metals on exposure, and the second that these changes were definitely proportional to the corrosion. The first object has been achieved by making a direct test on a wide range of non-ferrous metals and alloys, whilst the quantitative nature of the results has been investigated by observing the relationship between the resistance change and the diameter of the specimen, and also by a direct correlation of the increase in resistance and the corresponding change in weight of the same specimen.

Short Period Corrosion Tests on Typical Non-Ferrous Materials (*Winter*, 1926-27).

Fourteen sets of 22 S.W.G. (0·028 in.) specimens were exposed at the Royal Botanic Gardens, Regent's Park, where a site in the Meteorological enclosure was kindly provided by the Royal Botanic Society. The changes in the resistance of the specimens after 53 days' exposure, from November 18, 1926, to January 10, 1927, are given in Table III. Although the materials have been arranged in order of resistance change, this is not necessarily the true order of corrodibility, since, in the absence of any knowledge of the corrosion-time curve for the different materials, it would not be justifiable to base any conclusions as to their respective merits on the results of tests extending over such a limited period.

The results show conclusively that the resistance method can be successfully applied to a wide variety of materials. The average difference between the result for any one specimen and the mean for the series of three specimens of the same material is as low as 2 per cent. of the actual increase in resistance, which is very satisfactory accuracy for a " field test " on corrosion.

In order to test whether the corrosion product contributes to the conductivity, the resistance of the phosphor-bronze and the 60/40 brass specimens was redetermined after as much as possible of the corrosion product had been removed by scrubbing with a nailbrush, moistened in acetone. This did not produce any appreciable difference, and the corrosion product may therefore be assumed to be a non-conductor. (The " blanks " were also scrubbed.)

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* If we assume that the corrosion film is of uniform thickness at all points, and also that there is a sharp boundary between the corrosion product and the uncorroded metal. Admittedly these conditions are only approximately obeyed in practice. The point is that, in comparative tests on different metals of widely different densities, such as copper, zinc and aluminium, it would probably be preferable to calculate the loss in volume of a standard specimen rather than its loss in weight. Under ideal conditions the relative increase in resistance is directly proportional to the loss in volume.

TABLE III.—*Exposure Tests at the Royal Botanic Gardens.*
Changes in Resistance of 22 S.W.G. Specimens after 53 Days' Complete Exposure.
(Winter, 1926/27.)

	R_0 Ohms.	Percentage Increase in Resistance.				
		Blank.	Exposed Specimens (Corrected for " Blank ").			Mean.
1. 80/20 Copper-Nickel ...	1·15	0·04	1·42	1·52	1·53	1·49
2. Nickel I*	0·38	−0·04	1·61	1·69	1·72	1·67
3. Nickel II	0·38	0·00	1·68	1·72	1·80	1·73
4. Copper	0·08	−0·01	1·95	1·85	1·89	1·90
5. Phosphor-bronze ...	0·72	0·01	1·99	2·02	2·10	2·04
6. Nickel Silver (Seconds)	1·25	−0·11	2·15	2·12	2·15	2·14
7. 55/45 Copper-Nickel ...	2·15	0·04	2·08	2·17	2·22	2·16
8. 70/30 Nickel-Copper II*	2·10	0·00	2·34	2·34	2·47	2·38
9. 70/30 Nickel-Copper I	2·10	0·00	2·47	2·37	[3·00]†	2·42
10. Cadmium-copper (1 per cent. Cd.)	0·09	0·00	2·65	2·52	2·61	2·59
11. 70/30 Brass (As received)	0·28	−0·03	2·60	2·68	2·66	2·65
12. 70/30 Brass (Annealed)	0·28	0·03	2·81	2·87	2·97	2·88
13. 70/30 Brass (As received and fitted with soldered terminals) ...	0·28	−0·07	3·07	2·99	3·01	3·02
14. 60/40 Brass	0·30	0·00	3·64	3·84	3·88	3·79

EFFECT OF REMOVING PART OF THE CORROSION PRODUCT.
Increase in Resistance per cent. due to " Scrubbing."

	Phosphor-bronze.				60/40 Brass.		
Exposed Specimens	0·06	0·06	0·07	...	0·01	0·06	0·08
Blank		0·11		...		0·03	

The Effect of the Length of the Coil and of the Gauge of the Wire on the Resistance Changes Produced by Corrosion.

The relation between the resistance change produced by corrosion and the diameter of the wire was investigated by exposing copper specimens of four different gauges. Simultaneously with these tests, observations were also made on copper specimens of the same gauge, but comprising different lengths of wire. This was done to test whether any local corrosion occurred at the joints; in this case the percentage increase in resistance should be greater for the shorter specimens. The full list of specimens is given below:—

Series.	Number of Turns.	Length of Wire.	S.W.G.	Diameter.
		(Cms.)		(Ins.)
A	40	160	22	0·028
B	28	110	22	0·028
C	14	60	22	0·028
D	38	150	21	0·032
E	37	150	20	0·036
F	37	150	19	0·040

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* (Table III.) Since nickel and the nickel-rich copper-nickel alloys can only be amalgamated with difficulty, these materials were fitted with terminals of copper-nickel alloys with a higher copper content. Two series were prepared in each case, (I) with 55/45 and (II) with 80/20 copper-nickel terminals respectively. No appreciable difference was observed in the resistance changes of the two sets of specimens and the procedure may therefore be considered justifiable.

† Specimen damaged.

A rather curious observation was made during the initial resistance measurements. Although duplicate determinations agreed well in the case of the coils with the full number of turns, there was a definite increase in the resistance of the shorter coils (Series B and C) from one determination to another. This is shown in Table IV,

TABLE IV.—*Variation in the Initial Resistance of Copper Specimens.*

Series A
(40 turns. 22 S.W.G.).
$R_0 = 0{\cdot}1 \times 8/10$.
Values of α.

Date	9.7.26	9.7.26	12.7.26	Mean
A1	338	333	335	335
A3	335	329	331	332
A4	422	418	419	420
A5	313	311	312	312

Series B
(28 turns. 22 S.W.G.).
$R_0 = 0{\cdot}1 \times 5/9$.
Values of σ.

Date	9.7.26	10.7.26	12.7.26	13.7.26	15.7.26	18.7.26	Mean of last four
B1	329	320	318	310	311	319	315
B3	333	334	333	327	330	336	332
B4	398	392	390	388	390	395	391
B5	265	253	248	245	244	246	246

Series C (14 turns. 22 S.W.G.).
$R_0 = 0{\cdot}1 \times 4/14$.
Values of α.

Date	13.7.26	15.7.26	17.7.26	19.7.26	Mean of last three
C1	576	522	527	519	523
C3	972	953	956	952	954
C4	652	624	621	611	619
C5	787	760	766	763	763

Series D (38 turns. 21 S.W.G.)
$R_0 = 0{\cdot}1 \times 6/10$.
Values of α.

Date	14.7.26	16.7.26	19.7.26	Mean
D1	670	672	674	672
D3	947	944	945	945
D4	651	646	648	648
D5	536	532	535	534

Series E (37 turns. 20 S.W.G.).
$R_0 = 0{\cdot}1 \times 4/9$.
Values of σ.

Date	13.7.26	15.7.26	17.7.26	Mean
E1	67	68	68	68
E3	250	247	243	243
E4	203	200	200	201
E5	235	238	234	236

Series F (37 turns. 19 S.W.G.).*
$R_0 = 0{\cdot}1 \times 3/8$.
Values of σ.

Date	14.7.26	16.7.26	19.7.26	Mean
F1	544	544	543	544
F3	551	554	554	553
F4	318	316	318	317
F5	392	391	393	392

* See also Table I.

where a complete list of all the determinations, in chronological order, is given, After a few days the resistance of the shorter coils also reached a constant value. The variation in resistance is probably due to the stretching of the specimens, which leads to a permanent " set " in the case of the shorter specimens. (It will be remembered that when the specimens are prepared all the loops are touching each other, but that these are pulled apart during measurement and exposure. Incidentally, the " blanks " are stored in the stretched condition.) Although this variation in resistance has never been observed in specimens with the full number of turns, it is advisable to anneal them in the fully stretched condition, so as to minimise the effects of any subsequent stretching. This has been done in the later work.

The specimens were exposed on the roof of the Royal School of Mines at South Kensington. The stand used to support the specimens is described in the Appendix. Exposure was begun on July 21, 1926, and the tests are still in progress. The increase in the resistance has since been determined on three occasions :—

1. On October 30, 1926, after 101 days' exposure.
2. On March 1, 1927, after 213 days' exposure ; and
3. On August 23, 1927, after 381 days' exposure.*

The complete results are given in Table V, and the mean increase in resistance for each series (i.e., the average for the three exposed specimens, less the change in the " blank ") is shown in Table VI. Table VII illustrates the effect of the gauge

TABLE V.

Increase in the Resistance of Copper Specimens on Exposure to the South Kensington Atmosphere.

Gauge.	Specimen.	Percentage Increase in Resistance.			Gauge.	Specimen.	Percentage Increase in Resistance.		
		Oct. 30, 1926. 101 Days.	Mar. 1, 1927. 213 Days.	Aug. 23, 1927. 381 Days.			Oct. 30, 1926. 101 Days.	Mar. 1, 1927. 213 Days.	Aug. 23, 1927. 381 Days.
22 S.W.G. (160 cms.).	A1	0·93	4·07	4·97	21 S.W.G. (150 cms.).	D1	0·67	3·60	4·50
	A3	0·91	4·02	4·86		D3†	0·11	0·05	0·04
	A4	1·04	4·23	5·11		D4	0·73	3·85	4·86
	A5†	0·08	0·12	0·14		D5	0·71	3·74	4·74
22 S.W.G. (110 cms.).	B1	0·90	4·22	5·23	20 S.W.G. (150 cms.).	E1†	0·07	0·06	0·01
	B3†	0·12	0·09	0·13		E3	1·01	3·37	4·04
	B4	0·76	4·26	4·86		E4	0·80	3·41	4·24
	B5	0·90	4·56	5·60		E5	0·99	3·58	4·38
22 S.W.G. (60 cms.).	C1	0·91	4·20	5·10	19 S.W.G. (150 cms.).	F1	0·49	2·82	3·62
	C3†	0·08	0·03	0·04		F3	0·46	2·83	3·65
	C4	0·93	4·46	5·33		F4	0·47	2·88	3·81
	C5	[1·52]	[5·15]	[6·19]		F5†	0·12	0·12	0·20

TABLE VI.—*Corrected Mean Values of the Percentage Increase in Resistance.*

Series.	Percentage Increase in Resistance.		
	101 days.	213 days.	381 days.
A	0·88	3·99	4·82
B	0·73	4·26	5·10
C‡	0·84	4·30	5·18
D	0·59	3·68	4·66
E	0·86	3·39	4·21
F	0·35	2·72	3·49

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* The periods stated do not include the few days during which the specimens were undergoing examination in the laboratory.

† Blank stored in the Laboratory.

‡ Mean of two specimens only.

TABLE VII.—*The Effect of Gauge on the Resistance Change.*

Series.	Diameter (in.)	Relative diameter d.	101 days.			213 days.			381 days.		
			Per cent. increase in resistance ΔR.	$\Delta R \times d$ $= K$.	Per cent. deviation of K from Mean.	Per cent. increase in resistance ΔR.	$\Delta R \times d$ $= K$.	Per cent. deviation of K from mean.	Per cent. increase in resistance ΔR.	$\Delta R \times d$ $= K$.	Per cent. deviation of K from mean.
A	0·028	7	0·88	6·2	+13	3·99	27·9	−3·1	4·84	33·9	−5·6
D	0·032	8	0·59	4·7	−15	3·68	29·4	+2·1	4·66	37·3	+3·6
E	0·036	9	0·86	7·7	+40	3·39	30·5	+5·9	4·21	37·9	+5·3
F	0·040	10	0·35	3·5	−36	2·72	27·2	−5·6	3·49	34·9	−3·1
			Mean	5·5		Mean	28·8		Mean	36·0	

TABLE VIIA.—*Results after* 541 *Days' Exposure.*

Series.	Percentage increase in resistance.					Relative diameter d.	$\Delta R \times d$ $= K$.	Per cent. deviation of K from mean.
	1.	3.	4.	5	Mean value corrected for blanks. ΔR.			
A	6·78	6·69	6·99	0·14	6·68	7	46·8	−5·4
D	6·03	0·05	6·54	6·52	6·31	8	50·5	+2·0
E	0·03	5·40	5·74	5·79	5·61	9	50·5	+2·0
F	5·07	5·15	5·49	0·24	5·00	10	50·0	+1·0
B	7·17	0·09	6·74	7·54	7·06	Mean	49·5	
C	7·30	0·00	7·50	[8·43]	7·40			

on the resistance changes, and gives the values of the product K of the percentage increase in resistance and the relative diameter of the wire. Theoretically, K should be constant for all four gauges of wire at any given period. It will be seen that this is the case with the exception of the results for the first period of exposure. The individual values of K for the later periods are, on the average, within 5 per cent. of the corresponding mean values for the four gauges of wire. This figure is comparable with the probable experimental error.*

The results of this experiment show that the resistance change is inversely proportional to the diameter. It is therefore possible to use the electrical resistance method as an "acceleration test" by conducting tests on relatively thin wires, since the changes produced in the thicker wires used in practice can be calculated from the results of the experiment by simple proportion. In this respect the method constitutes an acceleration test of an ideal type, having the great advantage over many previous tests of this kind that the increased rate of attack is not obtained by exposing the specimens to artificial conditions, often radically different to those encountered in practice. On the contrary, the medium of exposure is unchanged, and the acceleration is produced by choosing the geometrical shape of the specimen in such a way that corrosion produces a maximum effect. In view of the present demand for accelerated corrosion tests in industry, it may be permissible to urge that this is by far the soundest way to diminish the time necessary for an experiment.

The anomalous results observed after the first hundred days' exposure may be partly due to slight structural differences between the extreme surface layer of the wire and the core. If the surface layer is attacked at a different rate from the body of the wire, the erratic results obtained in the initial stages of attack might be explained by assuming that this layer had a slightly different thickness in the different specimens.

It is interesting to note that these corrosion tests in the open show a seasonal variation in the rate of attack, similar to that which has already been observed when the specimens are exposed in a Stevenson screen.† This may be illustrated by calculating the mean daily increase in the value of the constant K for the four periods of exposure.

Period.	Value of K.	Increase.	Days.	Increase per day.
July-October, 1926 (summer) ...	5·5	5·5	101	0·055
November, 1926 March, 1927 (winter)	28·8	23·3	112	0·280
March-August, 1927 (summer) ...	36·0	7·2	168	0·043
August, 1927 February, 1928 (winter)	49·5	13·5	160	0·084

The rate of corrosion in winter is thus appreciably greater than in summer, although the difference between the two periods is not so pronounced as in the case of Stevenson screen exposure, where the ratio of the rates of attack may vary from six to thirteen, according to the material. This large difference in the corrosivity of the atmosphere is no doubt due to the increased pollution and greater humidity during the winter.

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* (Added February 10, 1928.) A redetermination of the resistance of the specimens on February 3, 1928, after 541 days' exposure gave the values shown in Table VIIA. It will be seen that the values of K for the four gauges of wire are in even better agreement than in either of the earlier periods of exposure.

† Trans. Far. Soc., **23** (1927), 184.

As regards the effect of the length of the coil on the resistance changes, the agreement between the results for the different specimens is sufficiently good to show that there is no appreciable local corrosion at the joints. Indeed, the slight differences observed, amounting to less than 8 per cent., may not be due to this cause at all, but rather to a closer packing of the loops in the longer coils, leading to a mutual protective effect, and in consequence to a reduced rate of corrosion.*

Correlation of Weight Increment and Resistance Change (Stevenson Screen Exposure).

The validity of the resistance method for studying corrosion may be tested by a direct observation of the weight changes of the specimens on which the resistance determinations are made. This is most conveniently done when the specimens are exposed in a Stevenson screen. In this case the increase in weight of the specimens can be readily determined; moreover, if the positions of the specimens in the screen are suitably chosen, it is possible of obtain a series of different weight increments for specimens exposed simultaneously, since the intensity of corrosion decreases from the bottom of the screen upwards.†

The first tests were made on 22 S.W.G. (0·028 in.) specimens: two sets of copper, one of 80/20 copper-nickel and one of phosphor bronze. In order to obtain as many points as possible on the weight-increment/resistance change curve, no "blanks" were kept and all the specimens were exposed. After the initial resistance measurements, the amalgam was removed from the terminals and the specimens were weighed. They were then exposed for a suitable period in a Stevenson screen on the roof of the Royal School of Mines. This was followed by a second weighing and a final resistance determination.

The results obtained are given in Table VIII, and the data are plotted in Fig.6. In all four cases the results are in fairly good agreement with a linear relationship between weight increment and resistance change, as demanded by theory. The straight lines should, however, pass through the origin, since zero weight increment should correspond to zero resistance change. In the absence of "blanks," it is not advisable to form any conclusions on this point, although the intercepts on the resistance change axis are probably too large to be attributed solely to experimental error.

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* The actual distance between two loops is approximately given by

$$\delta = \frac{3\cdot5 - nd}{n-1}$$

where n equals the number of turns and d equals the diameter of the wire. If the necessary calculations are made, the following values of δ are obtained :—

Series A ...	$\delta = 0\cdot061$ in.	Series D ...	$\delta = 0\cdot062$ in.
B ...	0·101	E ...	0·060
C ...	0·239	F ...	0·058

† This is due to the fact that most of the air enters by the bottom of the screen. Full details of the method of exposure in a Stevenson screen are given in the Meteorological Magazine, 62, 53 (1927).

TABLE VIII.—*Correlation of Weight-Increment and Resistance Change (Stevenson Screen Exposure).*
(22 S.W.G. specimens.)

Metal.	Specimen.	Weight-increment. Mgms.	Resistance change per cent.	
			Observed.	Calculated.
Copper I.	1	56·4	0·62	0·45
Exposed for 77 days,	3	81·4	0·82	0·65
October-December, 1926.	4	65·9	0·69	0·52
$P=31$ per cent. Cu.	5	30·0	0·37	0·24
Copper II.	1	61·3	0·76	0·38
Exposed for 55 days,	3	115·4	1·11	0·72
October-December, 1926.	4	62·5	0·71	0·39
$P=26$ per cent. Cu.	5	29·1	0·46	0·18
Phosphor Bronze.	1	65·1	0·94	0·49
Exposed for 41 days,	3	49·2	0·77	0·37
November-December, 1926.	4	91·4	1·16	0·69
$P=28$ per cent. Cu, 2 per cent. Sn.	5	29·1	0·59	0·22
80/20 Copper-Nickel.	1	111·4	1·12	1·15
Exposed for 48 days,	3	95·5	0·94	0·99
November, 1926-January, 1927.	4	131·0	1·21	1·35
$P=30$ per cent. Cu, 7 per cent. Ni	5	77·6	0·95	0·80

Theoretical values for the resistance change, corresponding to the observed weight increments, can be calculated if the percentage of metal in the corrosion product is determined by analysis. If the weight increment of the specimen is S, and the corrosion product contains P per cent. of metal, the weight of metal corroded is $W=SP/(100\text{-}P)$, since a weight increment of (100-P) gms. would result from the corrosion of P gms. of metal. The corresponding values of the resistance change may then be calculated by equation (3) on page 109.

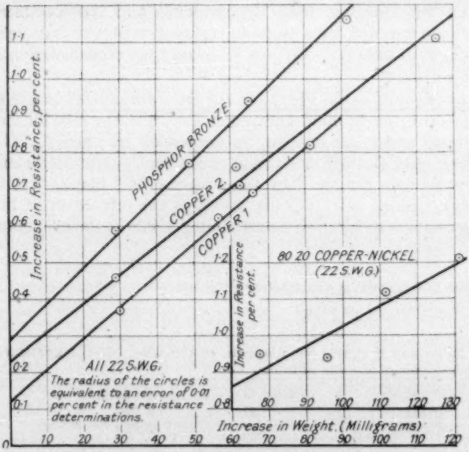

FIG. 6.—CORRELATION OF WEIGHT INCREMENT AND RESISTANCE CHANGE
(Stevenson Screen Exposure).

The observed and calculated values of the resistance change are given in Table VIII. (For the specimens under consideration, $A = 36$ cm.^{2}, $r = 0{\cdot}0355$ cm., and D was taken as 8·9 throughout.) It will be seen that both are of the same order of magnitude, but that the calculated figures are, on the whole, lower than those observed. This is not surprising, since the calculated figures necessarily represent minimum values of the possible resistance change for two reasons: First, any departure from ideal conditions—the uniform film of corrosion product—would lead to an increased resistance change per unit weight increment; secondly, the values of P found by analysis are almost certainly too low, since it is not possible to remove all the corrosion product, and the layer closest to the metal, which remains undetached, probably contains a higher percentage of metal. On the other hand, the *slopes* of the straight lines agree fairly well with the theoretical values.

Percentage Increase in Resistance per Mgm. Weight Increment.

——	Observed.	Calculated.
Copper I	0·0088	0·0079
Copper II	0·0078	0·0062
Phosphor bronze	0·0093	0·0076

(The figures for the 80/20 Copper nickel alloy are too erratic to be used for this calculation.)

The approximately linear relationship between weight-increment and resistance change has been confirmed by similar experiments on twelve 18 S.W.G. (0·048 in.) "silicon bronze"[*] specimens, in which the usual "blanks" were kept. The results after 96 and 157 days' exposure are shown in Figs. 6A and 6B respectively.[†]

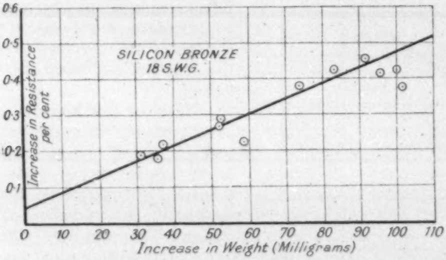

FIG 6A.—CORRELATION OF WEIGHT INCREMENT AND RESISTANCE CHANGE.

Result for "Silicon Bronze" after 96 Days' Exposure.

The encouraging feature of the results is the regular relationship between weight-increment and resistance change. The changes produced by corrosion in a metal are so complex that two different methods cannot be expected to give the same quantitative interpretation of its results; the fact that an approximate agreement is reached may be considered satisfactory. The results show that the resistance increases regularly with corrosion. Any irregularities due to pitting or other causes must therefore be of secondary importance, since the effect of a pit of dimensions comparable with the diameter of the wire would be to produce a large increase in resistance for a negligible increase in weight.

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[*] Copper with 0·8 per cent. of tin and 0·7 per cent. of zinc.

[†] Fig. 6B was added on February 10, 1928. It should be pointed out that the linear relationship between resistance change and weight increment cannot be expected to persist in the later stages of corrosion, because, owing to the condensation of moisture from the atmosphere, a saturated solution of corrosion product collects on the lowest parts of the specimens, thus affording every facility for intense local corrosion This may be the cause of the two anomalous results shown in Fig. 6B.

Correlation of Change in Weight and Resistance Change for Fully Exposed Specimens.

The correlation of resistance change and change in weight may also be carried out in the open, although in this case the experimental difficulties are much greater. The weight of metal corroded may be determined by applying a correction for the metal removed by the rain water, as in Vernon's experiments.* Let S be the observed increase in weight of the specimen itself, and E the weight of metal, per specimen, found in the rain water. If the metal removed by the rain were replaced on the specimen, an increase in weight equal to $E+S$ would be observed. This figure, therefore, represents the weight of non-metallic radicles contained in the corrosion product *on the specimen*, and if the percentage of metal in this corrosion product is P, the weight of corroded metal remaining on the specimen is $P(E+S)/(100-P)$. The total weight of metal corroded is thus $W=E+P(E+S)/(100-P)$. This value of W may be used to calculate the resistance change of the specimen, by the method already described.

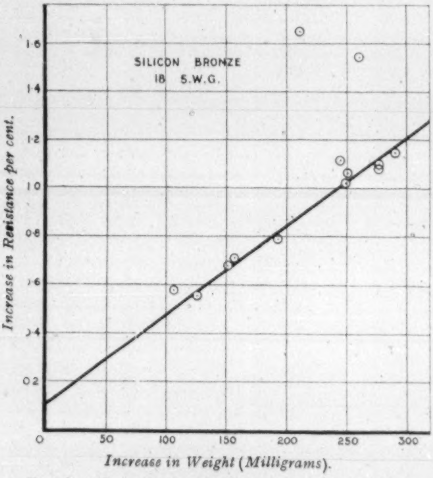

FIG. 6B.—CORRELATION OF WEIGHT INCREMENT AND RESISTANCE CHANGE.

Results for "Silicon Bronze" after 157 Days' Exposure.

Tests of this kind have been made on copper, nickel, 60/40 brass and 80/20 copper-nickel specimens, fully exposed on the roof of the Royal School of Mines. The wires used were 0·036" (20 S.W.G.) in diameter and 160 cms. long. The resistance measurements and the determination of the change in weight of the specimens were made in the same way as for the Stevenson screen experiments. During exposure the specimens of each material were supported over a separate enamelled pail, in which the rain water falling from the specimens was collected. At the end of the tests the amount of the respective metals in this rain water was determined. The analyses were complicated by the large volume of water collected (from 6 to 7 litres), and by the presence of considerable quantities of soot and other wind-blown pollution, which contained both iron and aluminium oxides. The specimens were exposed for 168 days, from March 2 to August 17, 1927.

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* W. H. J. Vernon, Trans. Far. Soc., 23, 166 (1927).

The percentage increase in resistance and the weight changes of the specimens are given in Tables IX and X, whilst the observed and calculated values of the resistance change are compared in Table XI. In the case of the two pure metals and the 80/20 copper-nickel alloy the results are in substantial agreement, the observed value being slightly the higher, which is understandable for the reasons already discussed. There is, however, a marked difference in the case of 60/40 brass, where the observed value is much lower than the calculated one. It is not advisable to discuss the matter very fully until further data are available, but there is nothing improbable in the result. Bengough and May have shown that the zinc present in solid solution is capable of reacting with the corrosion product of the brass to deposit metallic copper,* and similar examples of the "dezincification" of brass by atmospheric corrosion have been given by Vernon.† In the present case the results of the analyses support the view that copper has been re-deposited, since the 0·2041 gm. of zinc found in the corrosion product was originally associated with 0·3062 gm. of copper, whereas only 0·1122 gm. of copper was recovered. If this is so, the condition that the corrosion product should be a non-conductor is no longer fulfilled, and it is quite conceivable that the re-deposition of copper would cause a diminution of the observed increase in resistance.

TABLE IX.—(*Correlation of Change in Weight and Increase in Resistance for Fully Exposed Specimens*). *Values of the Percentage Increase in Resistance.*

—		Copper.	80/20 Copper-Nickel.	Nickel.	60/40 Brass.
Exposed Specimens	1	1·14	1·36	1·88	2·40
	2	1·15	1·39	1·92	2·25
	3	1·16	1·44	1·88	2·32
Mean		1·15	1·40	1·89	2·33
Blank		−0·02	0·06	0·00	−0·10
Net change ...		1·17	1·34	1·89	2·43

TABLE X.—*Change in Weight of the Specimens (Grams).*

—	Copper	80/20 Copper-Nickel.	Nickel.	60/40 Brass.
1	+0·0222	+0·0076	−0·0681	+0·0248
2	0·0231	0·0072	−0·0683	0·0181
3	0·0226	0·0112	−0·0701	0·0193
Mean	+0·0226	+0·0087	−0·0688	+0·0207

The correctness of this view has been supported by the results of some tensile tests on these wires, which were kindly made by Mr. W. E. Alkins, M.Sc., in the

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* G. D. Bengough and R. May, J. Inst. Met., 32, 149 (1924).

† W. H. J. Vernon, Trans. Far. Soc., 23, 170 (1927).

TABLE XI.—*Observed and Calculated Values of the Resistance Change.*

Metal.	S	E	$E+S$	P	$P(E+S)/(100-P)$	W	D	Resistance change.	
								Calculated from W.	Observed.
Copper ...	+0·0226	0·0232	0·0458	52·8	0·0512	0·0744	8·9	0·83	1·17
80/20 Copper-Nickel	+0·0087	Cu 0·0189 Ni 0·0197 0·0386	0·0473	Cu 58·6 Ni 0·7 59·3	Cu 0·0680 Ni 0·0008 0·0688	Cu 0·0869 Ni 0·0205 0·1074	8·9	1·20	1·34
Nickel ...	−0·0688	0·1227	0·0539	16·1	0·0103	0·1330	8·8	1·50	1·89
60/40 Brass ...	+0·0207	Cu 0·0088 Zn 0·1751 0·1839	0·2046	Cu 30·7 Zn 8·6 39·3	Cu 0·1034 Zn 0·0290 0·1324	Cu 0·1122 Zn 0·2041 0·3163	8·4	3·73	2·43

Research Laboratories of Messrs. T. Bolton & Sons, Ltd., at Oakamoor, N. Staffs. The breaking loads of the unexposed and exposed wires are shown in Table XII; each figure represents the mean of nine or more determinations. The percentage decrease in the breaking load of the brass is in much better agreement with the change in weight, and confirms the view that the observed resistance change is too low. It is interesting to note that the order of corrodibility of the four materials is the same, whether the comparison is based on the loss in weight, the resistance change, or the decrease in breaking load.[*] The tensile tests were made on a 1000 lbs. machine reading to 1 lb., which does not appear to have been sensitive enough to detect the small change in the breaking load of the copper specimens.

TABLE XII.—*Decrease in the Breaking Load on Exposure.*

Metal.	Breaking load (lbs.).		Decrease.	
	Unexposed.	Exposed.	Lbs.	Per cent.
Copper	40	40	Too small to	be detected.
80/20 Copper-Nickel	60·2	58·8	1·4	2·3
Nickel	76·0	73·6	2·4	3·2
60/40 Brass	62·0	58·0	4·0	6·5

The analyses of the 80/20 copper-nickel alloy seem to show that there is a tendency for the copper to accumulate on the specimen, perhaps owing to a selective solution of the nickel salts in the corrosion product by the rain water. This may be associated with the brown colour that develops on this alloy during exposure. There does not, however, appear to have been any marked copper re-deposition, since the two metals are fairly well balanced in the total corrosion product represented by W.

IV. CONCLUSION.

The main object of this Paper has been to describe a new, and perhaps more accurate, method of studying corrosion in the field. In experiments of this type it is difficult to obtain the rigorous agreement that is reached in more academic researches. When this difficulty is taken into consideration, it may be claimed that the results reported in this Paper show that the resistance method is directly applicable as a quantitative test of the atmospheric corrosion of metals. No doubt the method will be found to have its limitations—an indication of these may already have been seen in the case of the brass specimens discussed in the last section—but any corrosion test that can be applied quantitatively on a large scale under actual service conditions must undoubtedly be of practical value.

A series of tests by the electrical resistance method has been included in the field tests on atmospheric corrosion that were begun in June, 1927, by the Atmospheric Corrosion Committee of the British Non-Ferrous Metals Research Association at five stations in this country representative of rural, suburban, urban, industrial and marine atmospheres respectively. Admittedly some caution will be necessary in interpreting the results. For instance, it be will advisable to establish the shape of the corrosion/time curve for each material investigated before any comparisons may be based on the results of exposure tests over a limited period.

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[*] **It should be clear that this " order of corrodibility " can only apply to the given experimental conditions.**

Further experiments on the effect of gauge on resistance change will also be necessary to establish the law of similarity in each case, if the results obtained for thin wires are to be applied to the same materials in bulk. Subsidiary tests on the lines indicated in this Paper are, however, being continued, and extended to include a wider range of materials, and there is every reason to hope that the information obtained in this way will permit of an accurate interpretation of the results of the main tests when these are concluded.

Acknowledgments.

The author is indebted to the Council of the British Non-Ferrous Metals Research Association for permission to publish this Paper. He would like to express his appreciation of the very kind manner in which Prof. H. C. H. Carpenter, F.R.S., has supervised the work, of the encouragement and assistance which he has received from Dr. R. S. Hutton and Dr. O. F. Hudson, of the British Non-Ferrous Metals Research Association, and of the valuable advice given him by his colleague, Dr. W. H. J. Vernon, throughout the work. He would also like to thank Prof. A. O. Rankine, O.B.E., D. Sc., and Miss M. Bell, of the Physics Department of the Royal College of Science, for the loan of apparatus with which he was able to make some preliminary measurements.

Finally, acknowledgments are due to the Committee of the Royal Botanic Society of London for their kindness in providing a site for the exposure tests in their gardens, to the Director of the Meteorological Office for the loan of a Stevenson screen, and to Messrs. Thomas Bolton & Sons, Ltd., who kindly provided the copper, brass and bronze wires used in this work.

Metallurgical Department,
Royal School of Mines, S.W.7.
October, 1927.

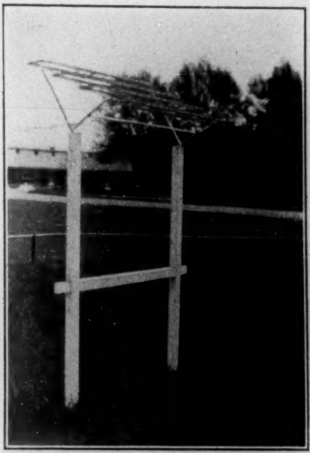
V. APPENDIX.

Design and Calibration of the Stands Used for Outdoor Exposure.

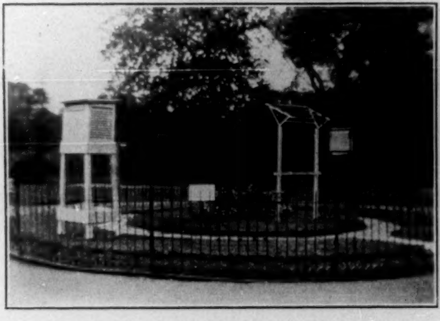
Since the corrosion of a specimen is largely determined by the volume of air circulating over it, it is necessary to pay considerable attention to the design of the stand used for exposure, so as to ensure that all the positions in the stand are equivalent in this respect.

The stand installed on the roof of the Royal School of Mines, and used for the tests described on page 116, consists essentially of four wooden frames, made from ½ in. square rod. The overall dimensions of these frames are 20 in. by 12½ in., and each accommodates three rows of specimens, as shown in Fig. 8. The frames are supported in a horizontal position at different heights from the ground, so as to minimise any possible protection of one set of specimens by another. The stand was calibrated by determining the weight increments of a number of specimens of three different metals, which may be referred to as *A*, *B* and *C*. The distance between neighbouring specimens was varied from one frame to another to ascertain whether there was any mutual protective effect on the part of the specimens. In view of the labour involved, it was thought sufficient to weigh a selection of the exposed specimens, and to fill the remaining positions with similar dummy coils.

The weight increments after 14 days' exposure are shown in Fig. 7. It will be seen that the packing of the specimens has no perceptible effect on corrosion.



STAND USED FOR THE RESISTANCE TESTS.

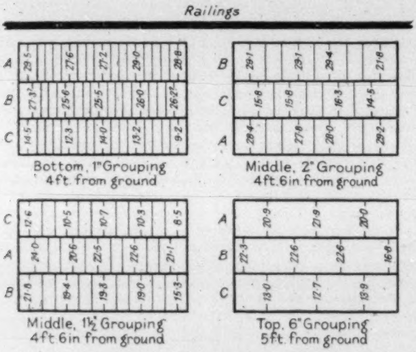


THE EXPOSURE STATION AT THE ROYAL BOTANIC GARDENS, REGENT'S PARK.

FIG. 8.

To face page 128

(The abnormal results at the ends of some of the rows are due to interference by the woodwork of the stand.) There is, however, a great difference between the results obtained at the back and front of the stand respectively. Thus, the average weight increment of the *B* coils in the right-hand frame near the railings is 29 mgms., whilst the corresponding figure for the specimens in the back left-hand frame is 19 mgms. The difference is probably due to the position of the stand at the edge of the roof, as a result of which the specimens at the back are more sheltered from the wind beating against the wall of the building. A roof is, therefore, not an ideal site for carrying out comparative corrosion tests, unless it is sufficiently wide to enable the specimens to be placed at a reasonable distance from the edge. In the present case it is doubtful if these local disturbances could be completely eliminated, and consequently the most recent experimental work has been carried out in the Royal Botanic Gardens, Regent's Park, where much more uniform conditions of exposure are obtained.



Bottom, 1" Grouping 4 ft. from ground

Middle, 2" Grouping 4 ft. 6 in. from ground

Middle, 1½" Grouping 4 ft. 6 in. from ground

Top, 6" Grouping 5 ft. from ground

FIG. 7.—CALIBRATION OF THE STAND ON THE ROOF OF THE ROYAL SCHOOL OF MINES.

A new type of stand has also been designed, in which any interference by the material of the stand itself is reduced to a minimum. The specimens are exposed on wooden frames of ½ in. square rod, 4 in. wide, by pushing the terminals into holes drilled in the wood. These frames are bolted to two triangular brackets made of ½ by $\frac{3}{16}$ in. flat iron, as shown in Fig. 8. (The whole stand is, of course, adequately painted.) The lowest frame is 4 ft. 9 in. from the ground, and there is a vertical and horizontal clearance of about 3 in. between adjacent frames. The results of a calibration of the stand, given in Fig. 9, show that all the positions in the stand are equivalent within the limits of the experimental error.

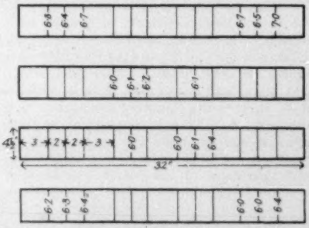


FIG. 9.—CALIBRATION OF THE STAND AT THE ROYAL BOTANIC GARDENS.

(The figures are the weight increments of copper specimens after 21 days' exposure in May, 1926.)

DISCUSSION.

Mr. J. H. COSTE said that the Author's method struck him *prima facie* as being satisfactory, unlike certain methods in which attempts were made to scrape off the corroded metal. Had the Author any data as to the composition of the air to which the specimens had been exposed?

Information on this subject could no doubt be obtained from the relevant committee of the D.S.I.R.

Dr. E. H. **Rayner** said that some time ago he had advised the Author against the use of bridge methods, on account of secular change in the resistance of wire joints; but the Author had overcome this difficulty with excellent results by means of the new method of jointing he had described. Had he allowed for the effect of thermal E.M.F., which in the case of copper-nickel alloys would be material in such precise measurements as those described? Difficulty is often experienced in obtaining a definite area of contact between mercury and a copper rod dipping into it. Varnishing of the copper does not help matters, but no difficulty arises if the copper be nickel-plated. It would be advantageous to include the annealing temperatures in the Paper for future reference. As the tests occupied a considerable period of time, it must be borne in mind that secular changes might occur in the standard resistances in the bridge.

Dr. R. S. **Hutton**, on behalf of the Non-Ferrous Metals Research Association, expressed appreciation of the encouragement given to the Author by the Physical Society's publication of his Paper. The problem of corrosion was so complicated that it would be most valuable to have a simple method applicable under widely varying conditions, like that described by the Author.

Dr. W. H. J. **Vernon** said that he had watched the work with great interest, and was impressed by the Author's accuracy and thoroughness. While the Society would be mainly interested in the method as such, from an economic point of view the important matter would be its application in obtaining corrosion data under a wide range of conditions. Arrangements were being made for observations at six stations engaged in research on atmospheric pollution. The method of accelerating results by reducing the diameter of the specimens was much superior to methods in which artificial corrosion was produced, since the results of the latter bore little relation to natural corrosion. Caution was necessary, since the later course of a corrosion process could not be inferred from its earlier stages; thus arsenical copper, for instance, is at first more rapidly corroded, but after long exposure much less so, than pure copper.

The **Author**, in reply, thanked Dr. Rayner for the suggestion as to the use of nickel-plated copper for the mercury contacts. He had eliminated the effect of thermal E.M.F. by adopting a careful routine, and had re-calibrated the standard resistances from time to time, in addition to using unexposed blanks as controls. By arrangement with Dr. J. S. Owens, the field work would be associated with the research which is being carried out on atmospheric pollution.

Prof. E. **Wilson** (communicated): I am very pleased to see a Paper by Mr. Hudson on the Atmospheric Corrosion of Metals, and appreciate the invitation to contribute to the discussion. On reading the Paper through I am still in difficulty as to how the variation in specific resistance due to exposure is to be allowed for in the absence of direct experiment. The following figures are taken from a Paper read before the Society,* and refer to specimens of high purity aluminium and duralumin, both of which were exposed to London atmosphere for eight years. If R_0 R_t were the electrical resistances of the wires at the commencement and at time t during exposure, ρ_0 ρ_t the corresponding specific resistances, a_0 a_t the corresponding cross-sectional areas, it follows that the ratio $R_t/R_0 = \rho_t/\rho_0 \times a_0/a_t$. It will be seen that the ratio a_0/a_t is much greater in the case of the high purity aluminium, and yet the ratio R_t/R_0 is the same for each specimen. If, therefore, the amount of corrosion is to be inferred from the change in resistance, it becomes a question as to whether the correction due to the change in resistance of the control specimen, which had been stored in the laboratory, could allow of the ratio a_0/a_t being determined with fair accuracy. The control specimens stored in the laboratory were tested, and it will be seen that no change was found in the case of the high purity aluminium, and the duralumin diminished in specific resistance. Whereas in the case of each of the exposed specimens the ratio ρ_t/ρ_0 was increased, the increase being greater in the duralumin.

——	Time t in years.	Unexposed.	Exposed.		
		ρ_t/ρ_0	ρ_t/ρ_0	a_0/a_t	R_t/R_0 observed.
High purity aluminium	8	1·00	1·061	1·127	1·15
Duralumin	8	0·954	1·109	1·038	1·15

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* Proc. Phys. Soc., 39, 15 (1926).

It might be worth while placing the control specimen in a tube filled with an inert gas and subjecting it to the same temperature changes as the exposed specimens in order to see if a better agreement could be obtained. I am glad to see that the Author has increased the diameter of the exposed conductors, and I should like to ask if special attention has been given to the possibility of error due to thermo-electromotive forces. In conclusion, I should like to congratulate the Author on his Paper, and I look forward to further contributions to what is admittedly a difficult subject.

AUTHOR'S reply: Prof. Wilson's chief point seems to be that changes may occur in the specific resistance of the exposed specimens, which will not be compensated for by observing the changes in resistance of similar specimens stored in the laboratory. His views are based on observations of the specific resistance of certain aluminium alloys before and after exposure. The final measurements were made after scraping off the corrosion product. Prof. Wilson will probably agree that the accuracy obtainable by this method is not very great, particularly in the case of an $\frac{1}{8}$-inch wire, as used in his experiments, so that the values of ρ_t/ρ_0 deduced in this way cannot be regarded as conclusive. He was perhaps unfortunate in having to deal with complicated alloys like duralumin, some of which are subject to age hardening, and presumably, therefore, to secular changes in resistance. In these cases one would expect the apparent specific resistance to increase on exposure, since Prof. Wilson's method of measuring the final resistance would not eliminate the effects of intercrystalline corrosion, to which some of these alloys are particularly susceptible. In the present investigation, however, the fact that the observed resistance changes agree with the values calculated from the accompanying changes in weight of the specimens shows that there can be no appreciable variation in the specific resistance of the materials.

It is conceivable, however, that in certain complicated alloys resistance changes might occur as a result of exposure outdoors quite apart from those due to corrosion, and I therefore agree with Prof. Wilson's kind suggestion as to the advisability of exposing the "blanks" to the same temperature changes as the exposed specimens, and I hope to test the matter experimentally in the near future.

XVII.—SOME METHODS OF ESTIMATING THE INTENSITIES OF SPECTRAL LINES.

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Received December **16, 1927.**

(Communicated by Prof. O. W. Richardson, F.R.S.)

ABSTRACT.

This Paper presents a critical account of several methods of spectral photometry applied to the special case of the band spectrum of helium. With the method finally adopted, the line spectrum is photographed in the usual way, after which the plate has to be calibrated. To do this the slit of the spectroscope is illuminated in a special manner by a tungsten filament lamp, so that upon development the plate exhibits a number of images of continuous spectra of progressively increasing density. From these images may be ascertained (*a*) the relation between intensity of light and density of image, and (*b*) the relation between plate sensitivity and wavelength of the incident light. Density measurements are obtained by a simply constructed selenium cell microphotometer, with which the density contour of each line can be investigated if necessary. Both visual and thermopile microphotometers are also described, and a list of papers dealing with the subject is given.

INTRODUCTION.

THIS investigation, which is preliminary to an inquiry into the distribution of intensity in the band spectrum of helium, was undertaken with the object of ascertaining the most convenient method for the purpose. In view of the increasing importance of intensities, it is desirable that, so far as technique is concerned, every possibility should be considered. Especially is it thought that any apparatus or method evolved with simplicity of construction as the end in view will be of service to others to whom the elaborate and expensive recording densitometers are not available.

It may be stated here that no attempt has been made to dispense with photography in an endeavour to gain directness ; the conditions in a discharge tube may vary so considerably in the time taken to examine directly only a few lines that at present it seems that the use of photography is unavoidable. Moreover, it is only in exceptional cases that discharges can be obtained of sufficient intensity to permit of direct investigation by means of the thermopile. Owing to the variation with wavelength of the sensitivity of plates, it follows that each one used must be standardised with a source of known spectral energy distribution.

The devices tried here arrange themselves naturally into two groups—in the first the length of the line image, which is dense at one end and fades away towards the other (the result of a progressive decrease of exposure), is taken as a measure of intensity, whilst in the second the spectrum is photographed in the usual manner, and the densities of the line images are translated into intensities by means of " density marks " on each plate. For convenience of exposition the neutral wedge apparatus is given first since its principles apply generally to the first group.

WEDGE METHOD A.

Of the two methods of using the Hurter-Driffield Swarzschild expression ⁽¹⁾ for the comparison of intensities, that in which the exposure time is kept constant is, on the whole, to be preferred, since in that case not only are the experimental conditions simpler, but any variations with time of the intensity of the source have no effect. Since the exposure time remains constant, it is, of course, the intensity which must be varied, and one of the simplest means of achieving this is presented by the neutral wedge placed immediately before the collimator slit. A very complete description of the use of such a wedge will be found in a series of papers by Merton and Nicholson, ⁽²⁾ on which the following abridged account is based.

Consider light of wavelength λ and intensity J_λ incident upon the wedge. If the material of the wedge has an extinction coefficient a_λ (for convenience referred to the base 10) for this wavelength, then the light which emerges at a place distant l from the thin end has an intensity J_λ' given by

$$\log J_\lambda' = \log J_\lambda - l a_\lambda \tan \theta$$

where θ is the angle of the wedge. Or if, instead, we refer to the density D_λ of the wedge at the thick end (L being the thickness).

$$\log J_\lambda' = \log J_\lambda - l D_\lambda / L$$

where $D_\lambda = \log \left(\frac{\text{incident intensity}}{\text{transm. intensity}}\right)$. There will be some point at which J_λ' becomes equal to the critical intensity which is just sufficient to affect the photographic plate. At the point on the plate corresponding to this point on the wedge the line image will end. By measuring the length of this image and dividing it by the magnification of the spectrograph, it is possible to find the point on the wedge at which the critical intensity is being transmitted. Suppose the distance from the thin end of the wedge is l_λ. Then if ${}_cJ_\lambda'$ represents the critical intensity, we have

$$\log {}_cJ_\lambda' = \log J_\lambda - l_\lambda D_\lambda / L$$

and

$$\log J_\lambda = \log {}_c J_\lambda' + l_\lambda D_\lambda / L$$

The quantity $l_\lambda D_\lambda / L$ is the logarithm to base 10 of a quantity defined in Merton's Paper as the " photographic intensity " ⁽³⁾ of the light λ. In order to be able to allow for the variation of sensitivity of the plate, it is necessary to use black-body radiation. For such radiation, if the temperature of the black-body be known, it is possible to calculate the values of J_λ in arbitrary units, taking as unity, for example, that at some convenient wavelength such as λ 4000 Å. To obtain the actual distribution in the light falling on the photographic plate these values have then to be corrected for the variation of the dispersion of the spectrograph. Having then photographed the black-body (with the same exposure time ⁽⁴⁾ and on the same plate as the line spectrum), we can measure the photographic intensities for a range of wavelengths. It follows, then, that for a wavelength λ

$${}_cJ_\lambda' = \frac{\text{Calc. int. of black-body at } \lambda}{\text{Phot. int. of black-body at } \lambda}\Bigg)$$

If now in the line spectrum a line of wavelength λ_1 is found to have a photographic intensity equal to X, then in order to obtain a true intensity from this (relative, of course, to that of the black-body at λ 4000 Å.) it is necessary to multiply X by the factor

$$\left(\frac{\text{Calc. int. of black-body at } \lambda_1}{\text{Phot. int. of black-body at } \lambda_1}\right)$$

A separate experiment is necessary to determine the values of D_λ. By means of Nicol prisms, two exposures of equal duration are given, the intensity in one case being cut down to one-tenth of that in the other by suitably orientating the prisms. If in one case the point on the wedge which is transmitting the critical intensity is distant l_λ' from the thin end, whilst in the other it is distant l_λ'' then

$$\log J_\lambda = \log_e J_\lambda' + l_\lambda' D_\lambda / L$$

$$\log (J_\lambda/10) = \log_e J_\lambda' + l_\lambda'' D_\lambda / L$$

or

$$D_\lambda = L / (l_\lambda' - l_\lambda'')$$

It is to be noted that both in the determination of D_λ and in the use of the wedge any absorption in the spectrograph is eliminated.

A wedge 7 mm. long, with a density ranging from approximately 0·2 to 4·2, was obtained, and with it a photograph of the helium band at λ 4650 Å. was taken. This band is probably the strongest in this spectrum, and so offers special advantages. Upon endeavouring to measure this photograph difficulties were at once encountered. In the first place, the wedge was too dense and called for rather long exposure times. The most serious difficulty, however, was a consequence of the fact that the ends of the lines were not well defined, and large errors of measurement were unavoidable. This can be remedied by increasing the density gradient of the wedge, but, unfortunately, not only does this prevent examination of weak lines, but at the same time the sensitiveness of the method is decreased. Two alternative methods of overcoming this difficulty which have been suggested by Merton are :—

(1) Enlargement through a process screen of a positive prepared from the original negative. The screen breaks the image up into dots, and it is easier to decide upon the last visible dot than upon the actual end of the line. This method has the disadvantage of introducing two extra photographic processes and a process screen.

(2) "Cutting" the negative with a solution of potassium ferricyanide, which removes chemical fog. This is a questionable cure, since there is always the possibility of non-uniform action of the solution on the plate.

From the plates which were obtained it was difficult to obtain consistent measurements even though resort was had to the ferricyanide treatment. Some representative results are given in Table No. 1. It seems probable that the method is of most use for quite strong lines in the visible part of the spectrum; undoubtedly in such cases, giving due consideration to the simplicity both of

TABLE I.—*Some measurements obtained using the neutral wedge method.*

Two separate measurements (*A* and *B*) of the same plate gave :—

Line No.	1	2	3	4	5	6
A	33	28	64	72	71	93
B	33	38	45	70	68	74

A wedge density of 4·2 was assumed in calculating these intensities.

construction and fundamental idea of the auxiliary apparatus required, the method leaves little to be desired. Unfortunately the construction of a wedge suitable for use in the ultra-violet presents some difficulty.(5)

WEDGE METHOD B.

This method was suggested during work with a Fabry-Perot étalon, in which the étalon had several times been used with the plates not parallel in order to separate the emergent beams in a plane parallel to the slit. If a short collimator slit is used with the étalon in this condition the appearance in the field of view of the observing telescope is somewhat modified in that all line images are bright at one end, and fade away towards the other. In this case of course the fading is discontinuous, the image consisting of a number of rectangles of decreasing intensity placed one above the other. Now suppose that light of wavelength λ and intensity J_λ is incident upon the étalon. The series of images for this wavelength have intensities proportional to

$$J_\lambda P_\lambda^2,\ J_\lambda P_\lambda^2 R_\lambda^2,\ J_\lambda P_\lambda^2 R_\lambda^4 \ldots,$$

where P_λ and R_λ are respectively the transmission and reflecting powers of one of the étalon films. If there are n of these images visible on the photographic plate it follows, since the intensity of the last visible image is the critical intensity for the wavelength λ, that

$${}_cJ_\lambda' = J_\lambda P_\lambda^2 R_\lambda^{2(n-1)}, \text{ using the notation of } (A)$$

or

$$J_\lambda = ({}_cJ_\lambda'/P_\lambda^2) R_\lambda^{2(1-n)}.$$

Comparison with the case of the neutral wedge shows that the "photographic intensity" in this case is $R_\lambda^{2(1-n)}$. Proceeding as before to standardise each plate with black-body radiation gives the factor

$$({}_cJ_\lambda'/P_\lambda^2) = \left(\frac{\text{Calc. int. for black-body at } \lambda}{\text{Phot. int. for black-body at } \lambda}\right),$$

and this may be evaluated over the whole range for which the plate is sensitive. Conversion from photographic to true intensities in the line spectrum then follows from the preceding section. To obtain the values of R_λ a separate experiment, analogous to that for the determination of D_λ of the preceding section, is required. Consider only a single wavelength and give two exposures of equal duration, one with intensity J_λ and, by means of suitably adjusted Nicol prisms, one with intensity $\frac{1}{2}J_\lambda$. If there are p images visible in the first case and q in the second, then

$$J_\lambda = ({}_cJ_\lambda'/P_\lambda^2) R_\lambda^{2(1-p)};$$

$$\tfrac{1}{2}J_\lambda = ({}_cJ_\lambda'/P_\lambda^2) R_\lambda^{2(1-q)};$$

$$\log R_\lambda = (\log \tfrac{1}{2})/2(p-q).$$

A somewhat easier though less satisfactory procedure would give two exposures of, say, $32t$ and t seconds respectively. If the first gives p and the second q visible images on the plate then, assuming that the reciprocity law(6) holds, there results

$$\log R_\lambda = \{0{\cdot}85 \log (1/32)\}/2(p-q).$$

It is admitted that this method possesses disadvantages. For example, it is possible that the last visible image for one line may be a little denser than the corresponding images for other lines. Without making measurements of density it is not possible to make accurate allowance for this, but probably the addition of one-half to the number of images would go far to compensate for it. On the other hand, the discontinuous decrease of intensity is not entirely a disadvantage, since in the case of a continuous decrease it is a difficult matter to decide upon the end of a line. A serious disadvantage of the scheme is the inflexibility which results from its dependence on the value of the reflecting power of the metal films. Although a low value of the reflecting power such as would be obtained by using thin films gives a more rapid decrease of intensity in the series of emergent beams, thus enabling the last visible image to be more easily decided upon, yet a lengthy exposure is necessary if two lines of nearly equal intensity are under examination, for most of the light is concentrated in the first few images. Any increase of reflecting power, giving a more gradual decrease of intensity in the emergent beams, implies thickening the films. The absorption losses will now be serious and the exposure time is again lengthened.

The one great advantage of the method lies in the simplicity of the apparatus required, and although when put to the test, it failed because films thin enough to allow of photography of the band lines with a reasonable exposure time had not enough reflecting power to make the method an accurate one, it is possible that it may prove of some use as a check method for strong lines which in the most favourable cases would be separated by only a few Ångstrom units. With quartz plates and platinum films the method permits of easy extension to investigations in the ultra-violet.

WEDGE METHOD C.

The devices described here are attempts to make use of the Swarzschild relation by varying the exposure time instead of the intensity. The appearance of the photographic image is exactly similar to that obtained with the neutral wedge. The first apparatus took the form of a toothed wheel which, when rotated before the slit of the collimator, varied the exposure time from a maximum at one end of the slit to zero at the other. Calculation showed that with a wheel 8 cm. in diameter and with some ten or more teeth it could be assumed that the exposure time when so varied increased linearly. This method proved to have a serious defect in that the discharge producing the band spectrum was of a semi-periodic nature, and it was not possible to prevent the rotating wheel from showing some sort of stroboscopic effect, which at once nullified its occulting action. The device was, therefore, modified, and finally took the form of a cam-driven shutter moving vertically before the slit. The motive power for the cam, which was designed to drive the shutter with constant velocity, was supplied by an electric motor geared down so that the final speed was about three revolutions per minute. Two or three satisfactory photographs were obtained. When an attempt to measure these was made, however, considerable difficulty was experienced in determining the positions of the ends of the lines, and records of the same plate differed so widely that it was clear that it was not worth while to spend time in elaborating the method. Some specimen measurements are given in Table No. II.

TABLE II.—*Some measurements obtained by varying the time of exposure along the slit.*

Line No.	1	2	3	4	5	6
A	100	100	65	48	31	11
B	80	100	55	40	25	8

DENSITOMETER METHODS.

In the methods which had so far been tried the difficulty of estimating the end of the line was always a source of inaccuracy, although considerable practice might have trained the eye and improved the consistency of the results. In the light of the experience which had been gained it seemed that a more profitable approach would be by measuring the density of lines in a spectrogram taken in the usual way; intensities could then be obtained from a density scale photographed on the same plate. The first part of the problem is one for microphotometry. The advantage of selenium in work of this kind is well known—its great sensitivity to light permitting of the construction of sensitive cells of very small size. Again, there is not with selenium that need for protection against the stray thermal radiation which is so troublesome a source of disturbance with the thermopile, nor does one require any auxiliary amplifying devices, as is the case with the photoelectric cell. The objectionable properties of selenium are well known. In spite of these it was decided to make use of a selenium cell.

FIRST (SELENIUM CELL) DENSITOMETER METHOD.

The photographic plate to be examined was placed in a carrier which could be traversed by means of a micrometer screw (one division on the divided head corresponded to a movement of 0·01 mm.) to and fro between the condensing and projection lenses. The latter was a Taylor-Hobson camera lens of focal length 115 mm. ($f/4{\cdot}5$), giving a twenty-five times magnified image at a distance of about 3 metres. A hundred candle-power half-watt lamp was used for a light source. The linear selenium cell, designed by Professor Thirring, consists of a narrow strip of selenium mounted between two brass blocks upon which the terminals are mounted. The dimensions of the active selenium surface (which is protected by a thin transparent plate) are 11 mm. by 0·17 mm.

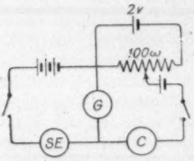

DIAGRAM 1.

The literature of the subject indicated that a procedure advocated by Fournier d'Albe(7) had met with some success. The cell was to be made part of a battery bridge; alternately illuminating and darkening the cell for equal periods of time then caused the galvanometer spot to travel to and fro over the scale, and when after a short time the amplitude of this became constant it was taken as a measure of the intensity of the incident light. In order to make certain that the alternations were regularly performed, a revolving shutter was constructed of three 60° sectors, and fitted to the spindle of a stop-clock. This shutter was placed immediately before the cell, and in this way periods of ten seconds light and darkness were automatically obtained. The circuit employed with the selenium cell is shown in Diagram No. 1. The cell SE had a dark resistance of the order of 20 megohms, and was balanced by the graphite line resistance C, value 1 megohm. The galvanometer G was of the Onwood moving coil pattern, with a resistance of 1053 ohms and a sensitivity of about 10^{-8} amps. per mm. at 1 metre. Usually in such a circuit as this a galvanometer of much higher resistance would be employed; but since in this particular case it was found that ample sensitivity had been obtained, the Onwood galvanometer (unless otherwise stated) was used throughout. Some trials were first made with a modification of the method; the cell was kept dark most of the time, and illumination was confined to periods of only 1/50 sec., the resulting kick of the ballistic galvanometer being taken as a measure of the light intensity. The results were not good, but it is possible that they could have been improved by the use of a more accurate shutter. A test was next made of the relation (which according to Fournier d'Albe is linear) between light intensity and amplitude of galvanometer swing. A knowledge of this relation is not necessary to the working of the method if density marks are on each plate; but unless it is known, however, it is not possible to obtain the actual densities of the various parts of the plate. A small pea-lamp was placed at various distances from the cell, and the corresponding amplitude for each position of the lamp obtained. These amplitudes were then plotted against arbitrary units of intensity. The curve was not linear, but showed a decrease in slope for increasing intensities, as shown in Diagram No. 2. It was thought that this might be due to reflected light reaching the cell; but, although careful screening removed all possibility of this, the shape of the curve remained the same. For this reason it was not possible, with this method, to convert easily plate readings into densities. This result will not be discussed further, for it seems that it is usual with commercial selenium cells containing metallic selenium. (8) The next step was to make measurements on some band-lines, of which several plates had been obtained on the Hilger constant deviation spectrometer. These lines were at this stage of the work not directly the subject of the investigation; two of the best plates, therefore, were chosen, and attention was confined mainly to these in an endeavour to obtain constant results. The two chosen plates came respectively from batches of the Ilford Panchromatic and Empress brands. The procedure adopted with these plates was as follows :—

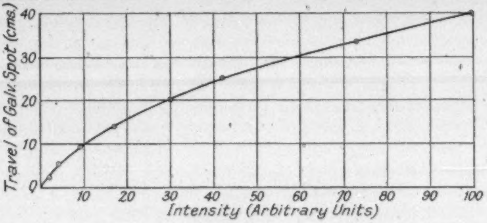

DIAGRAM 2.

Measurements were first made on the density marks (five in number), when the plate was removed and replaced so that the band-lines, some twenty in number, were projected on the plane of the cell, care being taken to see that the plate was so orientated that the cell came truly down the middle of each line. The plate was then moved along so that each line in turn fell on the cell, observation of the galvanometer spot (without the shutter rotating) showing when the cell was covered by the densest part of the line. Then the shutter was started, and usually after five or six alternations of intensity the amplitude of swing of the galvanometer spot became constant. At the end of a run the motion of the plate was reversed, and some of the first measurements repeated to make sure that the constants of the cell were unaltered. The measurements of the density marks provided material for the construction of an intensity calibration curve, from which, by interpolation, the intensities of the band-lines, in arbitrary units, could be obtained. The process is somewhat lengthy, but the results (of which specimens are given in Table No. III) both in consistency and accuracy were vastly superior to any that had been obtained previously. It was during these measurements that a fatigue effect was noticed when the cell was subjected to constant intense illumination, and some special experiments were made to ascertain the nature of this phenomenon. The cell was placed at a distance of about twelve inches from a small lamp which, with an ammeter, was supplied with current from an accumulator. The voltage was applied to the selenium cell, and the bridge was balanced about an hour before the test was made. The position of the galvanometer spot was quite constant during this period. The lamp circuit was then made, and readings of both the ammeter and bridge galvanometer were taken over a period of some hours. The existence of the fatigue was clearly demonstrated. In order to see to what extent the fatigue might be modified by previous treatment (with the object of fully fatiguing the cell before use) some separate experiments were made. These showed that much of the effect could be removed by a previous strong illumination, although to a slight extent it was always in evidence.

TABLE III.—*Some measurements obtained using the selenium cell as a photometer.*

Patch No.	1	2	3	4	5
A (March 4), mm.	126	96	54	35	29
B (March 6), mm.	127	98	55	37	30

The readings are the amplitudes of the swing of the galvanometer spot.

Line No.	1	2	3	4	5	*6	*7	*8
Intensity in Arbitrary Units.								
A (Jan. 5)	30	42	47	43	37	31	28	18
B (Feb. 13)	30	43	48	45	39	35	25	16

Both *A* and *B* were obtained from the same plate, measured on the dates given at the head of the columns.

A little difficulty was experienced with the varying opacity of the unexposed parts of the panchromatic plates, and it seems that it is this fault which will probably limit the accuracy of photographic methods. This necessitated the measurement of a number of unexposed parts of the plate and the adoption of a mean value. One source of error was found to arise from imperfect focussing of the lines on the cell. If the focussing was not attended to so that the best conditions prevailed the results were not consistent. The method, in view of the results obtained, is clearly a practicable one, and its only disadvantage lies in the fact that it is somewhat lengthy. Nevertheless, it was thought that existing knowledge of the properties of selenium pointed to the fact that it was better to use it as a judge of equality of illumination rather than directly to make its change of resistance a measure of the intensity of illumination. It was consequent upon this decision that a modification of the device used by Toy was constructed.

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* These lines are not fully exposed. In such cases as this the more horizontal part of the density-intensity curve of the photographic plate is being used, so that any errors in setting the line on the cell, or any irregularities in the film, are here relatively more important.

SECOND (SELENIUM CELL) DENSITOMETER METHOD.

The apparatus was arranged as shown in the accompanying Diagram No. 3. The selenium cell *A* and plate and projection system *B* are identical with those of the last method. At *C*, however, is a device for projecting an illuminated patch on the cell. The intensity of this patch can be regulated at will by sliding the shutter 1, which carries a scale moving over a pointer, before the slit in the screen 2. The selenium cell and the ground glass 3 are at conjugate points of the cylindrical lens 4. The illuminating lamp *C* is of the same size and type as that in the projection system and the two are so connected that by means of a switch it is possible to change over very rapidly from one to the other. A future apparatus would be constructed on a much smaller scale and in this case two shutter-controlled beams from the same lamp (instead of two separately controlled lamps) would be installed. It was not practicable to use only one lamp with the existing apparatus and in use the two lamps caused no difficulty.

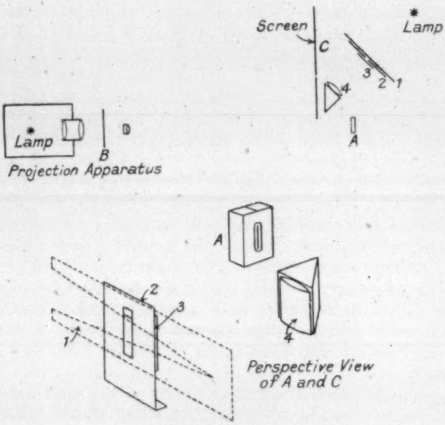

DIAGRAM 3.—ARRANGEMENT OF APPARATUS IN SELENIUM CELL PHOTOMETER.

The procedure adopted when measuring was similar to that of the preceding method; the plate was moved along until the image of the line under examination fell upon the cell, when further slight adjustments were required until the galvanometer showed that the cell was correctly placed in the densest part of the image. The shutter 1 was then placed in such a position that upon changing over rapidly from one lamp to the other no movement of the galvanometer spot occurred. When this condition is satisfied it is evident that the intensity of the shutter-controlled patch (of which the reading on the scale attached to the shutter 1 is a measure) has the same value as that of the portion of the image of the plate which falls on the cell. It was arranged that the minimum scale reading corresponded to that position of the shutter in which no light reached the cell, whilst the maximum reading (by suitably adjusting the position of the lamp at *C*) gave about the same illumination on the cell as the clear part of the plate under examination.

The method was simple of operation and comparatively rapid whilst in sensitiveness and accuracy it left nothing to be desired.

The only fault was its inability to become self-recording. As with the previous method some care was necessary when focussing otherwise consistent results were not obtained. However, it was remarked that, of two consecutive runs of the same plate, although the actual intensities on the arbitrary scale were not exactly the same, yet when reduced to terms of the strongest line a good agreement was obtained. Some typical results obtained with this method are shown in Table No. IV.

TABLE IV.—*Some measurements made with a Toy and Rawling type densitometer.*

Patch No.	1	2	3	4	5	6
A (March 27, 1926) ...	78	54	23	11	8	6
B (Jan. 6, 1927) ...	80	52	22	10	8	6

The figures are the readings on the densitometer scale.

Line No.	1	2	3	4	5	6	7	*8
Intensity in Arbitrary Units.								
March 27, 1926	32	45	51	47	42	37	28	18
Jan. 6, 1927	34	44	49	46	42	37	28	22

The apparatus, too, has further possibilities in that with it the density contour of a line may be obtained, from which in some cases the presence of an unresolved component might be inferred. For example, with the Hilger constant deviation spectrometer which as here adjusted was just able to resolve $Q1$ and $Q2$ of the He 4650 band, an interval of 0·9 Å.U., the lines $P4$ and $Q9$ (being separated by only 0·5 Å.U.) were not resolved. On obtaining the contour of the line $P4$, which to visual examination was single, the presence of another line was at once apparent. A control experiment on $P5$ showed that the contour for this line was quite symmetrical (*see* Diagram No. 4). Since the width of a line image when projected on the cell is about 1·5 mm., whilst that of the cell is only 0·17 mm., the reason for this sensitiveness is obvious. The superior sensitivity of selenium which permits of the construction of so small a detector is, of course, a contributory factor. It will be seen that the resolving power of this densitometer was quite sufficient for the spectrograms which were used. In order to be quite certain that these selenium cell densitometers were giving correct indications a model of the more usual thermopile type was constructed. A short description of this is included here.

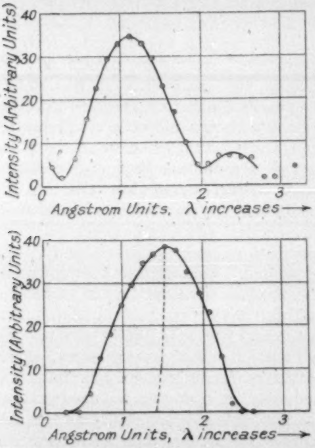

DIAGRAM 4.—THE CALCULATED RESOLVING POWER OF THE SPECTROMETER, FOR ZERO SLIT WIDTH, IS 8600. THIS CORRESPONDS TO AN INTERVAL OF 0·55Å.U.

The upper curve refers to $P5$.

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* This line is underexposed.

THIRD (THERMOPILE) DENSITOMETER METHOD.

In accordance with the general aim of this work the construction was simplified as much as possible. Much of the detail can be gathered from Diagram No. 5. The 100 candle-power half-watt lamp *A* is turned so that its semicircular filament lies in the vertical plane which contains the optic axis of the system. When in this position a very intense image (approximately linear) can be obtained on the slit *B* which is 0·04 mm. wide and about 10 mm. long. This slit can very conveniently be constructed of razor blades of the paper-steel type clamped in a holder similar to that shown. The slit now acts as source for the microscope objective *C*, 3 cms. focal length, which projects a reduced image of the slit on to the photographic plate *D*. The plate is carried in a traversing holder. The lens *E* is used to image the illuminated portion of the plate on the thermopile *F*, which with its brass mounting was taken from a Hilger infra-red spectrometer. The resistance of the thermopile was 8·6 ohms. The galvanometer was of the Broca type and had a resistance of about 12 ohms; the sensitivity when in use was of the order of 10^{-10} amp., but owing to a heavy magnet system the period at this sensitivity was rather long. Three concentric iron shields provided adequate magnetic screening. A cardboard tube lagged with cotton wool placed about the thermopile prevented disturbances due to draughts, but unfortunately this was not able to stop a slow drift of the galvanometer spot due presumably to radiation from the observer. It was finally necessary to make the instrument self-recording. This was done with the aid of a small motor, which drove the micrometer screw of the plate holder through a reduction gear, and a recording camera. Upon examining the records obtained with this apparatus it was found that, so far as the density marks were concerned, there was no reason to suspect those obtained by the preceding method. With the lines the agreement is bad, and from the photographic trace it is clear that this must be due to lack of resolving power of this densitometer. It is a difficult matter to limit the illuminated area on the plate to a strip smaller than a spectral line, and yet to pass enough energy to enable the galvanometer to be used with a moderate sensitivity. As a check on the measurements a plate was sent to Messrs. Adam Hilger for examination on their Moll densitometer. Unfortunately, only the low resolving power apparatus was available and the trace from this showed the same characteristics as the one already obtained—good agreement for the density marks but poor agreement for the lines. It is a surprising circumstance, however, that the intensities, when given in terms of that of the strongest line, have values in fair agreement with the rest. (*See* Table V.)

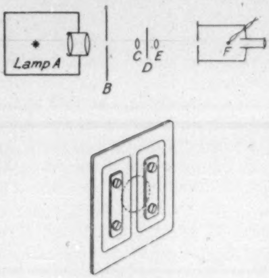

DIAGRAM 5.

It was now that, through the kindness of Professor L. C. Martin, of the Royal College of Science, an opportunity was obtained to use an optical densitometer.

TABLE V.—*Results obtained from the thermopile densitometers, compared with some obtained with a selenium cell.*

Line.	Moll. inst. A.	Lab. inst. B.	Se. cell. C.	A′.	B′.	C′.
Q3	42	60	71	100	100	100
4	37	54	65	88	90	92
5	31	45	58	74	75	82
6	29	39	48	69	65	68
7	18	23	29	43	38	41
P3	30	38	51	72	63	72
4	29	38	50	69	63	70
5	27	36	40	64	58	56
6	19	24	29	45	40	41
R1	35	48	59	83	80	83
2	33	47	57	79	78	80
3	30	43	54	71	72	76
4	26	38	47	62	63	66
5	22	30	35	52	50	49

The values A, B, C should be identical.

The column A′ is obtained by multiplying the values of column A by a factor which makes the intensity of $Q3=100$. The values B′ and C′ are derived similarly.

FOURTH (OPTICAL) DENSITOMETER METHOD.

For a complete account of the principle and construction of this apparatus reference should be made to Trans. Opt. Soc., 26 (1924-25), 109. Briefly, however, the apparatus (shown in Diagram No. 6) consists of a microscope containing a Lummer Brodhun prism, which divides the field of view into two parts. The central part of the field is occupied by the portion of the plate under examination illuminated by the beam which travels via condenser, reflecting prism, diaphragm and opal glass No. 1. The outer portion of the field is illuminated by the beam which comes via the neutral tinted wedge and opal No. 2. The neutral wedge is moved to and fro until both parts of the field are judged to be of equal intensity. The scale reading of the pointer attached to the wedge is then taken as a measure of the intensity, or if the wedge has previously been calibrated, the actual density of the image on the plate can be obtained. The apparatus, which was more or less an experimental model, proved quite sensitive, and the intensity values obtained with it agreed well with those obtained by other methods. Some specimen results are given in the adjoining Table No. VI. It was noticed, however, that measurement of more than a few lines was a little tiring, and with fatigued sight the accuracy fell rapidly. It is also somewhat difficult to match the two parts of the field if the tint of the wedge is not the same as that of the exposed plate.

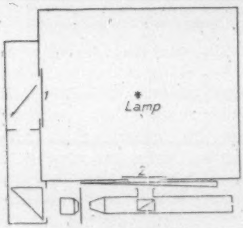

DIAGRAM 6.

TABLE VI.

Line	Q1.	Q2.	Q3.	Q4.	Q5.	Q6.	Q7.
Densitometer Reading, July 12 ...	124	110	106	110	120	131	137
,, ,, July 14 ...	122	109	106	111	119	129	135

The readings are of the scale over which the neutral wedge moves.

CONVERSION FROM MICROPHOTOMETER READINGS TO INTENSITIES.

The Calibration Marks.

These marks were obtained by means of a stepped aperture, which was illuminated by a tungsten filament lamp, and a cylindrical lens. In this way there were projected on to the slit of the spectroscope five images (of continuous spectra) of progressively increasing density.

The intensity scale was examined in the following manner. Two sets, say, A and B, of intensity marks were obtained on a plate, the intensities in set A being 4/3 those in set B. This was done approximately by varying the width of the spectrometer slit. The densities of each set of images were then measured and plotted against the intensity-mark number, and in each case a smooth curve drawn through the points so obtained. It is now necessary to search for points in A and B which have the same density. Suppose marks Nos. 1 and 1·5 in set A have the same densities as those of 1·5 and 2 respectively in set B. From this information it is deduced that intensity-mark 1·5 has 4/3 the intensity of mark 1, and that mark 2 has 4/3 the intensity of mark 1·5. Proceeding in this manner the relative intensities over the whole scale can be determined. The result of such an investigation for one of the apertures which was used is shown in Diagram No. 7.

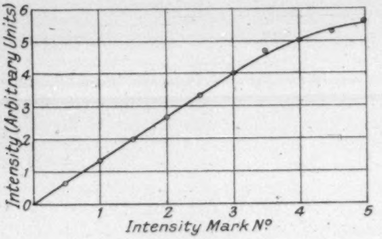

DIAGRAM 7.

The Change in Plate Sensitivity.

Since the photographic plate is not equally sensitive to all parts of the spectrum a further calibration is necessary, especially when the range of the line spectrum under examination is large, or when the lines fall in a region in which the sensitivity of the plate varies rapidly. It must be remembered too, that selective absorption will occur in the spectroscope, but this can be included in and determined with the characteristics of the plate. The method by which these characteristics are usually determined consists of photographing a source of known spectral energy distribution, and evaluating its distribution, neglecting the peculiarities of the plate. Comparison of the measured with the known distribution then enables a correction factor for each wavelength to be determined. The most convenient source is a black-body, and both the positive crater of the carbon arc and hollow tungsten filaments have been employed in this capacity. Much information on this point may be gathered from reference No. 37. An estimation of the sensitivity variation may also be made by making use of the fact that within the visible spectrum the energy distribution of a tungsten filament lamp is approximately that of a black-body at some slightly higher temperature known as the "colour temperature." The relation between true and colour temperatures is given in reference No. 37. The true temperature of the filament may be determined from its change of resistsance with temperature, or by means of a pyrometer. At high temperatures a sufficiently approximate formula for the calculation of the distribution is

$$J_\lambda \propto \lambda^{-5} e^{-a/\lambda T},$$

where $a = 1{\cdot}445$ cm. degree. If a prism spectrograph is used the dispersion of the prism must also be taken into account.

The Conversion to Intensities.

To illustrate the final steps of the work, some measurements of the λ 4650 helium band obtained with a Toy and Rawlings' type densitometer, are given in Table VII. From the calibration marks a curve is drawn connecting densitometer reading with intensity. The relative intensities of the spectral lines are then obtained by interpolation on this curve. It should be added that, under the circumstances in which the plate was obtained, there was no correction for change of sensitivity.

TABLE VII.—*Measurements on* λ *4650 helium band.*

Intensity	0	20	40	60
Densitometer Reading	80	52	22	10

Line.	Densitometer Reading.	Intensity.
Q 1	30	34
2	18	44
3	14	49
4	16	46
5	20	42
6	26	37
7	40	28
R 1	30	34
2	31	33
3	30	34
4	33	32
5	38	29
P 2	52	20
3	32	33
4	28	36
5	35	31
6	46	24
7	58	17

Comparison of Results.

A survey of the results gained by means of the methods previously described leads to the following estimates of the probable errors to be expected: Method A, probable error, 25 to 30 per cent.; method C, 10 per cent. First and second densitometer methods, densities 2 per cent., intensities 5 to 7 per cent. Fourth densitometer method, 7 per cent.

CONCLUSION.

From the results which have been obtained so far the following conclusions may be drawn. Firstly, that although these methods of which the neutral wedge is typical may be very useful with strong lines, with weaker lines such as are found in the helium bands it is better to photograph the lines in the usual way and endeavour to interpret intensities from densities. Secondly, that although it is not impossible to obtain density measures directly from the indications of the selenium cell, it is far better used, both as regards convenience and sensitiveness, in a secondary position merely as a detector of equality of illumination. And lastly, that although the method due to Toy may suffer from the disadvantage that it cannot easily be made self-recording, yet in sensitiveness and reliability it gives greater advantages than any other which has so far been tried.

In conclusion, the writer wishes to express his indebtedness to Professor W. E. Curtis, by whom the problem was suggested, for advice and criticism throughout the work.

REFERENCES.

(1) This expression states that density of photographic image $=\gamma \log (Jt^{p/i})$, where γ, p, i are constants, J represents intensity and t time.

(2) Phil. Trans., 216, p. 459 ; 217, p. 237 ; 220, p. 137.

(3) Since if we assume that ${}_{c}J_{\lambda}'$ is independent of λ, which amounts to the same thing as neglecting the variation of the light-sensitivity of the plate, the quantity $10^{l_\lambda D_\lambda / L}$ is a measure of J_λ.

(4) If this is not adhered to it may happen, if the index in $J.t^p$ is a function of λ as it sometimes is, that the values of ${}_{c}J_{\lambda}'$ calculated from the black-body cannot truly be applied to the line spectrum.

(5) T. R. Merton. Proc. Roy. Soc., 106 (1924), 378.

(6) For constant density $J_\lambda . t^p =$const, where $p \simeq .85$.

(7) Proc. Roy. Soc., A. 89 (1913), 75.

(8) Gudden and Pohl. Zeits. f. Phys., 35 (1925), 243.

(9) Proc. Phys. Soc., 36 (1924), 432.

BIBLIOGRAPHY OF DENSITOMETERS, MICROPHOTOMETERS, WITH SOME EXAMPLES OF SPECTRAL PHOTOMETRY.

P. P. Koch, Methods of Photographic Spectral Photometry, Ann. der Phys. 30 (1909), 843.

P. P. Koch, Recording Microphotometer, Ann der Phys. 39 (1912), 705.

King and Koch, Registering Microphotometer, Astrophys. Journ. 39 (1914), 213.

Richtmyer, Null Method of Photo-electric Photometry, Phys. Rev. 6 (1915), 66.

Stetson, Thermo-electric Photographic Photometry, Astrophys. Journ. 43 (1916), 325.

Baillaud, Photographic Photometry, Ann. der Phys. 5 (1916), 131.

Kunz, Photo-electric Photometry, Astrophys. Journ. 45 (1917), 69.

Benson, Renwick and Ferguson, A Photometer for Photographic Densities, Phot. Journ. 58 (1918), 155.

Kohler, Selenium Photometry, Elekt. Zeits. 40 (1919), 104.

Fabry and Buisson, A New Microphotometer, Journ. de Phys. 9 (1919), 37.

A. H. Compton, A Photo-electric Photometer, Am. Illum. Eng. Soc. Trans. 15 (1920), 28.

Meggers and Foote, A New Microphotometer for Photographic Densities, Opt. Soc. Am. J. and Rev. Sci. Inst. 4 (1920), 24.

Goos, A Recording Microphotometer, Zeits. Instrument 41 (1921), 313.

Moll, A New Registering Microphotometer, Proc. Phys. Soc. 33 (1921), 207.

Schoen, Adaptation of Thalofide Cell to Measurement of Photographic Densities, Opt. Soc. Am. J. and Rev. Sci. Inst. 7 (1923), 483.

Pettit and Nicholson, A Registering Microphotometer, Opt. Soc. Am. J. and Rev. Sci. Inst. 7 (1923), 187.

Toy, A Selenium Photometer, Proc. Phys. Soc. 36 (1924), 432.

E. A. Baker, A Photo-electric Photometer and Densitometer, Journ. Sci. Inst. 1 (1924), 345.

Siegbahn, A Registering Microphotometer, Phil. Mag. 48 (1924), 217.

Merton and Nicholson, Wedge Methods, Phil. Trans. 216 (1916), 459; 217 (1918), 237; 220 (1920), 137.

Beatty, On the Distribution of Energy in the Lines of the Balmer Series of Hydrogen, Phil. Mag. 45 (1923), 1079.

Sampson, The Estimation of Continuous Spectrum in Stars, Monthly Notices R.A. Soc. 83 (1923), 174; 85 (1925), 212.

Fassbender, Intensities in some Nitrogen Bands, Zeits. f. Phys. 30 (1924), 73.

Hansen, Standards of Intensity for Calibrating Photographic Plates, Zeits. f. Phys. 29 (1924), 356.

Harrison, Construction of a Precision Densitometer, Opt. Soc. Am. J. and Rev. Sci. Inst. 10 (1925), 159.

Albrecht and Dorneich, An Automatic Registration Photometer, Phys. Zeits. 26 (1925), 514.

Rosenberg, A Photo-electric Photometer, Zeits. Instrument. 45 (1925), 313.

Jones, A Method for Photographic Spectral Photometry, Opt. Soc. Am. J. and Rev. Sci. Inst. 10 (1925), 561.

Lambert and Chalonge, A Recording Microphotometer, Comptes Rendus 180 (1925), 924.

Martin, An Optical Microphotometer, Trans. Opt. Soc. 26 (1924), 109.

Dobson, A Photo-electric Photometer, Proc. Roy. Soc. 104 (1923), 248; Engineering 122 (1926), 403.

Goldschmidt, A New Photo-electric Photometer, Meteor. Zeits. 43 (1926), 241.

Kurrelmeyer, Use of Arcs in Photo-electric Photometry, Nature 117 (1926), 657.

A. Danjon, A Differential Microphotometer, Rev. d'Optique 5 (1926), 55.

Sewig, Intensities in Band-spectra. CN 3883, 4216. N$+$3 914, Zeits. f. Phys. 35 (1926), 511.

Frerichs, Spectral Photometry with Large Dispersions, Zeits. f. Phys. 35 (1926), 524.

Dobson, Griffith, Harrison, Photographic Photometry, Oxford Univ. Press.

Dorgelo, Photographic Spectral Photometry, Phys. Zeits. 26 (1925), 756.

Toy, A New Photoelectric Densitometer, Journ. Sci. Inst. 4 (1927), 369.

DISCUSSION.

Mr. J. O. C. VICK asked the Author whether he had tried the method of sharpening the end of the line image obtained with the wedge which consists in contact-printing from the original negative, and again printing if necessary from the copy, the process being repeated as often as may be required. In using the Toy and Rawlings densitometer, how had he controlled the cross-section of the two light pencils to be compared? He might find useful the new design of this apparatus, which employs a photo-electric cell associated with a valve amplifier, and is adapted to the photometry of fairly broad spectral lines.

The AUTHOR replied that he thought that repeated contact printing would destroy the sensitiveness of the wedge method as applied to investigations on spectral lines. As to the new apparatus, he thought that it was a great advance on Dr. Toy's earlier model.

Dr. L. C. MARTIN : I should like to thank Mr. Childs for his interesting and useful Paper. Has he made any attempt to apply the wire-grating method of obtaining spectra of known relative intensities ? Also, I should like to know how much value he attaches to the density contours of spectrum lines which can be obtained with densitometers at present available. The correction for finite width of slits is often likely to mask the true density, and with the photographic plates available it is very difficult to obtain results which are not seriously affected by the size of the grain. Trials of photographs of lines with known relative intensities have not always been very encouraging, especially with recording instruments.

AUTHOR'S reply to Dr. Martin : Dr. Martin raises several interesting and important questions. I have not tried the wire-grating method which he suggests. Although I understand that its use in astronomical investigations has met with some success, application to a problem such as that mentioned in the Paper would probably introduce difficulties into the photometry of the resultant negative, owing to the continuous change of intensity on going from one order grating image to the next. The density contours shown in the Paper are, of course, of little use from an intensity point of view, since integrating under the curves would probably be stretching the method a little too far. What was intended, however, was to demonstrate that a simple photometer, such as the one described, was capable of giving more information (as to non-symmetry) than could be gathered from visual examination. The effect of the grain of photographic plates is admittedly troublesome, especially with the fastest plates. Most microphotometers do much to mitigate the effect by integrating the transmitted light over a length of the line image. It must be remembered, too, that the effect of grain will probably be much the same in both line image and calibration mark

XVIII.—ELECTRICAL METHODS OF HYGROMETRY.

By P. W. Burbidge, *M.Sc., B.A.*, Professor of Physics, and N. S. Alexander, *M.Sc.*, Auckland University College, New Zealand.

Abstract.

An account is given of experiments performed to investigate the possibility of applying electrical methods to the measurement of humidity. Two methods are considered, depending on (1) the change in resistance of organic materials (cotton-wool and human hair) on exposure to water vapour; (2) the change in mobility of ions due to water vapour.

Results show that in the first case the change in resistance with humidity is not linear, the logarithm of resistance being proportional to the humidity, while in the second the effect produced is too small to permit of accurate measurement. Thus, in neither case does the method seem suitable for general use, though each could be applied in special cases.

Of the three types of hygrometer in general use for the measurement of atmospheric humidity—viz., the wet and dry bulb, dewpoint, and hair hygrometers—each has certain disadvantages, and none appears capable of a high degree of precision. The present investigation was undertaken with a view to determining the possibility of applying electrical methods to the measurement of atmospheric humidity. Two methods were considered, the first depending on the change in electrical resistance of organic materials on absorption of water vapour, the second on the variation in mobility of ions caused by water vapour.

I. Electrical Resistance Method.

Masson and Richards[*] investigated the mass of water absorbed by cotton on exposure to air of various humidities, a more detailed study of the same phenomenon being carried out later by Trouton and Miss Pool.[†] A considerable lag is indicated, and the relation between vapour pressure and mass absorbed appears to be parabolic, Masson later indicating an asymptotic approach to saturation values. These investigations showed that the mass of water absorbed depends on the relative humidity and not on the absolute vapour pressure.

Barr[‡] describes an electrical resistance hygrometer, but abandons it on account of the large lag, which, however, appears to be due to the form of the instrument.

In the experiments now described, a layer of pure cotton-wool, of such thickness as to be semi-transparent, was placed between two electrodes in the form of a grid (Fig. 1), the whole being mounted through insulating bushings inside a metal box with a close-fitting lid.

The air in this box could be brought to any desired humidity by circulating it by means of a fan over the surface of sulphuric acid solutions of known density contained in large shallow dishes.

The resistance was measured directly by means of a high potential (100-500 volts) and galvanometer. Ohm's law was obeyed, and no polarisation effect could be observed.

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[*] Proc. Roy. Soc. (1906).

[†] *Ibid.*

[‡] Proc. Roy. Soc. (Feb., 1922).

The variation of resistance with humidity was found to be such that, on plotting the logarithm of resistance against vapour pressure (at constant temperature), a straight line was obtained (Fig. 2). At high humidities the resistance decreased enormously, and trustworthy readings could not be obtained above 95 per cent. humidity.

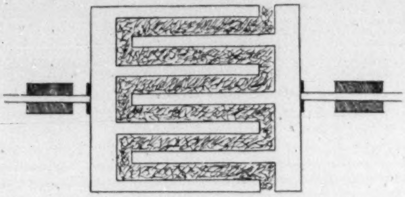

FIG. 1.—COTTON-WOOL RESISTANCE UNIT.

The lag appeared excessive, but this was attributed to the time taken for the air to come into equilibrium with the solution, so further experiments were made to eliminate this effect.

The specimen, as before, was contained in a small box which could be placed in communication with either of two larger boxes containing air at the desired humidity, the air being thoroughly mixed by means of a fan. Humidities were measured by means of a dew point hygrometer of a form due to Holtzmann,[*] by means of which the humidity of the air close to the specimen could be determined (Fig. 3).

The lag was found to be of the order of 5 to 6 minutes, varying with the thickness of the specimen, but fairly constant over wide ranges of humidity. A more accurate calibration of the specimen over a limited range confirmed previous observations, and showed an agreement within 2 per cent. between values obtained with rising and falling humidities.

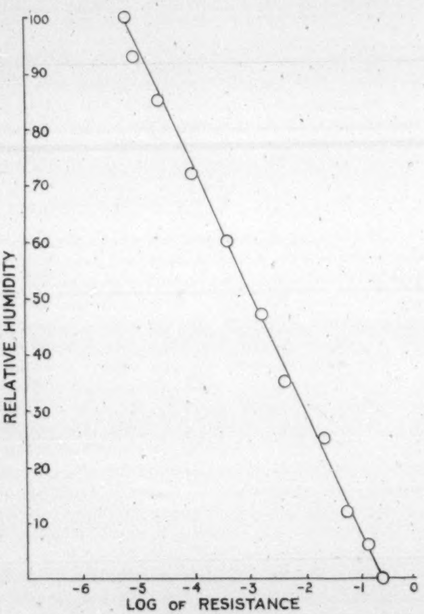

FIG. 2.—VARIATION OF RESISTANCE WITH HUMIDITY (COTTON-WOOL).

Temperature Effects.—The relative humidity was kept constant at 70 per cent., and the temperature of the air varied by circulating it over a heating coil, the temperature

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[*] Phys. Zeit. (Sept., 1924).

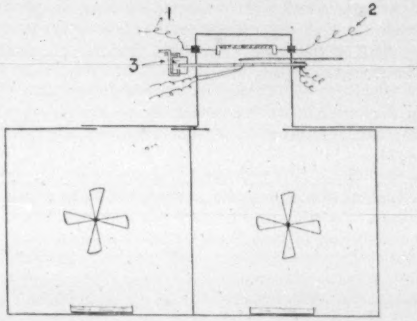

FIG. 3.—SECOND FORM OF HUMIDITY APPARATUS.

1. High potential lead. 2. Galvanometer lead. 3. Holtzmann hygrometer.

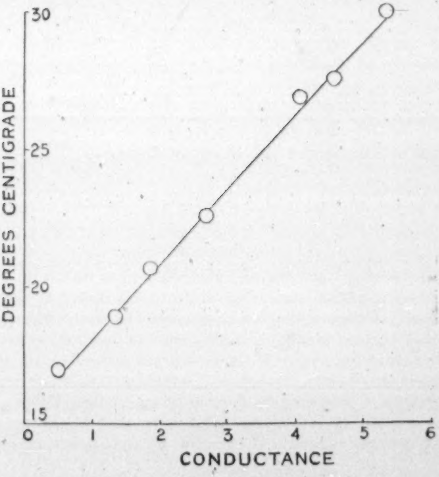

FIG. 4.—VARIATION OF CONDUCTANCE WITH TEMPERATURE (COTTON-WOOL).

being measured by means of a mercury thermometer with the bulb placed close to the specimen. A linear relation was observed between temperature and reciprocal of resistance, showing that the resistance depends more nearly on the absolute vapour pressure than on the relative humidity (Fig. 4).

The experiments were repeated with human hair freed from grease by repeated extraction with alcohol. The change in electrical resistance with humidity was found to be of a very different type from the change in length, the decrease in resistance being small from 0 to 50 per cent. humidity, but greater at high humidities (Fig. 5).

This seems to indicate that resistance and elongation are governed by separate conditions, the former probably a surface effect, the latter depending on the water-content of hair cells. In other respects its behaviour resembles that of cotton wool. Its calibration curve (log of resistance against humidity) is not linear, giving a greater proportional decrease in resistance at higher humidities. The curve obtained on plotting temperature against reciprocal of resistance is likewise not linear.

The results obtained indicate that an instrument employing this principle would be insensitive at low humidities, though it could be employed over a limited range. It would also be liable to disturbance by changes in temperature, and possibly by acid or saline impurities in the air.

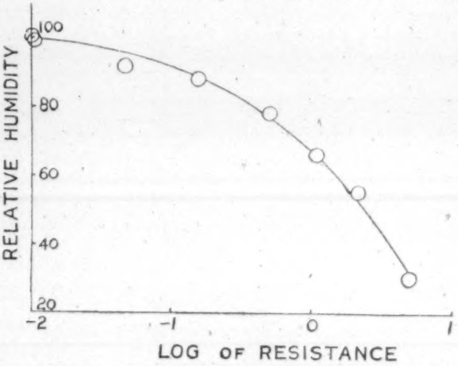

FIG. 5.—VARIATION OF RESISTANCE WITH HUMIDITY (HAIR).

After the commencement of this work, an abstract was noted in Science Progress, July, 1926, of a Paper by Albrecht,* describing a similar type of instrument.

II. IONISATION METHOD.

Tyndall and Grindley† investigated the effect of moisture on the mobility of ions, the mobility of negative ions being shown to decrease from 2·15 in dry air to 1·60 at saturation. If then ionisation is produced in air subjected to an electric field less than that necessary to give saturation, the ions will remain subject to a certain amount of recombination. If the mobility is decreased, the time for recombination, and hence its amount, should also increase to an extent depending on the water vapour present. According to Tyndall and Grindley, the positive ion when first formed has a mobility indistinguishable from that of the negative ion, and similarly affected by water vapour. The positive ion is, however, rapidly transformed into a slower moving type, the rate of transformation being decreased by water vapour. Hence in damp air it has a longer period of higher mobility, and thus possibly a higher average mobility. This would tend to oppose the effect of water vapour in diminishing the current.

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* Beitrage zur Phys. der Fr. Atm., 11, 4.

† Proc. Roy. Soc. (Feb., 1926).

Experiments were carried out to determine if the change in ionisation current could be made sufficiently great to enable the method to be applied to the measurement of humidity. A hygrometer on this principle would have no lag, an approximately uniform scale, and would measure the absolute amount of water-vapour present in the air.

The apparatus consisted of an ionisation can with two electrodes, the contained air being brought to the desired humidity by acid solutions as before. Two sources of ionisation, γ-rays, from radium and α-rays from uranium oxide freed from radioactive impurities, were used. Currents were measured by determining the rate of charge of a Wilson electroscope. Saturation curves were obtained as shown (Fig. 6). These indicate that the maximum difference in ionisation current obtainable between 0 and 100 per cent. humidity is too small to permit accurate measurement, particularly as the ionisation current would be affected by impurities such as hydrogen and sulphur dioxide, and by variations in temperature and pressure.

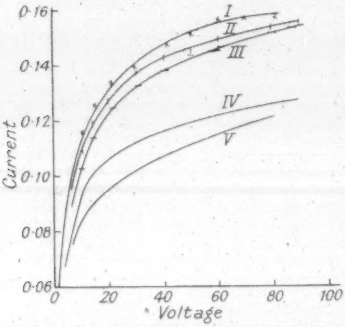


Fig. 6.—Relation between Ionisation and Humidity.
γ-Ray Ionisation—I Dry Air; II 56% Humidity; III Saturated Air.
α-Ray Ionisation—IV Dry Air; V Saturated Air.

In the Report of the N.P.L. for 1926, p. 64, reference is made to work[4] on the effect of humidity on the mobility of ions. (By opposing the motion of the ions by means of a constant air-current, effects on a larger scale than above are obtained, but the application of this principle to a simple form of hygrometer would involve the difficulty of maintaining a constant flow of gas.)

III. Conclusion.

Since the investigation showed that such instruments would not be of great value for the measurement of humidity, no further work was done. Although suffering from various disadvantages, the instruments would appear to compare favourably with other types in use, e.g., the hair hygrometer, especially in the region of higher humidities; their time lag is small, and both forms, especially the first, could be very easily made distant-reading or recording.

DISCUSSION.

Prof. A. M. Tyndall: The Author refers to some work done by Mr. Grindley and myself on the effect of water vapour on the mobility of positive ions. This work has recently been carried further, but the results so far obtained are of a complex character and are difficult to interpret. In this connection I should like to ask whether the Authors are inclined to attach any significance to the difference in the shape of the two curves IV and V in Fig. 6, or whether they think that the more rapid rise of curve IV is due to accidental causes.

Dr. Ezer Griffiths: The Paper is interesting, as it illustrates some of the difficulties encountered when an attempt is made to utilise the electrical properties of a hygroscopic material in the design of a hygrometer. With a loosely packed material, such as cotton-wool, imperfect electrical contact of the material with the electrodes is inevitable, and the conduction of electricity under such conditions is complicated by many variables. It would be interesting to know how accurately the line shown in Fig. 2 can be reproduced after a period of time. The Authors state that agreement within 2 per cent. is obtained between observations on rising and falling humidities. In a Paper by F. P. Slater, published in Proceedings B of the Royal Society, 1924, data are given for the change of electrical conductivity of a single cotton fibre with variations in humidity, and it was stated that equilibrium was not obtained even after the lapse of 24 hours after making a change in the humidity. It would therefore seem that the subject merits further investigation. The Authors refer to the Report of the N.P.L. for 1926, which contains a brief reference to some work on the effect of humidity on the mobility of ions. As this work was carried out by my colleague Mr. Awbery and myself, a few remarks concerning it may be of interest, as we have not yet published a detailed account. At the time we were seeking for a method of measuring fairly rapid changes of humidity, and it was suggested that the variation of mobility of ions with humidity might constitute the basis of a method. Since the relation between humidity and mobility was not then known (the only data available, so far as we knew, being the isolated observation of Zeleny that the mobility of the negative ion in dry air was 1·24 times the mobility of that ion in moist air), our initial efforts were directed to the determination of this relation. We tried the method described by Burbidge and Alexander, but did not find it satisfactory. We employed a thin layer of uranium oxide spread over an aluminium plate and covered by layers of thin aluminium foil, the number of layers being so adjusted that the α rays just emerged and produced an ionised layer of gas close to the plate surface. Whilst there was no difficulty in measuring the small ionisation currents produced, a very formidable difficulty was encountered in the variability of the contact potential effects. With no applied voltage, a small ionisation current was obtained which varied erratically. Changes of temperature of the electrodes had a marked influence. A considerable amount of work was done on the problem, but this simple method of experiment was ultimately abandoned in favour of a wind channel method. The principle employed is well known: the movement of the ions under the electric field is balanced by an opposing air blast. When moisture is present in the air stream several sources of error have to be guarded against, particularly electrical leakage over the surface of insulators. Zeleny in 1898 tried the air stream method. He employed a current of gas flowing through a tube, the end of which was fitted with a disc of gauze, which very nearly fitted it. He abandoned the method in favour of one in which the electric field on the ions was perpendicular to the air blast. This procedure has been followed by subsequent investigators. By making one modification in Zeleny's original method, we were able to adapt it for the determination of the variation in mobility with humidity. Instead of using a single piece of gauze to close the end of the tube, we divided it up into a central disc and a guard ring. The central disc was made a very accurate fit into the hole in the gauze, but was isolated electrically. Now it is a well-known fact that the velocity of the stream in the vicinity of the walls of a tube falls off to zero at the boundary. Consequently, the air-blast method fails if an attempt is made to work with the entire stream flowing through the tube, since there is no opposing blast to balance the electric field in the region of the walls. There is no difficulty in maintaining a constant stream of gas; we employed a small blower, and as indicator of the rate of flow a hollow sphere of glass floating in a conical tube fixed vertically. We found it impossible to blow a sphere with wall of uniform thickness, and this proved to be an advantage. It was observed that when the flowmeter was functioning the sphere would set itself with the thick lenticular segment lowermost. In this position it would ride quite steadily, remaining in the axis of the tube with a clear space all round for the air to flow past. A typical series of observations will be found on page 64 of the N.P.L. Report for 1926.

Now, for the purposes of practical hygrometry the use of a quadrant electrometer as a current measuring instrument is not feasible. By a valve amplification system we were able to obtain deflections of satisfactory magnitudes with changes of humidity on a sensitive pointer type of current indicator. What seems desirable, however, is a more powerful source of ionisation, as with high amplification of feeble currents surface leakages are apt to be troublesome.

Mr. R. S. WHIPPLE described a method, due to Prof. A. T. Mukarji, of Patna, for eliminating surface leakage in an electrometer. The whole of the apparatus is hermetically sealed in a dry enclosure and circuit changes are made by means of electromagnetic switches operated from outside the enclosure. Alternatively the apparatus might be so arranged that it can be manipulated from outside by means of permanent magnets.

Mr. J. H. COSTE said that the conductivity of cotton-wool was presumably due to mineral substances in the fibre, and as the ionisation of these varied with dilution the phenomena were complex. Further, the heating effect of the current would affect the water content of the fibre. He suggested the use of cellophane, which might be pleated and arranged between grid-shaped electrodes.

Mr. C. R. DARLING added that Cross and Bevan had found that whereas dry cellulose and pure water are non-conductors, moist cellulose has an appreciable conductance, as if cellulose had the power of ionising water.

Mr. D. N. MACDONALD said he was in search of a good method of indicating the disappearance of humidity in an evacuated space, in connection with a commercial process carried out in vacuo, in which the evolution of moisture continues until the process is complete, when it ceases.

Dr. EZER GRIFFITHS suggested that a dew-point method would be very sensitive ; a silver thimble could be let into the chamber and cooled from outside the latter, while the dew on its convex surface inside the chamber could be observed through a window.

XIX.—ON A METHOD OF CONSTRUCTING THE CAUSTIC CURVE FORMED BY REFRACTION AT A PLANE SURFACE.

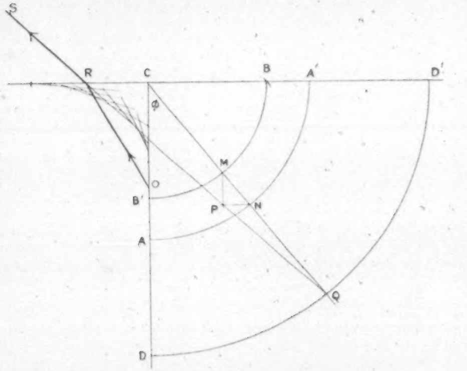
By C. J. Smith, *Ph.D., A.R.C.S.*, Metallurgical Research Laboratory, University of Sheffield.

Received 25 *December*, 1927.

Abstract.

The Paper describes a convenient method of constructing the caustic curve in question. The method depends on the properties of an ellipse which cuts the rays orthogonally.

In teaching the elementary principles of light, a problem which is frequently given may be stated as follows: Assuming the refractive index of a medium to be μ, draw the caustic curve formed by the refraction of light which has its source at a point in the medium. If trigonometrical tables be employed, the subsequent procedure involves the construction of small angles, a somewhat difficult task if great precision is required. The alternative method of constructing the paths of light rays by drawing circles of radii whose ratio is μ involves a repetition of the construction for each refracted ray whose path is traced. The object of the present note is to give a method which is rapid and accurate.



It is a well-known theorem in geometrical optics that the rays which emerge from such a medium are normal to an ellipse whose focus is the luminous point and whose semi-major axis is μ times the distance of the point below the surface of separation of the two media.

The following property of an ellipse is now used to construct these normals. If the ellipse has semi axes (a, b), then the equation to the normal at the point "φ" is

$$ax \sec \varphi - by \operatorname{cosec} \varphi = a^2 - b^2$$

The intersection of this line with the line which defines φ, viz., $y = x \tan \varphi$, is the point $(\bar{x}, \bar{y})$, where

$$\bar{x} = (a+b) \cos \varphi$$

$$\bar{y} = (a+b) \sin \varphi$$

so that the locus of $(\bar{x}, \bar{y})$ is a circle of radius $(a+b)$ whose centre is at the centre of the ellipse.

Now let O be the luminous point at depth OC below the surface of separation of two homogeneous media. With C as centre and radius CA, where $CA=\mu CO$, construct an arc AA', and with O as centre and radius CA draw an arc to intersect the surface in B. Draw the arc BB' and also the arc DD' where $CD=CB'+CA$. Through C draw any right line $CMNQ$ where $A\hat{C}Q=\varphi$. If this line intersect BB' and AA' in M and N respectively, then parallels to the axes through these points give P, a point on the required ellipse, whilst PQ is the normal to the ellipse at the point P, so that the emergent ray is RS and the ray in the medium is OR. By variation of φ more rays can easily be constructed, and the envelope of QR is one half of the required caustic.

DISCUSSION.

Mr. W. R. Bower (communicated): Mr. C. J. Smith has presented an interesting solution of an important problem in optics. His construction does not, however, fix the cusp of the caustic, nor the critical angle, and, like most methods, it is unfortunate with small angles.

A description of another method is appended, one that is rapid and accurate and very simple, but so far as I know is not in common use.

Caustic by Refraction at a Plane Interface.—Let XY represent the plane surface separating media M_1, M_{11};

μ (supposed less than 1), the index of refraction from M_1 to M_{11};

S, a radiant in M_1;

H the intersection with XY of its normal through S.

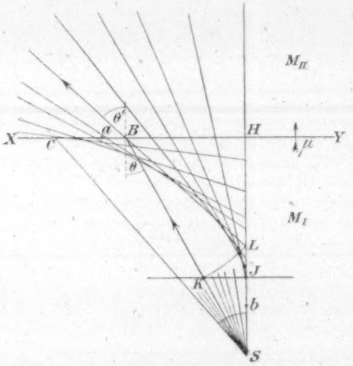
Between H and S take J, so that $HJ=\mu\times HS$. Through J draw a parallel to XY.

Draw a ray from S incident at some point B, on XY and cutting the J-parallel at K. With B as centre, radius BK, draw a circular arc cutting SH in L. Draw LB, this is the refracted ray corresponding to SB. For

$$\mu=\frac{HJ}{HS}=\frac{BK}{BS}=\frac{BL}{BS}=\frac{BH}{BS}\bigg/\frac{BH}{BL}=\frac{\sin\theta}{\sin\theta'},$$

θ and θ' are corresponding angles of incidence and refraction.

Similarly for other rays. The caustic appears as the envelope of the continuations of the refracted rays.



Also drawing shows that when B is close to H, L is very near to J, and J is the limiting position—cusp—when B is at H.

When L is at H, $BH/BS=\mu$, and the angle BSH is then the critical angle.

To find the limiting point of incidence C and the critical angle CSH; make $Ha=HJ$ and $ab=HS$. Draw SC parallel to ba.

For $aH=\mu\times ab$. $\therefore$ $CH=\mu\,.\,CS$. $\therefore$ $\sin CSH=\mu$.

The Caustic by Refraction at a Spherical Interface. may be obtained as an envelope as follows (no figure): From a point N draw concentric circles of radii r (the surface), $\mu\,.\,r$, r/μ. From a radiant S draw any incident ray intersecting at B, Z, the circles of radii, r and $\mu\,.\,r$ respectively. Join NZ, intersecting the circle r/μ at Z'. Z,Z' are the aplanatic points. The line through BZ' is the refracted ray when BZ is the incident. Similarly for other rays through S in this meridianal plane (of the paper). By further construction points on the caustic are obtained. Ref., "A Graphic Method of Optical Imagery," Proc. Phys. Soc., Vol. XXV (1913), p. 176.

Mr. A. Adamson, lecturer in physics in the University of Manchester, also communicated the same construction as that given by Mr. Bower (above) for plane surfaces, and emphasised the simplicity of the proof and its suitability for presentation to elementary students.

XX.—ON SOME MISAPPLICATIONS OF THE LAW OF ERRORS, AND ON THE INTRINSIC ERROR IN FOCOMETRY.

By T. Smith, *M.A., F.Inst.P.*

Received December 2, 1927.

ABSTRACT.

In some physical measurements the uncertainty due to a single cause is not infinitesimal, and the precision of the mean of a large number of observations given by the law of errors is not then physically significant. An example is afforded in attempts to identify the position of an optical image. According to geometrical optics, this lies in a definite surface; but, owing to the physical properties of light, this surface cannot be identified experimentally. The use of a double cylindrical lens as a means of limiting more narrowly the space in which this surface lies is discussed.

WORKERS in a national standardising laboratory ever have in mind the need for high accuracy in all the measurements it undertakes. Unfortunately, the attainment of great accuracy nearly always involves the expenditure of much time, and therefore the cost of very precise work is high. In some measurements there is a natural limit to the accuracy attainable by what we may describe as a frontal attack, and this happens very commonly when light necessarily enters into the experiment. A familiar example is found in photometry where the radiant energy of a beam of light is evaluated by means of the visual sensation it excites. The accuracy attainable in such experiments is limited because the visual sensations to which two similarly constituted beams of radiant energy give rise will not differ unless the ratio of the energies of the two beams differs from unity by more than a certain finite quantity. It is often supposed that the uncertainty in the value of the quantity to be measured (in this case the radiant energy) to which all observations made under such circumstances are subject, can be reduced or even removed by taking a sufficient number of observations and applying the law of errors to them. This conclusion, however, rests upon a misapprehension. The theory on which the law of errors is based requires the uncertainty attributable to any single cause to be indefinitely small, and this condition is violated by observations of the type now considered. The use of formulæ (whether derived from the violated theory or not) for the uncertainty will only be justified if extended experience shows that we are not in actual fact led thereby to incorrect conclusions. The kind of experience needed to give an answer is not, however, of a common kind. It is gained when a new method of measurement is discovered which is free from the source of uncertainty present in previous methods, or when the range of uncertainty of a new method is small in comparison with those of the old. It has been the experience of a number of workers, when such opportunities of criticising the applicability of the law of errors have presented themselves, that the deductions which would have been made from the law, had it been legitimate to suppose it applicable, have in fact been in error by many times the amount the law would have indicated. A graphical representation will make the position clear. The value of the quantity which it is the object of the experiment to determine may be represented by the distance of a variable point on a straight line, from a fixed origin.

Let T in Fig. 1 represent the true value. Any individual observation may give a value different from T owing to, let us say, systematic personal errors, accidental errors to which the law of errors is properly applicable, and to a single other cause which introduces uncertainty about which we can only say that it does not exceed the finite limits $\pm\eta$. Since in practice the total error may always be treated as a comparatively small quantity, any single observation, as a result of these several sources of error, will be represented by some point between two limiting points A and B. We will assume the personal error to have been removed in some way we need not consider here. By multiplying the observations we may reduce the value of the probable error of the mean, in so far as this is due to infinitesimal accidental errors, to as small an amount as we wish, and we are then left with a finite region CD, where $CT=TD=|\eta|+|\varepsilon|$, ε being some finite quantity, within which we are able to say it is probable the real value lies. The theory of errors, however, disregards the finite quantity η, and only throws some light on the magnitude likely to be attained by the (usually) smaller quantity ε.

When the special position of the uncertainty η has been disregarded, it has often been inferred that the true value probably lies within a very small range FG about a mean value E. This conclusion has been thought to be adequately supported by the convergence of a very great number of observations. When, however, a new method of measurement has been found in which the range of uncertainty IJ of a single observation is very much less than AB, the range in the old method, it has not infrequently been discovered that the mean point E reached in the older observations lies well outside the interval IJ, the distance EH between the mid points of FG and IJ being several times as great as FE or EG. Such experiences show that, under the conditions we have assumed, no reliance should be placed on a small calculated value for the probable error of the mean.

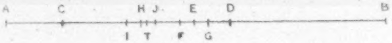


FIG. 1.

It may indeed well happen that the uncertainty in the quantity under measurement is much greater when the observations show only slight deviations from the mean than when they are considerable. This arises from the fact that in practice no reading varies from the true value by more than an amount which is well known for many types of observation. Thus, suppose the length X represents the greatest distance from the real point at which settings are actually made, and let UV represent the range of uncertainty about the true point T due to the single cause. At (a) are shown five closely spaced observations, through which lines are drawn extending to the distance X on either side. The only conclusion we can draw from these observations with certainty is that the real value lies between the limits P and Q. On the other hand, from the five observations (b) we can conclude that the real value lies between the limits R and S. It would ordinarily be inferred from the theory of errors that the error in the mean of set (a) was probably much less than that in the set (b). This example illustrates that the use of the law of errors, when its fundamental assumptions are not all satisfied, may lead to quite erroneous conclusions. Clearly the most satisfactory way of attaining an accurate measurement is to increase the accuracy of a single observation, not to increase the number of observations of lower accuracy.

The kind of difficulty we have just discussed occurs when measurements depend on the accurate focussing of an image. It was laid down by Lord Rayleigh, and subsequent workers have agreed with his conclusion, that as the focus is altered the image of a point source of light hardly changes its appearance until the movement has introduced variations in the phases of the constituent ethereal wavelets exceeding one quarter of a period. Thus, physically, the focus extends through a finite space, say in the neighbourhood of the axis of a lens. Moreover, from the very nature of this criterion, it follows that the extent of this space cannot be affected by magnifying the image which is to be focussed, or like methods. On the other hand, the mathematical focus is a definite point, and it is therefore not altogether surprising that a testing laboratory should at times be pointedly requested to determine the focus within narrower limits than these natural tolerances for "perfect" focus. The difficulty may, of course, be surmounted by resorting to the Hartmann, or some equivalent method, of locating the focus. Usually the cost of such methods prohibits their use. It happens, however, in this, as well as in some other cases where a similar difficulty occurs, that the uncertainty of a single setting can be reduced, without altering the method of observation, by displacing the range within which good settings can be secured in both directions, leaving only a small common region within which good settings can be made. To illustrate this, let us suppose that light from a point source converges to the mathematical focus *P*. Physically, a good focus is found anywhere between *E* and *F*. If now a positive lens is introduced

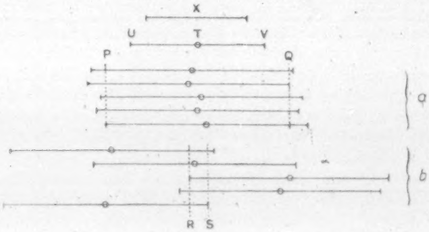


Fig. 2.

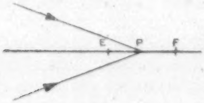


Fig. 3.

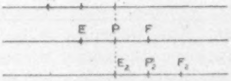


Fig. 4.

in the path of the converging light to the left of P the range EF will be moved towards the left; on the other hand, a negative lens will cause a movement in the opposite direction. In either case the magnitude of the displacement will depend on the power of the lens we introduce and its distance from P. If we have a means of producing equal displacements of both signs, and can adjust at will the magnitude of the displacement, we shall obtain the state of affairs illustrated in Fig. 4, where the mathematical focus P is the only common point of the two displaced ranges E_1F_1 and E_2F_2 obtained by the use of positive and negative lenses respectively. In practice lines are used as objects instead of points, and the sharpness of the focussing is judged by the rapidity of the transition from dark to light in a direction normal to the line. The use of lines as objects enables cylindrical lenses to be interposed in the path of the converging beam without prejudice to the formation of sharp images. In particular, we may use an object consisting of two lines crossing at right angles, and employ a lens with two cylindrical surfaces having their axes parallel to the two object lines, the one surface increasing, the other reducing, the convergence of the light. If such a lens is gradually moved away from the image, a position will be reached when one or both line images will become noticeably less well defined. The plane in which the focus is examined may be adjusted until the change in the appearance occurs simultaneously in the images of both lines. A very small movement of this plane in either direction is then sufficient to introduce an unmistakable difference in the appearance of the two line images. That is to say, we are enabled by such a device greatly to reduce the intrinsic uncertainty of a single setting. We proceed to consider the theory of a lens of this kind.

It will be assumed that the distance of the double-cylindrical lens from the plane in which the mathematical image is formed will be small, so that all rays are refracted near the axis of the lens, and its aberrations may therefore be neglected. Further, it is unnecessary to consider the aberrations of the lens forming the image at P; we merely suppose that P is the position where the range of phase variations is least, and that any inaccuracies of focus will become evident when the phase differences exceed a definite amount. We are thus virtually replacing the emergent wave converging towards P as its "best" focus with an ideal spherical wave with P as centre. The chief difference between the real and the ideal lens for our present discussion is that the amounts by which the cylindrical lens must be displaced from the focus to cause obvious deterioration in the image may differ in the two cases. A quantitative difference of this kind is obviously unimportant. The theoretical investigations can readily be carried out with the aid of either the characteristic function or the eikonal. The use of the most general forms for these functions, though leading readily to the interesting focal line properties of any optical instrument, is unnecessary when we intend to impose the condition that the principal axes of curvature of all the surfaces lie in the planes $y=0$ and $z=0$. It is not without interest to remark that the general method of constructing the characteristic function or the eikonal from those of its elementary constituents may fail when particular forms are assumed initially. An example is afforded by the combination of two cylinders with their axes at right angles to one another. The reasons for this failure are not obscure, but we may note that there is no failure if the formulæ established for the general case are employed to calculate the coefficients of the compound system. In the cases we shall consider the coefficients denoted by

$\mathscr{G}$ and $\mathscr{H}$ for each constituent surface are zero; it follows that in the complete system we shall have*

$$\mathscr{G}=\mathscr{H}=\mathscr{S}=\mathscr{T}=\mathscr{U}=\mathscr{V}=0,$$

so that the eikonal takes the form

$$\mathscr{E}=\frac{\mathscr{A}M^2+\mathscr{C}M'^2-2MM'}{2\mathscr{J}}+\frac{\mathscr{B}N^2+\mathscr{D}N'^2-2NN'}{2\mathscr{I}}+\dots$$

and the characteristic function is given by

$$V=\frac{(\mathscr{C}y^2+\mathscr{A}y'^2-2yy')\,\mathscr{J}}{2(1-\mathscr{A}\mathscr{C})}+\frac{(\mathscr{D}z^2+\mathscr{B}z'^2-2zz')\,\mathscr{I}}{2(1-\mathscr{B}\mathscr{D})}+\dots$$

As the characteristic function has been more extensively employed in problems involving the time taken by a disturbance to travel along non-stationary paths, we will illustrate the use of the eikonal in this problem. To fix ideas we will suppose that the coefficients present in $\mathscr{E}$ relate to a compound system consisting of a symmetrical lens, which is the part under test, and the two cylindrical surfaces having their axes in the planes $y=0$ and $z=0$. The surfaces of these cylinders are assumed to be at a fixed distance apart. We may denote the optical value of this interval by 2τ, and the powers of the two surfaces by i and j respectively. The power of the symmetrical lens and the magnification at which it is used are not essential factors of the problems; but, as we have included this lens in the system, we will denote its power by κ, and suppose that the object lines are infinitely distant. The direction of any particular object point is then defined by the variables M, N. Let the mid-point of the cylindrical system be optically distant $\varepsilon+1/\kappa$ from the principal point of the symmetrical system, and ξ from the second eikonal reference point. Then in the two principal planes we have the following powers and distances:—

κ		κ	
	$\varepsilon+\frac{1}{\kappa}-\tau$		$\varepsilon+\frac{1}{\kappa}+\tau$
i		j	
	$\xi+\tau$		$\xi-\tau$

so that

$$\mathscr{I}=\kappa\{1-i(\varepsilon-\tau)\},$$
$$\mathscr{J}=\kappa\{1-j(\varepsilon+\tau)\},$$
$$\mathscr{C}=-\{\mathscr{J}(\xi-\tau)+\kappa(\varepsilon+\tau)\},$$
$$\mathscr{D}=-\{\mathscr{I}(\xi+\tau)+\kappa(\varepsilon-\tau)\}.$$

It follows that the terms in $\mathscr{E}$ which are affected by changes of focus are

$$-\tfrac{1}{2}\left\{(\varepsilon+\xi)(M'^2+N'^2)+\frac{j(\varepsilon+\tau)^2M'^2}{1-j(\varepsilon+\tau)}+\frac{i(\varepsilon-\tau)^2N'^2}{1-i(\varepsilon-\tau)}\right\}.$$

---

* For notation employed see Trans. Opt. Soc., Vol. 29, 1927-28, p. 71.

We may identify the second reference point determined by ξ with the intersection of the experimentally selected image surface with the axis. The variables M' and N' will vary between limits determined by the pupil of the examining system, including the eye. Sometimes the symmetrical system under test will limit the radiation rather than the examining system; but in general the facts will be represented with sufficient accuracy by assuming that the effective radiation extends over the directions given by

$$0 \leqslant M'^2 + N'^2 \leqslant \alpha^2,$$

where α is a constant. Now the eikonal gives the value of the stationary path between that wavefront of the incident light which passes through the first reference point and the plane having the direction M', N' which passes through the second reference point. If, as we may, we suppose $M = N = 0$, this second reference point represents the centre of an out-of-focus image, and, when the focussing error is small, light by a non-stationary route will reach this point in a time which differs negligibly from that taken to reach the plane normally by the stationary route. It follows that the variations of phase we have to consider are given by the terms of the eikonal which have been written at length above.

In the absence of the cylindrical system we insert $\varepsilon = i = j = 0$, and the experimental focus may be at any point on the axis within the range determined by the condition

$$\tfrac{1}{2}\alpha^2 \mid \xi \mid \leqslant \tfrac{1}{4}\lambda,$$

that is to say, the range of uncertainty is λ/α^2. When the cylindrical lens is inserted the first term of our expression represents an ordinary out-of-focus effect, due to the want of coincidence of the second reference point with the ideal geometrical focus, and the remaining two terms represent additional variations of path due to refraction at the cylindrical surfaces. These will be opposed in sign, since the signs of i and j differ. Any axial displacement of the cylindrical system alone is represented by a change of ε without any change in the value of $\varepsilon + \xi$. If then τ is small, we can cause the phase variations due to the cylindrical refraction to assume selected values by moving this part of the system to the proper position on the axis. By altering the distance of the second reference point from the lens we make a common addition to both cylindrical effects.

Changes in the appearance of the image patch which extends in the direction $y = 0$ will follow from modifications of the coefficient of M'^2, and similarly alterations in the patch which extends in the direction $z = 0$ correspond to changes in the coefficient of N'^2. We know that when variations about the position of zero phase difference are made no change in appearance occurs until the variation amounts to about $\lambda/4$. In positions more remote from this intensity maximum much smaller movements of the image lead to detectable changes in the light distribution. Let us suppose that when the path variation amounts to $n\lambda$ a change $\pm\eta\lambda$ can be seen. The two focal lines will then present different appearances unless the variations of path can be represented by

$$\pm(n \pm \eta)\,\lambda.$$

Of the signs outside the bracket it is easy to ensure that one refers to the one line and the other to the perpendicular line. Now the actual path variations amount to

$$\tfrac{1}{2}a^2\left[\varepsilon+\xi+\frac{j(\varepsilon+\tau)^2}{1-j(\varepsilon+\tau)}\right] \text{ and } \tfrac{1}{2}a^2\left[\varepsilon+\xi+\frac{i(\varepsilon-\tau)^2}{1-i(\varepsilon-\tau)}\right]$$

respectively. If then

$$\frac{i(\varepsilon-\tau)^2}{1-i(\varepsilon-\tau)}\sim\frac{j(\varepsilon+\tau)^2}{1-j(\varepsilon+\tau)}=\frac{4n\lambda}{a^2}$$

we shall have

$$2(\varepsilon+\xi)+\frac{i(\varepsilon-\tau)^2}{1-i(\varepsilon-\tau)}+\frac{j(\varepsilon+\tau)^2}{1-j(\varepsilon+\tau)}\leqslant\frac{4\eta\lambda}{a^2}.$$

In practical applications it will be permissible to neglect the departures of the denominators from unity, and we may assume that the powers of the cylinders are equal in magnitude but opposed in sign, so that $i+j=0$. The equations then become

$$i(\varepsilon^2+\tau^2)=\frac{2n\lambda}{a^2}$$

and

$$\varepsilon+\xi\leqslant\frac{2\eta\lambda}{a^2}+2i\varepsilon\tau.$$

It at once follows that if the ratio τ/ε is sufficiently small (and this can obviously always be secured), we may, in the most unfavourable event, reduce the axial focussing range approximately to

$$\frac{4\eta\lambda}{a^2}.$$

The criterion by which the position of the image is selected is essentially photometric. The importance of good eyesight in the observer is doubtless increased in the modified test. The position on which the eye is focussed is to be taken as the focus of the lens under test, apart from the correction due to the axial length occupied by glass in the cylindrical system. As a numerical example we may suppose $i=0{\cdot}4$D, $\tau=\frac{1}{2}$ mm., and the system under test a telescope objective of aperture $f/15$. This gives $a=1/30$, and with visible light the natural sharp focussing range is about $\frac{1}{2}$ mm. The relation between ε and n is approximately

$$\varepsilon=50n^{\frac{1}{2}}\text{ mm.},$$

and the total focussing range is at most

$$2\left(\eta+\frac{n^{\frac{1}{2}}}{100}\right)\text{mm.}$$

Taking $n=1$, the cylindrical system is displaced five centimetres from the image, and the contribution of the second term to the focussing range is only 0·02 mm. If η should prove to be as great as 1/25 when $n=1$ (and approximate theoretical considerations lead us to anticipate a much smaller value) the focussing range would only be one-fifth as great as that in a direct examination. We are thus led to the conclusion that modifications in the experimental conditions will enable even the uncertainty attributable to the structure of light itself to be reduced to a negligible magnitude.

DISCUSSION.

Dr. L. C. Martin: I have been greatly interested in this Paper in connection with my work in ultra-violet microscopy, where the depth of focus for the ultra-violet radiation is correspondingly shorter than with visible light of longer wave-length ; but the ultra-violet focus must be estimated with respect to a visual focus of some kind. The possibility of obtaining greater precision in focussing is therefore an important question ; but can this be accomplished without an object of a special type, as, for example, by dividing the field and displacing the focus in one half ?

Mr. Smith will be interested in a Paper published by Dr. Bevek, of Messrs. Leitz, in Germany (Marburger Sitzungsberichte 61 (1927), p. 189), in which an experimental examination of the depth of focus in the microscope is described, which was undertaken with a view to finding the relation (for the special case) between the extreme errors of setting corresponding to the visual criterion of " focus " and the " probable error " as calculated from the usual expression. The deviations from the error distribution characteristic of a pure " probability theory " case, due to the existence of the visual criterion, are clearly shown by the results. The theory of errors is not quite invariably misapplied in this connection.

Capt. C. W. Hume asked whether it was not permissible to apply the law of errors to the determination of the limits E, F, Fig. 3, of the threshold, and then to calculate the true value P by assuming an appropriate relation ? Would not the probable error in P be thus reduced to that in E and F ?

Dr. F. E. Smith said that he had not yet read the Paper, but, prima facie, the author's conclusion seemed surprising. If two equal sources of light had to be compared photometrically it might be that no difference of illumination was visible until the ratio of one illumination to the other was 4 : 5 at one end of the threshold and 5 : 4 at the other, but the determination of these two limits depended only on the accuracy with which the scale could be set. After analysing a number of published observations, he felt that the more careful the observer the narrower would be the limits of his error.

The Author: I am glad to hear about the investigation mentioned by Dr. Martin, and hope to have an opportunity of seeing the Paper in which it is described. Capt. Hume's proposal is not free from special difficulties ; the existence of the definite word " threshold " must not lead us to imagine that this abstraction corresponds to percepts equally definite. The threshold may conceivably be less definite than the region of uniform sensation. In practice the difficulty can be overcome in a more satisfactory way. In reply to Dr. F. E. Smith, it is not suggested that the mean derived from a large number of observations is necessarily, or even in a majority of measurements, in serious error ; there are, however, definite instances in which it is known to have been considerably in error. If we rely upon the theory of errors, we take risks (despite all care in observations) from which we are freed by the substitution of more accurate measurements. This is in no way inconsistent with Dr. Smith's remark that the good observer obtains better results than the bad observer : in effect, this is our definition of a good observer. In the precise work contemplated in this discussion all observers employed are assumed to be good observers.

XXI.—THE TERMINAL VELOCITY OF DROPS.

By William D. Flower, *B.Sc.*

Received February 1, 1928.

(*Communicated* by F. J. W. Whipple, M.A.)

Abstract.

The distance a drop of given volume has to fall through in order that it may attain a constant terminal velocity has been determined for both water and methyl salicylate. A relation connecting the distance of fall for constant velocity and the volume of the drop has been obtained.

The terminal velocities of drops of water and of methyl salicylate have been determined for drops 0·2 cm. to 0·55 cm. in diameter. A relation connecting the terminal velocity of a drop and its volume has been obtained.

THE object of these investigations was to obtain data relating to the behaviour of liquid drops falling freely through the atmosphere.

Considering the case of any one drop falling freely through the atmosphere, it is evident that after it has fallen through a certain distance the change in its velocity will be very small, and to a very near approximation the velocity will be constant. This velocity is known as the " Constant Terminal Velocity " of the drop.

No determinations appear to have been made of the distance through which a drop must fall before it attains a constant terminal velocity, but there have been various determinations of the value of the terminal velocity.

The terminal velocity of water drops has previously been determined by three investigators. (*a*) The first was Lenard,(1) who measured the velocity of an upward stream of air which would just suspend the drop. (*b*) Then Mache(2) showed that the range of the terminal velocities of rain drops agreed with Lenard's results. Mache photographed rain, and, knowing the time of exposure and the length of the streaks on the negative, he calculated the various terminal velocity values.

(*c*) Later, Schmidt(3) used two circular discs with a narrow sector cut out of each attached to the spindle of an electric motor, the speed of rotation being so arranged that the drop whose velocity was required passed through the two openings. From a knowledge of the angular velocity of the discs and their distance apart the velocity of the drop could be calculated.

The Determination of the Distance of Fall for Constant Terminal Velocity.

The first part of the investigation consisted in determining the distance a drop of known volume has to fall in still air before it attains an approximately constant velocity.

The method employed was to liberate a drop of measured volume from a point vertically above the pan of a ballistic torsion balance, the path of a drop being increased up to a point beyond which no increase in momentum was recorded on the balance when the drop struck the pan.

EXPERIMENTAL DETAILS.

The apparatus employed was a torsion balance, shown diagrammatically (Fig. 1). The beam of the balance *AB* was very light in construction, and carried at one end an open framework of thin glass rods, on which a filter paper was placed to catch the falling drop, a fresh piece of paper being used for each drop. To ensure that each successive piece of filter paper was placed in the same position on the framework, holes were punched in the paper through which could be placed the projections on the beam.

It was found from experience that the filter paper employed absorbed the drops with a negligible amount of splashing. In order to obtain consistent results, the filter papers used in any one experiment had to be all of the same weight. In order to check that this was the case, the necessary holes were punched in a number of papers and one placed on the framework, and the position of the counterpoise *W* adjusted so that the scale reading of the pointer *P* was zero. The remaining papers were then in turn placed in position, and any which gave a readable deflexion were rejected.

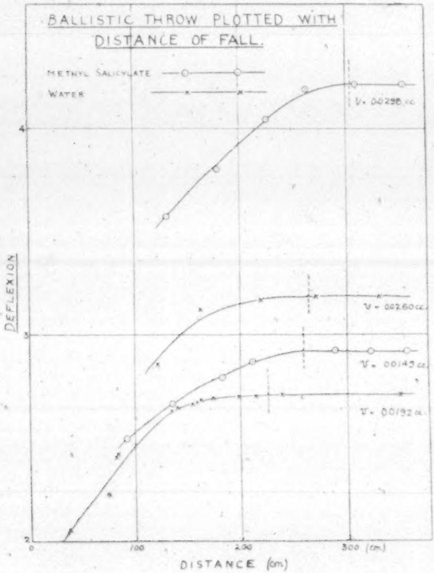


FIG. 2.

The method consisted in allowing drops from a burette to fall on to the filter paper from varying heights and observing the ballistic throw of the balance in each case. It was found that when a series of drops in succession were allowed to form slowly on the end of a tube and detach themselves under gravity there was very little variation in their volume, and any secondary drops formed were so small as to produce no appreciable effect upon the balance.

The individual ballistic throw was taken for a number of drops at each height, and the mean of these values was plotted against the corresponding height (Fig. 2).

A series of drops from the same height were found to give values in agreement to within one or two per cent., the greatest variation being under five per cent. From the curves connecting height of fall and ballistic throw the distance through which a drop of certain volume has to fall in order that it shall attain its constant terminal velocity, may be obtained.

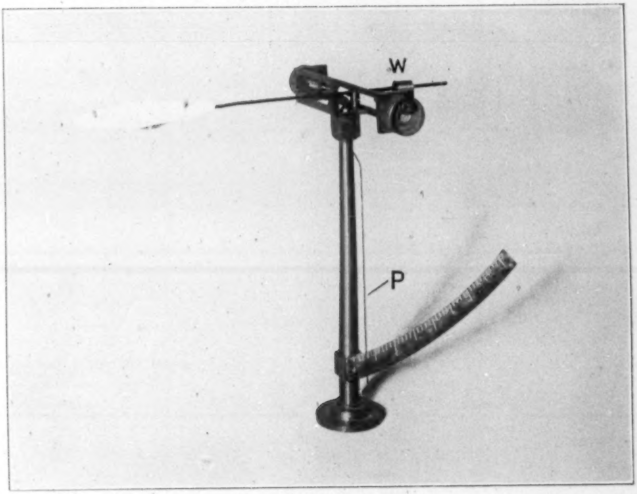


FIG. 1.

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Two liquids were used during the course of these experiments—namely, water and methyl salicylate. The second liquid was chosen because its surface tension and viscosity differed appreciably from those relating to water.

The results obtained for drops varying in volume from 0·2 c.c. to 0·7 c.c. in the case of water, and 0·15 c.c. to 0·65 c.c. in the case of methyl salicylate, are given in Table I (Fig. 3).

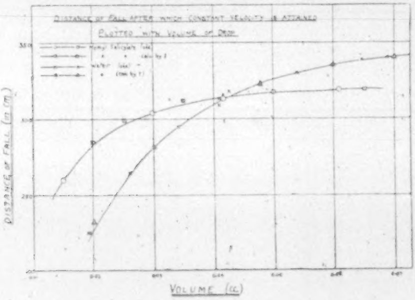

FIG. 3.

TABLE I.

Water.		Methyl Salicylate.	
Density ... 1·0 gm./c.c. Surface tension 73·5 dynes/cm. Viscosity ... 0·01142 c.g.s.	At 15°C.	Density ... 1·19 gm./c.c. Surface tension 38·8 dynes/cm. Viscosity ... 0·03836 c.g.s.	At 15°C.
Volume of drop.	Distance of fall for constant velocity.	Volume of drop.	Distance of fall for constant velocity.
C.c.	Cm.	C.c.	Cm.
0·0688	340		
0·0645	338	0·0608	319
0·0538	330		
0·0425	318	0·0414	313
0·0415	316		
0·0408	313	0·0298	304
0·0408	309		
0·0340	295	0·0149	260
0·0260	264		
0·0194	225		
0·0192	225		

DISCUSSION OF RESULTS.

There is no obvious relation connecting the volume of a drop and the distance it has to fall through in order to attain its constant terminal velocity; but from the results given in Table I an empirical relation has been deduced, which may be written as follows:

$$z=c(1-e^{-Bv}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus if

$$\left.\begin{array}{ll} v\rightarrow 0 & z\rightarrow Bcv \\ v\rightarrow \infty & z\rightarrow c \end{array}\right\}$$

In this equation Z is the distance a drop of volume v c.c. has to fall through in order that it shall attain its constant terminal velocity, whilst c and B are constants depending upon the liquid employed.

The values of c and of B are 319·2, 0·5527 respectively in the case of water, and 348·2, 1·142 in the case of methyl salicylate. It is interesting to note that the value of the constant c for the second liquid may be obtained by multiplying the value for water by the square root of the density, and the value of B by multiplying the corresponding one for water by the product of the square root of the density and the inverse ratio of the surface tension of the second liquid relative to water at 15°C. These ratios, however, have no theoretical basis and cannot therefore be assumed to apply to the general case.

It will be seen (Fig. 3) that the above equation gives fair agreement for the distance Z over the range of drops investigated, the largest variation being in the case of the smallest drops used.

THE DETERMINATION OF THE CONSTANT TERMINAL VELOCITY.

This part of the investigation involved the determination of the constant terminal velocity of drops of each of the two liquids used in the preceding experiments and for a similar range of volume of drop. In the present determination a photographic method has been employed, the final development of which depended essentially on allowing a number of drops, all of the same size, to fall just in front of a highly and evenly illuminated vertical ground glass slit, and photographing them by means of an ultra-rapid cinema-camera. An illuminated time-indicator, placed by the side of the slit, is also included in the photographs.

The resulting photograph (Plate I) thus consists of a succession of images of a particular drop as it passes in front of the slit and of the time indicator. Hence, by measuring the distance between successive positions of the drop, and noting the time interval between each, it is possible to calculate the required terminal velocity. Since the time of exposure is appreciable, the drop appears as a short line on the negative.

EXPERIMENTAL DETAILS.

The slit used was 30 cm. long and 4 cm. wide, and a number of fine parallel lines, a centimetre apart, were drawn across it to enable the distance that the drop had fallen, between successive photographs, to be easily measured. The time indicator consisted of an electric motor (the speed of which was controlled by an external resistance) carrying a disc, the radius of one half being 7·5 cm. and that of the other half 8·0 cm., rotating in front of another disc which was fixed.

This latter disc had a radius of 9·5 cm., and a semi-circular slit cut in it of radii 7·5 cm. and 8·0 cm.; the periphery of the disc outside the slit being divided into

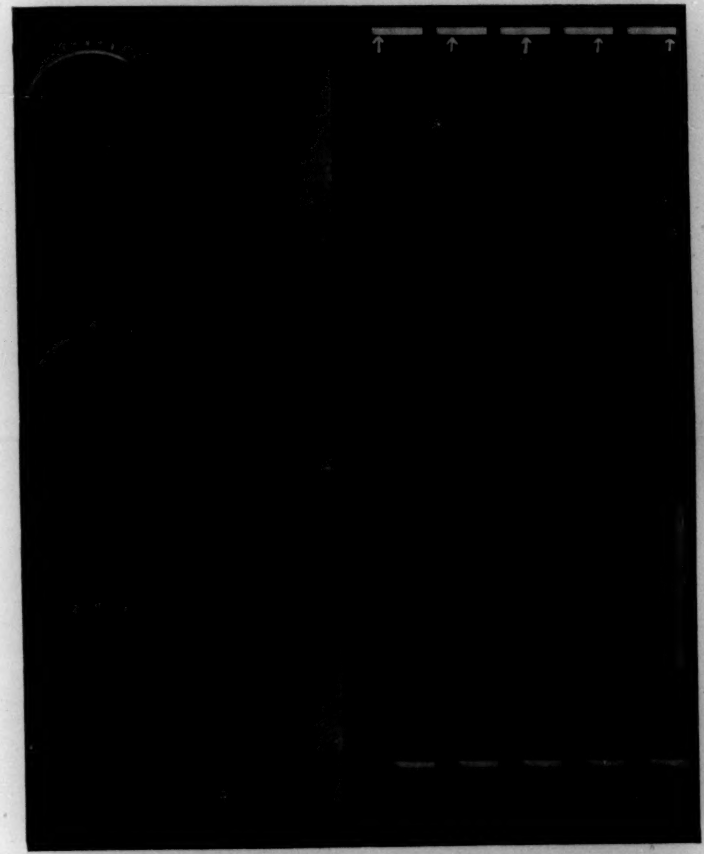

PLATE 1.

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ten equal divisions. A piece of tracing paper was fastened behind the slit and division marks to give even illumination.

At any instant a certain portion only of the illuminated semi-circular slit, determined by the position of the rotating disc, was visible. A tachometer driven by a flexible drive from the motor spindle enabled the speed of the rotating disc to be determined with an accuracy of 0·25 per cent. Hence, this speed W being known in r.p.m., the time interval represented by the displacement of one division on the periphery is found to be $3/W$ sec. The distance H having been measured, a drop has fallen between two photographs, and the number of time divisions corresponding to the terminal velocity may be determined. To obtain a succession of drops of the same size, two methods were employed.

1. The attachment of a jet of appropriate size to a Mariotte tube of 500 c.c. capacity.
2. The attachment of a jet of appropriate size to a reservoir of large cross-section 30 sq. cm., so that the variation in head of the liquid during an experiment was very small.

In order to reduce the length of film employed to a minimum, the rate of flow was arranged to be as rapid as possible consistent with the independent formation of the drops.

The jets used were of drawn out glass tube, and by varying the size of the jet drops of different volumes were obtained. The dropping apparatus was mounted at a height of more than 350 cm. above the slit, since all of the drops used, both of water and methyl salicylate, attain a constant terminal velocity after having fallen through this distance (vide (1) Part I).

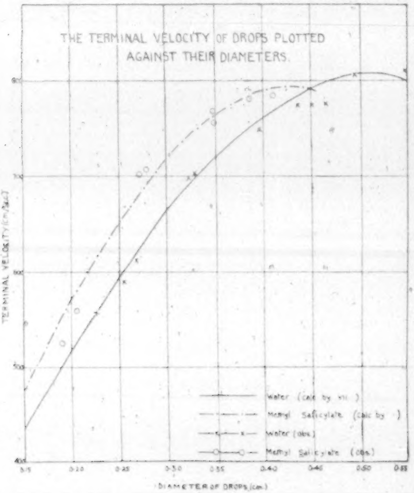

FIG. 4.

The drops were allowed to fall through a long pipe in order to eliminate as far as possible promiscuous disturbances of the air. To obtain the best possible contrast on the negative, both liquids were dyed a deep red, Rhodamine Red being the dye used.

The drop size was obtained by measuring the volume occupied by a large number of drops and taking the mean. By observing the drops during their formation through an Ashdown rotoscope, some were seen to have a small satellite drop following them. In some cases two satellites were observed. These satellite drops were caught on an absorbent paper, "Attracta Paper," and their volume estimated from the size of the splash produced. Hence the volume of the drop photographed was obtained, and from this its diameter calculated, which latter was taken as that of a sphere of volume equal to that of the drop.

The terminal velocities were obtained for drops varying in volume from 0·005 c.c. to 0·07 c.c. in the case of water, and 0·0035 c.c. to 0·04 c.c. in the case of methyl salicylate, and are given in Table II (Fig. 4).

Values of the terminal velocity obtained by Lenard and by Schmidt are given for purposes of comparison.

TABLE II.—*Water.*

Volume of drop. C.c.	Diameter of drop. Cm.	Terminal velocity.		
		(Flower) Cm./sec.	(Lenard) Cm./sec.	(Schmidt) Cm./sec.
0·0872	0·550	...	800	...
0·0830	0·548	810·7	...	...
0·0669	0·510	806·4	...	...
0·0655	0·500	...	800	...
0·0504	0·465	777·6	...	...
0·0477	0·450	...	800	...
0·0471	0·450	774·3	...	...
0·0414	0·435	774·9	...	...
0·0335	0·400	...	770	...
0·0287	0·385	748·4	...	...
0·0225	0·350	...	740	740
0·0178	0·328	702·4	...	...
0·0168	0·322	697·2	...	...
0·0141	0·300	...	690	692
0·0100	0·267	611·5	...	...
0·0086	0·254	589·5	...	...
0·0082	0·250	...	640	638
0·0060	0·226	556·7	...	...
0·0042	0·200	...	590	577
0·0018	0·150	...	570	498

Methyl Salicylate.

Value of drop. C.c.	Diameter of drop. Cm.	Terminal velocity. Cm./sec.
0·0377	0·419	784·5
0·0300	0·386	781·1
0·0222	0·349	754·7
0·0216	0·348	767·4
0·0213	0·347	767·4
0·0155	0·309	729·3
0·0109	0·277	706·9
0·0101	0·270	701·5
0·0045	0·205	559·9
0·0036	0·190	522·7

DISCUSSION OF RESULTS.

The probable errors in the above determinations may be dealt with under the following headings :—

(1) Determination of drop size.
(2) Accuracy of measurement of the time interval.
(3) Accuracy of measurement of the distance fallen by a drop.

(1) The volume occupied by a certain number of drops was measured when

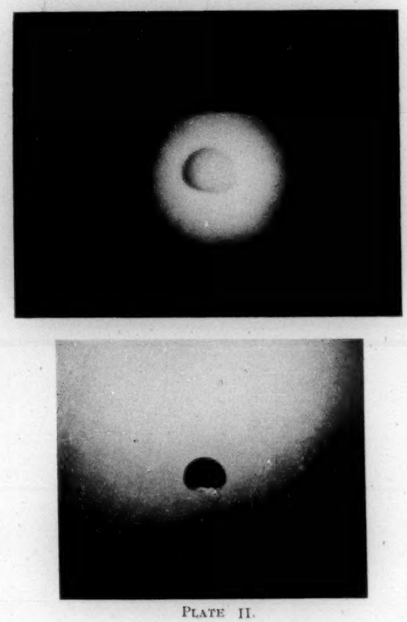

PLATE II.

[To face page 173

the drop-rate was about 100 per minute, whereas the drop-rate during an experiment may be as large as 200 per minute in the case of the larger size drops.

Donald(4) and Fildes and Smart(5) have shown that there is an increase in drop-size with drop-rate, hence it is seen that the drop whose velocity was measured was slightly larger than the one for which the diameter had been calculated. Curves connecting the drop-rate and drop-size were therefore plotted for a number of drops, and the error introduced may be safely estimated as not greater than six per cent.

(2) It was possible to measure to 0·1 division on a negative with accuracy, and since this, at the speed used (1500-1800 r.p.m.) represents less than 0·002 second—not more than 2 cm. per sec.—if it is assumed that the terminal velocity of any drop is not greater than 1,000 cm. per second, it may be neglected.

(3) The error introduced here may be computed by calculating the probable error of the arithmetic mean of the whole series of observations for one drop. In this way the greatest percentage probable error of the velocity was found to be less than ±1·5 per cent. The probable mean errors for four velocity determinations taken at random are given below (Table III).

TABLE III.

Liquid.	Number of observations.	Terminal velocity in cm./sec.	Percentage probable error.
Water	14	806·3	±0·77
	8	774·9	±0·35
Methyl salicylate ...	16	729·3	±0·658
	10	706·9	±1·37

From a consideration of the preceding, it is estimated that the present results are accurate to within ±5 per cent. It will be observed from Fig. 4 that the results for water drops above 3 mm. in diameter differ slightly from those obtained by Lenard, yet, from here to the lower limit of the present determination, there is a large discrepancy in the values. This may possibly be due to errors inherent in Lenard's method, for he was only able to keep his drops suspended for from 2 to 4 seconds before they glided out of the airstream, after which he measured the air velocity at the point of suspension by means of a small spherical cup anemometer. It is quite possible that this latter operation would disturb the air flow and a fictitious value for the velocity be recorded.

THEORETICAL CONSIDERATIONS.

The constant terminal velocity of fall of very small drops—0·1 mm. or less in diameter—may be computed by Stokes's well-known equation ;(6) but for drops with larger diameters this relation ceases to hold. These larger drops become distorted during their fall, and expose a larger surface to the air than would the corresponding spheres of equal volume. The accompanying photographs (Plate II) show the distortion brought about in a water drop 0·464 cm. in diameter after it had acquired its terminal velocity. Neither the question of the distortion of the drop, nor the effect of this upon its terminal velocity, appears to lend itself to simple mathematical treatment, and the equations of fall for these larger sized drops are but little more than empirical.(7) It is possible, however, to write down an equation of motion for a drop falling freely through air, provided that certain assumptions are made. If it is assumed that the resistance to the motion of the drop due to the air is proportional to the square of the velocity, and that the effect of the air which is dragged along with the drop is negligible, the equation of motion for constant terminal velocity will be

$$g(\rho-\sigma)v-kV^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where V is the constant terminal velocity,
ρ the density of the liquid,
σ the density of air,
v the volume of the drop,
k the coefficient of resistance depending upon the size of the drop.

Now, since σ is small compared with ρ, it may be neglected in the above equation.

Thus $$V^2=g\rho v/k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If now the value of k be determined by substituting the observed value of the terminal velocity in (3), it is found to increase with the size of the drop (Table IV). The value of k for solid spheres of the same volume, deduced from values of the terminal velocity of steel balls obtained by Richardson,(8) are given for purposes of comparison.

TABLE IV.—*Water.*

Volume of drop. C.c.	Diameter of drop. Cm.	Terminal velocity. Cm./sec.	k Gm./cm.	k Gm./cm. (Steel Spheres).
0·0830	0·548	810·7	12·40 × 10^{-5}	8·0 × 10^{-5}
0·0669	0·510	806·4	10·07	7·0
0·0504	0·465	777·6	8·17	5·8
0·0471	0·450	774·3	7·70	5·5
0·0414	0·435	774·9	6·76	5·2
0·0287	0·385	748·4	5·02	4·2
0·0178	0·328	702·4	3·53	3·5
0·0168	0·322	697·2	3·38	3·2
0·0100	0·267	611·5	2·62	2·7
0·0086	0·254	589·5	2·32	2·5
0·0060	0·226	556·7	1·92	2·0

Methyl Salicylate.

Volume of drop. C.c.	Diameter of drop. Cm.	Terminal velocity. Cm./sec.	k Gm./cm.	k Gm./cm. (Steel Spheres).
0·0377	0·419	784·5	7·15 × 10^{-5}	5·0 × 10^{-5}
0·0300	0·386	781·[illegible]	5·74	4·2
0·0222	0·349	754·7	4·54	3·8
0·0216	0·348	767·4	4·18	3·8
0·0213	0·347	767·4	4·23	3·8
0·0155	0·309	729·3	3·40	3·0
0·0109	0·277	706·9	2·55	2·8
0·0101	0·270	701·5	2·39	2·7
0·0045	0·205	559·9	1·67	1·9
0·0036	0·190	522·7	1·54	1·8

A relation between k and r, the radius of the drop, has been obtained which is given below:

$$k^2(P-r)=Qr^3 \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (4)$$

where P and Q are constants depending upon the liquid employed. The values of P and Q are respectively 0·339, 555×10^{-10} in the case of water, and 0·285, 415×10^{-10} in the case of methyl salicylate. If the value of k, in terms of radius r, from (4), be substituted in (3), the terminal velocity will be given by

$$V^2=(4/3)\pi\rho g[r^3(P-r)Q^{-1}]^{\frac{1}{2}} \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (5)$$

By differentiating this equation it is found that V has a maximum value when $r=0{\cdot}75P$ cm. From this it will be seen that water drops for which $r=0{\cdot}25$ cm., and methyl salicylate drops for which $r=0{\cdot}21$ cm., have the most rapid fall. Lenard also found that there was a maximum value for the terminal velocity when the radius of the drop was 0·25 cm. in the case of water.

The explanation of the above, given by Lenard, is that the friction of the air causes deformation of the drops, so that instead of retaining the shape of spheres they became flattened out, thus presenting an increased surface to the air through which they are falling. This deformation is slight for the smaller drops, but increases rapidly in the case of the larger drops. After a certain size of drop has been attained, any further increases in volume produces a greater flattening, and instead of the velocity being increased it is slightly decreased.

ACKNOWLEDGMENTS.

My thanks are due to Mr. J. D. Fry for his suggestions and for the interest he took during the progress of this work, and to Mr. F. J. W. Whipple I am indebted for his assistance in the final preparation of this Paper.

REFERENCES.

(1) Lenard, P., Met. Zeit., 39, p. 249 (1904).
(2) Mache, H., Met. Zeit., 39, p. 378 (1904).
(3) Schmidt, W., Akad. Wiss. Wien. Sitz. Ber., 118, 2a, p. 71 (1909).
(4) Donald, R., Proc. Roy. Soc., 86, p. 199 (1913).
(5) Fildes, P., and Smart, W. A. M., Brit. J. Exptl., Path 7, p. 68 (1926).
(6) Phys. and Maths. Papers 3, p. 59.
(7) Liznar, Met. Zeit. 31, p. 339 (1914).
(8) Richardson, L. F., Proc. Phys. Soc., 36, part 2, p. 78 (1924).

DISCUSSION.

Mr. J. H. COSTE: The relatively low terminal velocities found by Mr. Flower are interesting in connection with the dissolved oxygen content of rainwater which E. Hannaford Richards has shown to be frequently less than corresponds to saturation at the temperature of collection.

It has to be remembered that raindrops are formed in a relatively attenuated atmosphere, where the partial pressure of oxygen is low, and hence the amount needed for saturation of water is less than at the ground level. Nevertheless, it is somewhat remarkable that a drop should not become saturated with atmospheric gases during its fall. The process used by Richards for determination of dissolved oxygen was trustworthy, and the errors of collection would tend to bring the water into closer equilibrium with the air.

The following are some of Richards' results for the oxygen dissolved in rainwater collected at Rothamsted in 1915: January (80 per cent. saturation), 79·2 The remaining results are for 90 per cent. saturation, as follows: January, 87·0; February 85·7; July, 81·0; 84·5, 89·7,

August, 88·2 ; November, 89·5. There were 26 results in all, and only 6 over 98 per cent. of saturation. It should be noted that 1,000 ml. of distilled water dissolves about 10·2 ml. of oxygen from moist air at 0°C. and 6·4 ml. at 20°C. The table is from Jour. Agric. Sci., VIII, iii, 1917, p. 334, and the figures have been corrected by me for a pressure of dry air of 760.

Dr. W. N. Bond : Assuming that the circulation of the liquid inside the drop has little effect on the terminal velocity, certain results may be predicted. We obtain by dimensional considerations

$$g(\rho-\sigma)v=V^2r^2\sigma\,.\,f\left(\frac{Vr\sigma}{\mu}\quad\frac{r^2(\rho-\sigma)g}{T}\right)$$

(where T is the surface tension).

Firstly, for small drops the product involving T may be omitted. Thus, for small drops of two liquids falling in air, if Vr be the same, we might expect $V^2/r(\rho-\sigma)$ also to be the same.

Secondly, the radius near which the transition due to flattening of the drop occurs should be of the order given by $r^2(\rho-\sigma)g=\mathrm{T}$. A critical radius of this order has been found for air bubbles rising in water, there being a maximum terminal velocity for such sizes of bubble. Also in the case of the terminal velocity of drops and bubbles moving in viscous liquids (the kinetic energy being negligible) I have found in some conjoint work a critical radius of this order, but not involving a flattening of the drops. (Phil. Mag., April, 1928.)

In Mr. Flower's experiments, since $Vr\sigma/\mu$ is not greatly different for the drops of maximum terminal velocity in the two cases, I should have expected the critical sizes to have been related by the proportionality r (critical) $\propto\sqrt{\mathrm{T}/(\rho-\sigma)}$.

Mr. R. G. Lunnon (communicated) : Mr. Flower's methods are ingenious, and his results are of real value. They show the resistance for small drops to be much greater than had been supposed, for the values of the resistance coefficient ($W/\rho v^2d^2$) are between 0·15 and 0·20 for this range of Reynolds' numbers for perfect spheres, and the present values, for deformed spheres, are from 50 per cent. to 100 per cent. greater. From the photographs, it does not appear that the flattening was very great; and I think that an increase of 20 per cent. in the transverse diameter might account for the extra resistance. Were there no indications of the drops oscillating about a mean spherical shape ? It may be noted that the v^2 resistance law is not applicable even for perfect spheres, with these speeds ; the power of v should be between 1·4 and 1·7. It would be useful to know how the satellite drops were separately measured, and how large they were.

Mr. W. A. Benton suggested that the velocity might be found by measuring the ballistic deflection, though the relation between these quantities would not be linear.

Author's reply : Replying to Dr. Bond, I would like to point out that from the experimental values obtained [r (critical, water)/r (critical, methyl salicylate)] is 1·2, and if we substitute in his expected proportionality we get the value 1·5, which is more or less in agreement. Replying to Mr. Lunnon, the drops were observed only for a distance of about 30 cm., and there were indications of the drops oscillating about a mean spherical shape. It was thought that as the drops became distorted the oscillations would decrease and tend to be about a mean flattened-spherical shape. This might explain why the distortion in the cases shown is not quite the same in each photograph. The satellite drops were blown by a gentle horizontal stream of air away from the line of fall of the larger drops on to the absorbent paper, and thus they could be measured separately. The largest satellite drop measured had a diameter $<0{\cdot}1$ cm. Replying to Mr. Benton, I might say that the original intention was to obtain the terminal velocities from the ballistic deflection, but since the calibration curve was very steep the method, with the form of apparatus used, was not practicable and for that reason the photographic method was employed.

XXII.—ON THE LONGITUDINAL WAVE ALONG A ROD.

By SATYENDRA RAY, *M.Sc.*, Lucknow University.

Received February 8, 1928.

(*Communicated by* PROF. LALJI SRIVASTAVA, Government College, Ajmer, India.)

ABSTRACT.

That longitudinal waves are propagated with the same velocity for all wavelengths only when the waves are " geometrically similar " is here proved in exactly the same manner as for either strings, air columns or transverse waves along stretched strings. The expression for the velocity is of course different.

HOUSTOUN points out in his Introduction to Mathematical Physics, §98, that the longitudinal vibrations in a rod of any section are mathematically the same as those for the transverse vibration in a stretched string. An error due to contraction of the cross-section during elongation, and vice versa, is, however, pointed out as leading to an error that " can be neglected."

We may proceed to correct for this error for a rod of any section of area a. The elongation per unit length is $\partial\xi/\partial x$ at any section A, and the resultant force on a slab bounded by two planes A and B, distant dx apart, is equal to

$$Ea\left(\frac{\partial\xi}{\partial x}+\frac{\partial^2\xi}{\partial x^2}\,.\,dx\right)-Ea\frac{\partial\xi}{\partial x}=Ea\frac{\partial^2\xi}{\partial x^2}\,.\,dx$$

where E is Young's modulus. We equate this to the expression $m\,.\,\partial^2\xi/\partial t^2$, where m is the mass of the slab. In obtaining an expression for m we remember that the cross-section a, owing to a contraction equal to σ times the elongation in the linear dimension, becomes

$$a(1-\sigma\partial\xi/\partial x)^2$$

We therefore have for the equation of motion

$$\rho a\left(1-\sigma\frac{\partial\xi}{\partial x}\right)^2\,.\,dx\,.\,\frac{\partial^2\xi}{\partial t^2}=Ea\frac{\partial^2\xi}{\partial x^2}dx$$

or

$$\frac{\partial^2\xi}{\partial t^2}=E\,.\,\frac{\partial^2\xi}{\partial x^2}\Big/\rho\left(1-\sigma\frac{\partial\xi}{\partial x}\right)^2$$

This expression leads approximately to the form

$$v=v_0(1-\sigma\partial\xi/\partial x)^{-1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

for the velocity v of propagation, where $v_0{}^2=E/\rho$

In case ξ is of the form $\xi=\imath\sin 2\pi x/\lambda$

$$\frac{\partial\xi}{\partial x}=\frac{a}{\lambda}\cos 2\pi\frac{x}{\lambda}$$

so that the value of $\delta\xi/\delta x$ in (1) is a function of a/λ.

We note that, as in the case of waves along electric tubes of force, air columns and stretched strings vibrating transversely, v is the same for all values of wavelength if the ratio a/λ between the amplitude and wave length be constant—i.e., if the waves be geometrically similar.

XXIII.—ON THE DAMPING OF MERCURY RIPPLES.

By J. J. MANLEY, *M.A.*, Research Fellow, Magdalen College, Oxford.

ABSTRACT.

This brief communication is descriptive of a method devised for suppressing wavelets set up by temporary vibrations upon mercurial surfaces.

IN a paper* recently read to the Society I dealt with an interferometer pressure gauge in which the reflectors were the summits of two mercurial columns. Attention was drawn to the necessity for supporting the instrument upon a bench of great stability; mention was also made of the ease with which minute ripples are formed upon the mercury by passing vehicles, whether near or somewhat distant.

As a result of numerous trials, it is now possible to describe a plan whereby the difficulties arising from transitory vibrations may be overcome.

Experiment and theory agree in showing that when a glass plate of ordinary thickness is placed upon mercury the glass, like other solids, quickly makes contact with the wall of the containing vessel. But when the plate does not exceed 2 mm.† in thickness its behaviour is completely reversed. Such a plate instantly moves away from the wall, and, provided the width of the cistern holding the mercury is but slightly in excess of that of the disc, the latter is at once automatically and correctly centred upon the pool.

This extremely convenient and helpful behaviour of a thin plate is not apparently generally known, and all my efforts to discover some allusion to it have hitherto failed.

In view of the fact just mentioned, boxwood cisterns, shallow and circular, and having diameters ranging from 27 to 35 mm., were prepared. These were partially filled with pure mercury, and during experiments with any one of them the mercury was almost covered with a parallel-plane glass disc 25 mm. in diameter. Trials were made in the following way.

First, the cistern to be tested was placed upon the interferometer bench and a powerful beam of light directed upon the glazed mercurial surface, whence it was reflected to a distant screen. Next, the cistern was somewhat violently disturbed, and the time required by the mercury to come to rest noted. Proceeding thus with the several cisterns, it was quickly found that the most rapid and effective damping resulted from the use of one whose diameter closely approximated to that of the disc—a result in accord with theory.

Use was now made of another cistern having an internal diameter of 26 mm.

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* Proc. Phys. Soc., Vol. 40, Pt. 2, p. 57 (1928).

† The thickness of the plates used for the experiments here described was 1·99 mm. Plates having a thickness of 2·2-2·4 mm., according to the kind of glass, exhibit when placed upon mercury complete neutrality: they are neither attracted nor repelled by the walls of the containing vessel. Hence the necessity for restricting the thickness to 2 mm. or less. The thinner the plate the more prompt its movement to the centre of the pool.

Thus the annular clearance between cistern and disc was reduced to 0·5 mm. With this apparatus the effects of the most severe shocks were suppressed within two seconds or less. The efficacy of this particular cistern was still further tested by viewing the reflected image of a pair of cross-threads through a reading telescope; and again the damping, as thus determined, was equally rapid and complete. But, in order to ascertain to what degree of perfection the damping had been raised, it was obviously necessary to try the device by means of the interferometer itself.

Accordingly, yet another cistern similar in all respects to that just dealt with was made, partially filled with mercury, and, like the other, equipped with a parallel-plane glass disc. These twin cisterns were then used as the two reflectors of the interferometer.

The result of a trial was disappointing, for from the movements of the interference fringes it was seen that transitory vibrations caused the glazed mercury to oscillate in a complicated manner; and, further, that the oscillations, though small, were slow in dying away. Hence the next task was an attempt to introduce some additional device whereby this final difficulty might be overcome. Success was attained in the following way.

First, the glass discs were cleaned with alcohol, and then to some 6 or 8 equally spaced points upon the circumference of each plate was applied a droplet of an alcoholic solution of shellac. Next, a thread known as "silk twist," having a thickness of 0·5 mm., was immediately carried round the disc and through the several portions of shellac. As the solution hardened the thread was firmly secured. Finally, the internal diameter of the cisterns was suitably enlarged for the reception of the encircled plates. The annular clearance between plate and cistern was now less than 0·5 mm. The use of silk twist was here introduced for the following reasons.

When the thread is gently rubbed its smoothness is either increased or lessened, according to the direction of the stroke. A decrease in smoothness is due to numerous projecting filaments of great fineness and delicacy. In order that advantage might be taken of this property, the encircling twist was appropriately rubbed, and the discs thus armed with outstanding filaments or tentacles; and these, when the discs were *in situ*, lightly but surely clung to the inner face of the opposing wall. It will therefore be seen that, although the floating discs were thus effectively moored, they were yet free to respond to and follow any changes in the level of the mercury.

On subjecting the now completed device to a series of tests with the interferometer it was seen that the passing of ground tremors produced nothing more than simple and momentary swayings of the floating discs. Generally, the movements were such that the interference fringes oscillated evenly. Initially represented by one or two fringes only, the oscillations quickly died away.

From the results of the final exacting tests the conclusion reached was that the plan evolved for the suppression of mercurial ripples was effective, and sufficient to meet the demands of the experimenter.

We conclude by observing that an excessive control is indicated by a failure of the plates to respond to temporary vibrations. Inadequate control is, on the other hand, shown by a lack of the necessary damping. In the one case the silk tentacles are too many and in the other too few. The first named defect may be removed by a fine platinum wire used as a smoothing tool, and for convenience applied in turn to several evenly distributed regions of the attached silk twist. The other defect is similarly remedied with the aid of a very thin wooden rod which, unlike the wire, acts when passed over the thread in the right direction, as a roughing tool, and so increases the number of projecting filaments or tentacles. The art of regulating the number of active tentacles may be acquired with ease.

Finally, it may be remarked that the method of damping described above, might with considerable advantage be used in other instances; as, for example, in the case of artificial horizons.

XXIV.—EXPERIMENTS WITH MERCURY JETS AND THE PHENOMENA EXHIBITED AT THEIR IMPACT WITH STEEL AND GLASS.

By Professor WILL C. BAKER, *M.A.*, Physics Department, Queen's University,
Kingston, Ont., Canada.

Received March 9, 1928.

ABSTRACT.

As a light sphere is retained in a vertical jet of fluid in virtue of the change of momentum of the fluid produced by its adhesion to the sphere, it was thought that a steel sphere would not be retained in a vertical mercury jet, as there is no "wetting" of the steel by that fluid. Experiment showed that a given bicycle ball might or might not be retained by such a jet, as the speed of the jet (at a given angle of incidence) rose above or fell below a critical value for that ball.

Conditions were simplified by the use of cylindrical and of plane surfaces of steel, and an approximately constant time of adhesion between mercury and steel was found for various speeds of impact. This led to the explanation of the phenomenon in terms of the well-known instability of jets.

IN a note "On the Retention of a Ball by a Vertical Water-jet,"* the author showed that the phenomenon was essentially due to the change of momentum imparted to the water by its adhesion to the already wet ball.† It appeared that a consequence of this would be that, since mercury "wets" neither glass nor steel, balls of these materials would not be retained in a vertical mercury jet. A trial showed, however, that under certain conditions such balls could be retained, and experiments were begun in order to investigate the phenomena.

The jet was produced and controlled in the following way :—

From a glass separating funnel—which was used as a mercury reservoir—a quill tube ran downwards for about 75 cm. It was then bent upwards to a glass stop-cock lubricated with graphite. Just above the stop-cock the tube turned horizontally to a nozzle that allowed of adjustment to any angle in a vertical plane. The nozzle was sometimes of steel and sometimes of glass. To determine the velocity of the jet as it left the nozzle under any given adjustment of the stop-cock, the stream was caught in a small beaker for a known number of seconds, and the required velocity was computed from the volume issuing per second and the measured area of the nozzle. A better method of estimating the effective area of the nozzle was developed later. (See below.)

Instead of measuring the force exerted by the jet on the ball, as had been done in the case of the water jet,‡ it was found convenient to use the jet itself as an indicator of the forces acting. The vertical jet was allowed to impinge at different speeds and at different angles of incidence on a fixed ball. At low speeds the jet flattened out in a small nearly elliptical sheet after it struck the sphere ; it then gathered itself together and left the surface at an angle as shown in Fig. 1a. This behaviour of the jet implied the action on the ball of a force directed away from the axis of the jet. At high speeds the elliptical sheet of mercury formed a sort of cap on the surface of the ball, and the jet left from the upper end of the cap as indicated in Fig. 1b. Obviously the reaction in this case urged the ball *towards* the axis of the jet. Intermediate speeds showed cases where the fluid apparently left the surface in the line of impact as shown in Fig. 1c.

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* Science, Vol. LXIV, No. 50, p. 161.

† The same explanation was put forward by Osborne Reynolds (*see* Collected Papers, Vol. I, p. 1), but no experimental demonstration of its correctness was there given.

‡ Loc. cit.

To get simpler conditions, a horizontal jet was used with a polished cylinder of steel (radius 0·7 cm., axis vertical). At the highest available speeds the fluid would adhere to the steel through an arc of 150° before gathering itself into a spray as it left the surface.

The mercury against the cylinder was a thin flat elliptical sheet bounded by a thicker roll at the edges. The results were not sensibly different when the jet and cylinder were placed in a jar and the air exhausted with a motor-driven rotary oil air pump, and this fact was taken to show that the phenomenon did not depend on the jet's setting up reduced pressures by any entrainment of the air in contact with its surface. The higher the speed of the jet the longer was the major axis of the "ellipse of contact," and as the relation between these quantities was apparently linear, it was thought that perhaps the mercury adhered to the steel for a definite time—about 1/200 of a second—before breaking away. (See below.) There were strong electrical effects due to the breaking of the jet into spray as it left the cylinder, but while the spray had a powerful effect on an electroscope when the air was present, the same electroscope showed no measurable potential difference between the steel nozzle and the jet, nor between the jet and the steel cylinder. It was thought that any electrical effect sufficient to cause the observed change of momentum in the mercury would surely have been great enough to affect the electroscope. A glass cylinder freshly cooled in air from a bright red heat showed the same phenomena as did the steel, so the idea of the adhesion being due to films of air on the glass was set aside.

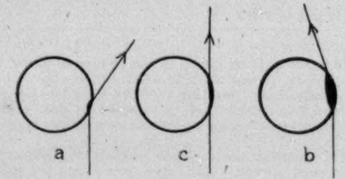

Fig. 1.

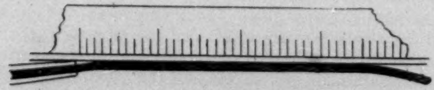

Fig. 2.

To simplify conditions further the jet was caused to impinge on the under side of a flat horizontal steel blade (Fig. 2). For high speeds the mercury spread out into a flat elliptical sheet as before, and at the further end gathered into a jet which, of course, resolved itself into drops almost at once. A paper scale attached to the upper side of the blade made it possible to read the distance over which the mercury ran in contact with the steel. At the highest speeds available (about 375 cm./sec.) the major axis of the ellipse was about 2·3 cm. long, but as the speed was reduced the dimensions of the elliptical sheet diminished until at 125 cm./sec. its length was only 1 mm. A further cutting down of speed at this point produced a sharply defined change in the behaviour of the jet. The flattened portion disappeared, and the mercury ran along the under side of the steel for 3 or 4 cm. before leaving in a stream of drops. The length of this type of run on the steel also diminished with slower jet-speed, until at about 40 cm./sec. its contact with the steel was of the order of 1 mm. The column of mercury running on the under side of the steel was usually a nearly uniform cylinder right up to the point where it broke away. Under some conditions, however, waves could be seen running along it which at certain speeds settled into systems of a stationary type, three or four complete waves being sometimes obtained. These were of a rapidly diminishing amplitude, but were of an apparently constant wave-length. When the impact was on glass it could easily be seen that a vibrating jet was adhering to the solid; for points at which the depth of the jet was least were points at which its breadth was greatest. The fact that three or four complete waves were formed shows that there was a real adhesion between the mercury and the glass. Again, if a copper wire (say 1 mm. diameter by 8 or 10 cm. long) be well amalgamated and laid on a piece of clean plate glass, with a small drop of mercury to fill up the space between the wire and the glass, and if this be gently tipped to let the excess of mercury run off along the wire, the whole system may be inverted without the wire dropping from the glass. The wire and adhering mercury are held up against their weight by the adhesion of the fluid metal to the glass. The function of the copper wire is to lend its rigidity to the system so that the mercury cannot draw itself up into a drop and cut off its contact with the glass by the "sphincter" action of the tension about the circle where the fluid meets the glass. This fact, as well as those reported in a note "On the Adhesion of Mercury to Glass Surfaces,"* show that the adhesion is not a temporary phenomenon, as suggested above, and that the effect of adhesion for definite small times in these experiments has still to be sought.

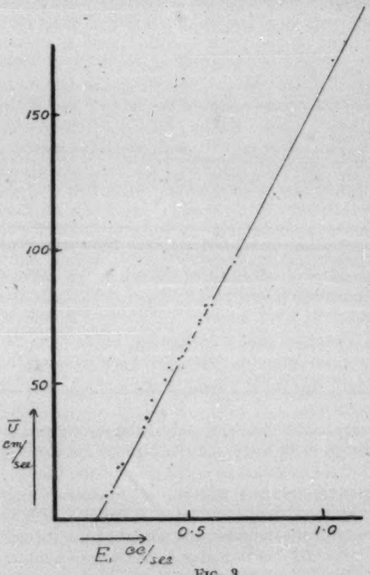

FIG. 3.

The jet velocities as noted above were determined from a nozzle area computed from the average of a series of measured diameters of the orifice. To check these, and to find whether any contraction lowered the effective area of the cross-section, the jet was allowed to discharge horizontally past a sheet of squared paper, and the initial horizontal speed of the jet was deduced from the co-ordinates of the observed parabolas described by the drops. These velocities, for the range of speeds available, were then plotted against the volume issuing per second. A perfectly linear graph was obtained (Fig. 3), but it did not pass through the origin, so that the newly formed drop has no horizontal velocity for an efflux of 0·15 c.c./sec. This loss of speed the graph shows to be the same at all speeds. It is accounted for by the fact that as each drop forms at the end of the issuing jet it is subjected to an impulse $2\pi T\int r dt$ of sensibly constant integral value, due to the surface tension (T) around the neck of the forming drop. The observed constancy of this impulse indicates a fairly constant functional relation between r and t (time) in the integral just given at all speeds (for a given nozzle).

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* Science, Vol. LXVII, No. 1725, p. 74.

Further, from the graph we see that the horizontal velocity of the drops ($\bar{v}$) is given by an equation of the type

$$\bar{v}=(E-c)K/a,$$

where K and c are constants, a the effective cross-section of the nozzle, and E the efflux in c.c. per sec. Also $a=dE/dv$, which gives a method for the determination of the effective area of the jet. Then E/a gives the velocity with which the mercury issues from the nozzle.

Next, the jet was set so as to deliver the cylindrical column of mercury to the under side of the steel blade as nearly horizontally as possible (as in Fig. 2), and the squared paper was employed, as above, to determine the horizontal velocity of the drops as they broke from the blade after runs of various lengths—i.e., from different nozzle velocities. The graph for the velocities of the drops ($\bar{v}$) plotted against the efflux (E) was of the same nature as that shown in Fig. 3. The effective area of the nozzle and the nozzle velocity (v) were therefore determined as above, The nozzle velocity when plotted against the efflux gave a line parallel to the plot of $\bar{v}$· but passing through the origin. This shows that the difference between v and $\bar{v}$ was constant at all speeds. When v and v are plotted against the distance run on the steel, the same type of graph was obtained.

Thus the measurements show no appreciable loss of speed as the mercury runs along the blade,* but they reveal a change in velocity, constant at all speeds, due to the formation of the drops at the end of the cylinder, as in the case of the free jet.

We are thus driven to the view that the mercury adheres to the steel as long as it remains in a cylindrical form. As soon, however, as the mercury gathers itself into spheres, owing to the instability of the cylinder, the action of the surface tension " closes the drop off " by the " sphincter " action about the circle of contact between the mercury and the steel. That is, the force of adhesion between the molecules of the fluid and those of the solid is usually† less than the force due to T/r

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* This agrees on the one hand with the apparent constancy of the diameter of the cylinder as it passes along the blade, and on the other hand with the apparent constancy of the wave-length of the standing waves when they are present.

† *Usually*, for there is the strange phenomenon of " sticking " reported in a note " On the Adhesion of Mercury to Glass " (Science, Vol. LXVII, p. 74, January 20, 1928). In the experiments there described the air-mercury edge would at times adhere to the glass so as to take an abnormally large pull to cause a change in the line of contact. For instance, in making a series of observations on the shrinking of the circle of contact from some given circle to a new stable position (due to a given displacement of the lens (*see* Fig. 5) of the article quoted), the edge often swept past points at which in a previous trial it had stuck, and at which it could not be got to stick again.

when the circle of contact is small. These views are strengthened by the fact of the adhesion to glass of the amalgamated wire mentioned above, and of the adhesion of the mercury to the surfaces of glass lenses. When the fluid is " closed off," as described, the glass is left " dry," since the cohesion of the mercury is greater than its adhesion to the glass. These considerations show why the run of the mercury on the steel is proportional to the speed, for it depends on the time required for the cylinder to break into drops ; the case of the apparently constant time of adhesion of the flat elliptical films is probably fundamentally the same even if the action be much more complex. They give, too, the reason for the results indicated in Fig. 1a, b and c, for if the time of contact be approximately constant the jet for slow speeds will be thrown off before it reaches the " equator " of the ball, while for greater speeds it will be well past that line before being torn away by the formation of drops. It will consequently leave the sphere in this case in an entirely different direction, as indicated in Fig. 1. The differences in the reaction of the jet on the ball are directly dependent on these changes in direction.

DISCUSSION.

Dr. W. H. Eccles offered an alternative explanation of the ball-and-jet phenomenon. If the jet strikes the ball at a point away from the vertical axis the stream distributes itself unequally over the surface of the ball, and the thinner region of the stream loses velocity faster than the thicker region. Hence the pressure exerted by the former is the greater in accordance with Bernoulli's theorem, and tends to restore the ball to the symmetrical position. The Author had experimented with steel and mercury in order to obtain an unwettable surface, but could he not more easily have waxed the surface of one of the balls ordinarily used ? The Paper reminded the speaker of a small slow-motion, variable-speed motor which he had once constructed, in which a water-jet of adjustable volume fell vertically on to a ball fixed to the lower end of an arm, the upper end of which was fixed to a shaft. The ball oscillated in the jet, and the consequent reciprocating motion of the shaft was rectified by means of ratchet mechanism.

Mr. T. Smith said that Dr. Eccles' explanation should be tested quantitatively, since the low-pressure region of the stream rises higher, and so covers a wider area than the low-velocity region. The correctness of his explanation cannot, therefore, be decided qualitatively.

Dr. Ferguson said that he did not think that Osborne Reynolds, who first discussed the problem, had considered Bernoulli's theorem as a possible explanation. It was interesting to note, however, that Reynolds had described an experiment in which a disc forming the bob of a pendulum had been made to vibrate in a water-jet in the way mentioned by Dr. Eccles.

XXV.—THE ELASTIC CONSTANTS OF GLASS.

By Edgar Philip Perman and William Donald Urry, University College, Cardiff.

Received March 18, 1928.

Abstract.

(1) The coefficients of compressibility of soda-glass and Jena 16^{III} glass have been determined at six temperatures ranging from 30°C. to 80°C. (2) From experiments on the effect of external pressure only, Poisson's ratio has been determined, and hence the modulus of rigidity and Young's modulus.

In measuring the coefficient of compressibility of a liquid by the application of equal internal and external pressure to a bulb containing the liquid, it was necessary to know the bulk modulus of the glass employed. We have, therefore, determined the coefficients of compressibility of the glass used in constructing the piezometer bulbs, and we have carried out a second series of experiments, from which values have been obtained for the bulk modulus, rigidity modulus and Young's modulus.

The two methods of experiment consist of :—

(1) The measurement of the direct linear compression of a long glass cylinder.

(2) The application of external pressure only to a piezometer bulb, and the measurement of the compression produced.

Two specimens of glass have been used :—

(*a*) Jena (red line) 16^{III} standard glass.

(*b*) A typical soda glass such as is commonly used in experimental work.

Summary of Previous Work.

In the work of Regnault[*] on the compressibility of liquids pressure was applied in turn to :—

(1) The outside only of the piezometer bulb.

(2) The inside and outside of the bulb simultaneously.

(3) The inside only of the bulb.

Three equations were obtained, but the final solution involved the unjustifiable assumption that Poisson's ratio for glass $=\frac{1}{4}$. The most direct method is that suggested by Amagat,[†] and used later by Buchanan and Tait.[‡] It was used by Amagat up to a pressure of 2,000 atmospheres.

It consists of the direct measurement of the longitudinal contraction of a glass rod under hydraulic pressure. This was observed directly in the experiments of Buchanan and Tait, but by an electrical contact method in the experiments of Amagat.

Another method is to apply a linear tension to a bulb of the glass closed by a graduated capillary. The increase in the internal volume is measured by the

[*] Memoirs de l'Institut de France, 21, 429.

[†] Comtes Rendus, p. 727 (1889).

[‡] Roy. Soc. Edin. (1880).

descent of liquid in the capillary. In the last two methods the value obtained is of course one-third of the bulk modulus.

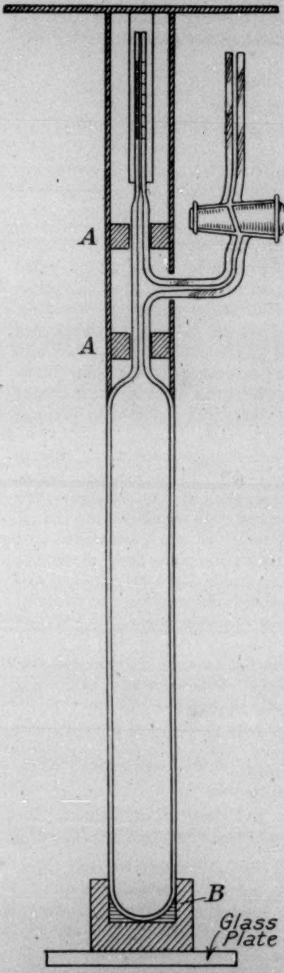

DIAGRAM 1.—GLASS TUBE ARRANGED TO MEASURE COMPRESSIBILITY OF GLASS.

METHOD I.

We have employed a modification of this last method. In applying a linear tension it is necessary to strain some portion of the bulb where irregularities occur, such as a seal between the bulb and the capillary, or an irregularity necessary for clamping. This complicates the evaluation of the factor $V \, . \, dP$ at the various cross-sections. Consequently we have attempted a modification of the above method. A long cylindrical bulb, whose uniform length is great compared with the hemispherical end, is subjected to compression, the force being applied below the capillary seal, which is under no strain. A brass tube of external diameter equal to that of the bulb, and about nine inches in length, is filed to fit the hemispherical end closing the bulb. To the centre of this end of the bulb is sealed a capillary, which in turn is sealed to the indicator capillary and the side tube water level adjuster. To the other end of the brass tube is soldered a table to carry the mass. The brass tube itself is suitably slotted for the side capillary to pass through and to obtain a view of the indicator capillary within. (Diagram 1.)

It will be seen that the pressure is applied at a point below which no serious irregularity occurs. In fact, the conditions approach the ideal conditions of a flat-ended cylinder, the only divergency being one hemispherical end. The factor $V \, . \, dP$ then is made up of :—

(1) $V_1 \, . \, dP_1$ for a flat-ended cylinder; and

(2) $\int_{r=0}^{r=R} V \, . \, dP$, where R=the radius of the hemispherical end.

Owing to the great length of the cylinder compared with the radius of the hemispherical end, the application of this integral gives a result differing negligibly from that obtained in using the total internal volume and the mean value of dP. The latter method has therefore been used.

EXPERIMENTAL PROCEDURE.

The compressibility coefficient for the glass is given by

$$c=3(dV/V)\div\{Mg/\pi(R^2-r^2)\}$$

where dV is the volume decrease,
V the total initial volume,
R the external radius,
r the internal radius,
Mg the force applied.

Prior to a set of experiments, V was first determined by filling with distilled water up to the point of seating of the brass tube and adjusting at 30·00°C. From weighings and density tables V was obtained. The entire apparatus was then filled with water, and the brass tube, fitted with rubber guiders *AA* lubricated with liquid paraffin, placed in position. The base of the cylinder was fitted into a metal base *B*. The apparatus was immersed in an electrically controlled thermostat constant to 1/300°C. ; with this sensitivity the water level in the indicator capillary remained constant for a far greater time than that required to perform a complete reading.

A mass of 5,000 grammes was placed on the platform at the top of the tube in order to produce the compression. The radii were carefully determined before making the apparatus by the mean of several readings with vernier callipers. The errors due to the expansion of the glass with temperature are negligible, the change in pressure caused by the expansion on heating from 20° to 80° being less than 1 part in 1,000. Having adjusted the level of the water in the indicator capillary, previously thoroughly cleaned from all traces of grease, the load was alternately applied and removed many times. It was found that freshly made bulbs could not be made to give constant readings for some weeks. Examples of this were noticed by Phillips* in some experiments on the creep effects in glass.

The decrease in the volume is measured by the rise of water in the capillary. This was ascertained by means of graduation marks on the capillary subdivided by a fine reading cathetometer and expressed primarily in cms. of capillary.

The values of the coefficient of compressibility here given have been reduced to compressibility per atmosphere.

COMPRESSIBILITY OF SODA GLASS.

The glass employed was supplied by Mr. W. J. Nelson, of Streatham. We are indebted to Mr. E. E. Cox, of this college, for an analysis, which gave the following results :—

SiO_2	66·00	per cent.
Al_2O_3 and trace of Fe_2O_3	4·99	,,
CaO (and SrO)	7·49	,,
Na_2O	13·05	,,
K_2O	8·30	,,
	99·83	,,

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* Proc. Phys. Soc., 19 (1905).

Four series of experiments with this glass were carried out. The data for one of them is here given sufficiently fully to give an idea of the accuracy attained.

SERIES I.

Capillary: 0·00 145 c.c.s. per cm. Load: 5,000 g. (This corresponds with $dP=5{\cdot}78$ atmosphere).

Radii: External, 1·325 cm. (mean of 8 readings); internal: 1·220 cm. (mean of 5 readings); internal volume: 94·6 c.c.s.

——	30°	40°	50°	60°	70°	80°
	mm.	mm.	mm.	mm.	mm.	mm.
Mean rise in capillary ...	3·06	3·08	3·10	3·13	3·16	3·17
Mean deviation from mean ...	0·01	0·015	0·01	0·01	0·01	0·01

The results of the four series may be summarised as follows :—

Series I. $c=243 + 0{\cdot}2 \ (t-30)$
,, II. $c=246 + 0{\cdot}16 \ (t-30)$
,, III. $c=244 + 0{\cdot}18 \ (t-30)$
,, IV. $c=241 + 0{\cdot}2 \ (t-30)$
Mean $c=243{\cdot}5+0{\cdot}185(t-30)$

Extrapolating to 0°C. this becomes

$$c=238+0{\cdot}185t,$$

a formula which should be of use to experimenters using a glass of this kind.

JENA GLASS.

Three series of measurements were made with 16^{III} glass, but the results of the third series alone are given, as the others were not constant.

$$c=229+0{\cdot}28(t-30).$$

Extrapolating to 0°C. this becomes

$$c=220{\cdot}6+0{\cdot}28t.$$

METHOD II.

The application of external pressure only to a piezometer bulb.—The relation between the decrease in volume on the application of pressure P_1, the elasticity constants, and the dimensions of the piezometer bulb is given by the expression

$$dV=\pi a^2 l\left\{\frac{P_0a^2-P_1b^2}{k(b^2-a^2)}+\frac{b^2(P_0-P_1)}{(b^2-a^2)n}\right\}$$

where l is the length of tube,
dV the change in the internal volume,
P_0 the internal pressure,
P_1 the external pressure,
k the bulk modulus of the glass,
n the rigidity of the glass,
and a, b, the internal and external radii respectively,*

when $$P_0=O,\ dV=\frac{\pi a^2b^2lP_1}{b^2-a^2}\left[\frac{1}{k}+\frac{1}{n}\right] \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

Now Poisson's ratio is given by

$$\sigma=(3k-2n)/2(3k+n) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2)$$

the only assumption being that the glass is isotropic.

From (1) and (2) we get

$$\frac{5-4\sigma}{1-2\sigma}=\frac{3(b^2-a^2)dV}{c.V.b^2.P} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

having put $1/c$ for k, where c is the compressibility per atmosphere.

From equation (3) and the results of experiments by Method II the value of σ has been derived, c being known from the results of Method I.

The work of Amagat is important in this connection, since he has verified the use of these equations for glass.†

Amagat states that the diminution of c with increasing pressure is within the limits of experimental error. This conclusion is of much importance, for it makes it possible to apply values of c found at low pressures to the whole range of pressures used in the determination of c for liquids.

APPARATUS.

The glass piezometer bulb was placed inside a steel cylinder so that a capillary tube sealed to one end of the bulb projected through the rubber packing. The bulb was filled with water, so that the level in the capillary could be read by means of a cathetometer. The whole apparatus was suspended in a thermostat after filling the remaining space in the steel cylinder with mercury, and this was connected by a steel tube to a Budenberg gauge and screw pressure apparatus filled with mercury.

The apparatus is described only briefly here, as a full description will be given when dealing with the compressibility of solutions in a later paper. The thermostat was constant to 0·01°C., and no variation (while pressure was constant) was observed in the water level in the capillary. The capillary was calibrated from an etched mark just outside the piezometer; it showed 0·02 c.c.s. per cm. Measurements were made at 30°, 40°, 50°, 60°, 70° and 80°C.; the numbers obtained at 80° are here given as a sample :—

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* Poynting and Thomson, Properties of Matter, p. 117 (1922).

† Comptes Rendus, 1889, 2, pp. 228 and 727.

Volume of bulb 25·68 c.c.s. Temp. 80°C.
External radius, 1·295 cms. Internal, 1·19 cms.

Mean dP	Mean dV	Mean σ	Mean deviation of σ from mean value.
13·93	0·01417	0·213	0·005

The results at the other temperatures are put together in a table at the end of this Paper.

Now $\sigma=(3k-2n)/2(3k+n)$, where k=the bulk modulus and n=the rigidity modulus. Rearranging, $n=3k(1-2\sigma)/2(1+\sigma)$. Values of n have been calculated and are given in C.G.S. units.

Again, Young's modulus $q=9nk/(3k-n)$. Values of q have also been calculated in C.G.S. units.

This work was undertaken originally to determine the compressibility of glass, but as it has been found possible at the same time to determine other constants with fair accuracy, it was thought worth while to publish them together. The final results are given in the following table :—

——	Soda glass.	Jena 16^{III}.
c 10^{-8}	$238{\cdot}0 \quad +0{\cdot}185\ t$	$220{\cdot}6 \quad +0{\cdot}28\ t$
k $1\text{-}0^{11}$	$4{\cdot}241 \quad -0{\cdot}00296t$	$4{\cdot}577 \quad -0{\cdot}0051\ t$
σ	$0{\cdot}2207-0{\cdot}00012t$	$0{\cdot}2475-0{\cdot}00068t$
n 10^{-11}	$2{\cdot}99 \quad -0{\cdot}002\ t$	$3{\cdot}29 \quad -0{\cdot}0036\ t$
q 10^{-11}	$7{\cdot}27 \quad -0{\cdot}005\ t$	$7{\cdot}97 \quad -0{\cdot}009\ t$

These relations have been extrapolated to 0° to make them available over the range 0° to 80°C., and may be used to 100°C. without appreciable error.

COMPARISON OF RESULTS.

It will be noticed that the compressibility of soda glass is slightly greater than that of Jena glass; in each case it increases slowly with rise of temperature. The results have been converted into bulk moduli (k) in C.G.S. units in order to compute n and q.

Our value for Young's modulus is here compared with the values obtained by Bell at 0°C.[*] for various types of glass by an acoustical method. He also observed that his results with newly worked glass were not consistent.

Type of Glass.	q 10^{-11}	Type of Glass.	q 10^{-11}
Soft German	6·95	Soft Jena	7·16
Baird and Tatlock, soda	6·91	Soda (Nelson, London) ... ... (extrapolated from our results)	7·30
Powell & Sons, gauge tube ...	7·36		

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[*] Bell and Chree, Proc. Phys. Soc., 19, 516 (1905).

Straubel found 0·228 for σ in Jena 16^{III} at room temperature. Berkeley* gives values for the compressibility of Jena glass at 0° and 30° obtained by himself from Amagat's absolute values for the coefficient of compressibility of mercury, and his own values for the apparent compressibility. He gives also a value obtained by Straubel. The numbers are :—

Temp.	Berkeley.	Straubel.	Perman and Urry.
0°C.	$223\ 10^{-8}$	...	$221\ 10^{-8}$ (extrapolated)
30°C.	$229\ 10^{-8}$	...	$229\ 10^{-8}$
(Room temp.)	...	$228\ 10^{-8}$	...

Compressibility plotted against temperature gives a straight line, so that extrapolation is easy.

No hysteresis effects were observed in any of these experiments, as the pressure was not continued a sufficient length of time.

DISCUSSION.

Mr. J. P. Andrews: The only value given in the Paper with which the authors' value of Poisson's Ratio for Jena 16^{III} glass may be compared is that of Straubel (1899), which is considerably lower. It may be pointed out that Straubel throughout his experiments, which were elaborations of Cornu's interference method, bent his glass bar over knife-edges whose maximum separation was 7 cms., the bars themselves sometimes reaching 3 cms. in width. It is known now that the separation should be greater than three times the width of the bar. For this reason, Straubel's values are certainly too small, and part of the difference between these and the authors' is to be accounted for in this way. It is, however, improbable that the whole difference is made up of this error.

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* Phil. Trans., A. 218, 295 (1919).

XXVI.—A VALVE-MAINTAINED HIGH-FREQUENCY INDUCTION FURNACE AND SOME NOTES ON THE PERFORMANCE OF INDUCTION FURNACES.

By G. Eric Bell, *B.Sc.*, *A.Inst.P.*, Physics Department, National Physical Laboratory, Teddington, Middlesex.

Received March 8, 1928.

Abstract.

This paper is divided into two parts. In Part I the electrical design is given of a valve-operated high-frequency induction furnace, together with some details of its performance, and in Part II a theory of the behaviour of induction furnaces in general is developed, and some experimental results supporting the theory are given.

Part I.—A Valve-operated High-frequency Induction Furnace.

The principles of heating by high-frequency (or ironless) induction are too widely known to render it necessary to discuss them here; it will suffice to say that the substance to be heated (which must be an electrical conductor or be contained in a conducting crucible) is surrounded by a coil in which alternating current of a suitable frequency is flowing, the eddy-currents induced in the substance serving to heat it. Several attempts have been made to propound a theory of heating by high-frequency induction, notably by Northrup,* Burch and Davis,† and Fischer.‡ The subject is referred to in Part II of this paper.

There are at least four accepted ways of producing the necessary alternating current in the inductor-coil, viz. :—

(*a*) The spark-oscillator, as used on a commercial scale by Northrup.
(*b*) The high-frequency alternator, as used extensively in Germany and in America.
(*c*) The thermionic valve oscillator.
(*d*) The Poulsen arc, which may be useful for giving high output at comparatively low frequency.

The high-frequency furnace to be described was required to be of a flexible type, capable of meeting the varied demands of a research laboratory. The valve-oscillator type lends itself admirably to the purpose, since it is possible to vary the frequency and other constants over a wide range, and also to obtain a very steady, delicately-controlled output. It may be added that the furnace was required in the first instance for the purpose of making measurements of the melting points of pure metals in a vacuum or a controlled atmosphere.

The valve-maintained induction furnace does not appear to have been used extensively on a large scale commercially, and there is practically no information published on the design and operation of such furnaces. The following notes dealing with the construction of a valve-maintained high-frequency induction furnace of 12 to 15 k.v.a. input may consequently be of interest.

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* Northrup, E. F., Trans. Am. Electrochem. Soc., Vol. XXXV, p. 69 (1919).
† Burch and Davis, Phil. Mag., Vol. I, p. 768 (1926).
‡ Fischer, W., Zeits. f. techn. Phys., Vol. VII, p. 513 (1926).

The choice of a suitable frequency presents some difficulty. Theoretical investigations suggest that above a particular frequency the efficiency of an induction furnace is constant, although it falls off rapidly below this frequency. This point is dealt with more fully in Part II of this paper.

The actual value of the frequency above which the efficiency is constant is a complicated function of the geometric dimensions of the charge, its electrical resistivity and its permeability; and, in general, it is found that the critical frequency is comparatively low.

When we consider the construction of an actual furnace, however, other considerations enter. Low-frequency resonating systems demand high capacity condensers capable of passing heavy currents, and, since these are both bulky and expensive, it may be advantageous to operate on a much higher frequency, even though the efficiency may be rather less. This has been done in the furnace described in this paper, and the desired results have been obtained satisfactorily at frequencies ranging from 700 to 1,000 kilocycles per second. For the immediate purpose in hand, charges of metal not greater than about 2 kilogrammes had to be legislated for, and it transpired that the appropriate inductor coil had an inductance of about 12 microhenries. The corresponding capacity is about 0·003 or 0·004 microfarad; this can easily be obtained using comparatively inexpensive air-dielectric condensers in the oscillatory-circuit.

The choice of circuit for the valve-oscillator was governed by the following principles :—

(1) Reliability; (2) safety in operation; (3) flexibility; (4) simplicity, especially simplicity of the inductor coil, which in an induction furnace becomes the furnace coil surrounding the material to be heated.

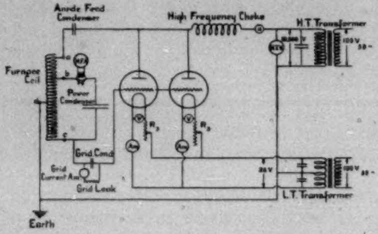

FIG. 1.—DIAGRAM OF HIGH-FREQUENCY CIRCUIT.

A modification of the well-known Hartley circuit, employing a centre-tapped coil, satisfies these conditions, and was therefore adopted. Preliminary experiments were made with other circuits, but none seemed to exhibit any marked superiority over this simple circuit. The "parallel-feed" system ensures safety in operating, since the furnace coil is at earth-potential from the low frequency point of view, and the high-frequency potential developed across it is not dangerous. The high-frequency circuit used is shown diagrammatically in Fig. 1. It will be seen that two valves (of which details are given later) are used, directly in parallel, in preference to the "back-to-back" or "push-pull" system, which is in some cases more satisfactory. This procedure was adopted owing to the limitations of the power supply available. Fig. 1 is self-explanatory, but it may be useful to record the values adopted for various components. The high-frequency choke is a single layer solenoid wound on a skeleton former 12 inches in diameter, and consisting of about 250 turns of 22 S.W.G. silk-covered copper, spaced ten turns to the inch, giving an inductance of about 7,000 microhenries. The anode feed condenser, which serves to prevent the flow of low-frequency current without hindering the high-frequency current or appreciably changing the phase relations, has a capacity of 0·002-0·004 microfarad. The grid condenser has a capacity of 0·005 microfarad, and the grid-leak, which is wire wound and oil cooled, has a resistance of 10,000 ohms. The bypass condenser across the H.T. transformer has a capacity of 0·002 microfarad, and those across the L.T. transformer are each 0·01 microfarad.

DETAILS OF COMPONENTS.

(*a*) *Valves.*—These are silica transmitting valves of the type O.C.2·5 KW. supplied by the Mullard Radio Valve Company. The following are their chief operating characteristics :—

Rating	3 kw. continuous anode dissipation.	Filament current ...	40 amps.
		Total emission	3 amps.
		Amplification factor ...	62.
Max. anode voltage	10,000 volts.	Impedance	42,000 ohms.
Filament voltage ...	17·2 volts.		

The electrode seals must be cooled by an air blast, which is supplied by blowers. In the present furnace two blowers connected to two independent supply circuits are used, and, since each blower alone is capable of supplying the necessary cooling air-blast, the valves should not be damaged by failure of one of the blowers. Signal lamps on the control panel indicate that the blowers are operating, and an electro-magnetic relay incorporated in the main circuit-breaker makes it impossible to switch on power until the blower circuits are complete, thus ensuring that the filaments of the valves cannot be heated until the air-blast is operating.

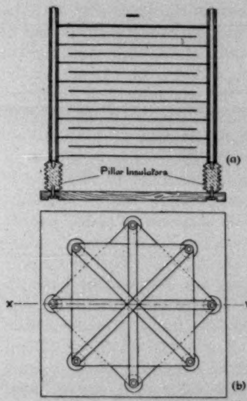

FIG. 2.—DIAGRAM OF CONDENSERS USED IN OSCILLATORY CIRCUIT. Plan (*b*) and Section through *XY* (*a*).

(*b*) *High-tension Supply.*—This is supplied from a 20 K.V.A. oil-cooled transformer, and is not rectified. The high-tension voltage is regulated by means of variable resistances in series with the primary of the transformer. An alternative—though, for this purpose, a less favourable—means of control would be by a suitably designed auto-transformer or an induction regulator. The regulating resistance is in two parts. The first part is designed to carry about 20 amps., and has a total resistance of 30 ohms, thus serving to keep down the H.T. voltage to about 3,000 volts until the filament voltage is properly adjusted. This portion is then short-circuited by means of a link-switch, and the second rheostat of 2 ohms total resistance, divided among sixteen studs, serves to give the necessary fine regulation.

(*c*) *Filament Heating Current.*—This is supplied by a 1½ K.V.A. transformer stepping down from 100 to 25 volts. Regulation is obtained partly by a rheostat included in the primary circuit of this transformer, and partly by fine adjustment rheostats in each filament circuit.

(*d*) *Condensers.*—All condensers, except those in the output oscillatory circuit, are mica-dielectric condensers. Those in the main oscillatory circuit are air-dielectric condensers designed and made at the Laboratory. Their general construction can be seen from Fig. 2. The plates (of which there are fourteen in each condenser) are of zinc sheet $\frac{1}{8}$ inch thick and 15 inches square, and the separation between the plates is 1 inch. Each condenser has a capacity of about 0·001 microfarad. Care has been taken to avoid points and sharp edges in the construction of the condensers.

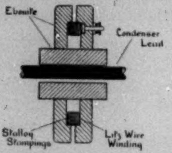

FIG. 3.—SECTION THROUGH DYE CURRENT TRANSFORMER.

(*e*) *High-frequency Ammeter.*—The various instruments used on the switch-board are standard A.C. instruments. Mention may be made, however, of the ammeter used for measuring the high-frequency current, for which purpose an ordinary hot-wire ammeter giving a full-scale deflection for 0·8 amp. is linked with the oscillatory circuit through a current transformer of the type described by Campbell and Dye.* This consists essentially of a laminated annular core of stalloy on which are wound a hundred turns of "Litz wire." The actual construction is shown in Fig. 3. The large central hole in the ebonite casing of the transformer is threaded on to one of the leads from the coil to the condenser, and the ends of the Litz winding are connected to the ammeter. The transformer thus steps down the current in the ratio of 100:1, and the ammeter has been recalibrated to read current directly when used in conjunction with this transformer.

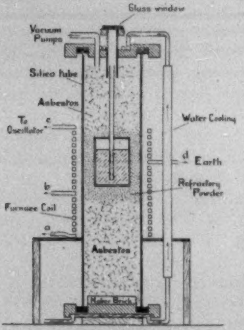

FIG. 4.—DIAGRAM OF FURNACE PROPER, SHOWING CHARGE IN POSITION AS ARRANGED FOR DETERMINATION OF MELTING POINTS.

(*f*) *The Furnace Proper.*—This is shown in Fig. 7, and also diagrammatically in Fig. 4. It consists of a silica tube round which is the inductor coil. This coil may be of solid copper strip or of flattened copper tubing wound edgewise, through which cooling water may be passed. In this furnace the coils are designed to have an inductance of about 12 microhenries. The one most frequently used (for loads of one or two kilogrammes of metal) consists of about fifteen turns of 6 inches internal diameter, space $\frac{3}{16}$ inch between turns; of these, eight turns are included between *b* and *c* (Fig. 1), and carry the main oscillatory current. The presence of the charge within the coil causes the load-impedance between the anodes of the valves and earth to be very low, as it increases the effective resistance of the oscillatory circuit. Since such a load is undesirable, it is necessary to increase the impedance of the load artificially by connecting the valves (via the feed condenser) not to the point "*b*," but

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* Campbell and Dye, Proc. Roy. Soc., A, Vol. XC, p. 621 (1914); and Nat. Phys. Lab. Collected Researches, Vol. XII, p. 14 (1915); Dye, Inst. Elec. Eng. J., Vol. LXIII, p. 597 (1925).

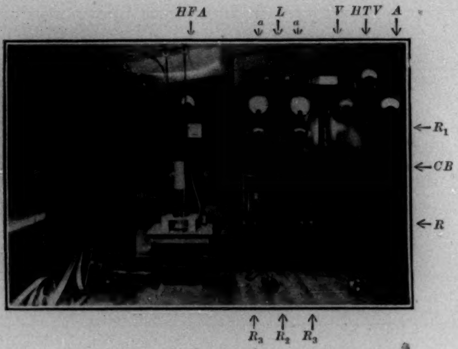
HFA
a
L
a
V
HTV
A
R1
CB
R
R3
R2
R3

FIG. 6.—GENERAL VIEW OF SWITCHBOARD AND FURNACE.

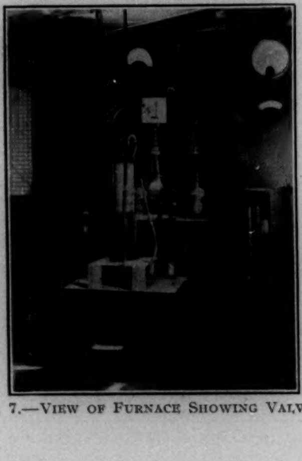

FIG. 7.—VIEW OF FURNACE SHOWING VALVES.

[*To face page* **197**

to some other point "*a*" on the coil. The best position of the "anode tap" (as the adjustable contact *a* is called) must be found by trial under working conditions. As the anode tap is moved away from "*b*" it is in general found that, up to a certain point, the output rises and the anode current falls; beyond this point the output falls, as also does the anode current. The best position of the anode tap is that which gives the maximum high-frequency current together with a reasonably low anode current. The tapping point *d* is approximately the mid-point of that portion of the coil between *b* and *c*. The silica tube is closed at each end by brass end pieces seating on rubber gaskets. The upper end-piece has a window for sighting into the metal, and also a tube through which the enclosure may be evacuated. The specimen to be heated is placed within the tube, as shown, being carefully positioned to be in the centre of the effective portion of the coil, and packed round with suitable lagging.

General Lay-out.

Fig. 5 shows the arrangement of the circuits used on the low-frequency side of the equipment. To the left of the diagram is the main circuit-breaker, which controls the main circuit feeding the primary of the high-tension and filament current transformers, and is itself controlled by two subsidiary circuits. One of these subsidiary circuits must be completed before the main switch can be thrown in, and includes switches linked mechanically with the blower switches, and also a short-circuiting contact on the door to a safety cubicle in which the equipment other than the furnace proper is housed. It is thus impossible to switch on power until the door giving access to the cubicle is properly fastened and the blowers are operating. The other subsidiary circuit consists of a coil and trip connected to conveniently placed emergency pushes, and so arranged that when these are pressed all A.C. power is immediately switched off.

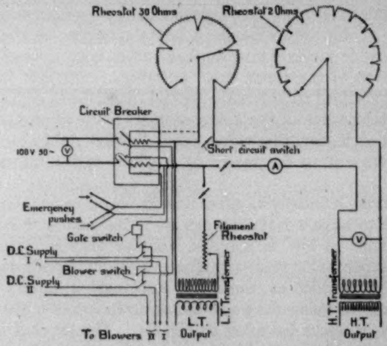

Fig. 5.—Diagram of Low-frequency Circuit.

A general view of the switchboard and control panels is given in Fig. 6. The main circuit-breaker (*CB*) is seen on the panel on the extreme right of the photograph; above the circuit-breaker are an ammeter (*A*) and a voltmeter (*V*) for measuring the power input, and above the switchboard, well out of reach, is an electrostatic voltmeter (*HTV*) giving the high-tension voltage. Immediately above the circuit-breaker is the control handle of the 30 ohm rheostat (R_1), the link switch for short-circuiting it being at the bottom of the panel. Just above this switch is the control handle (*R*) of the main 2-ohm rheostat.

On the adjacent panel are the instruments pertaining to the filament-heating circuits and the blowers. The centre rheostat (R_2) is in series with the primary of the filament-current transformer, and the two outer ones (R_3, R_3) are in series respectively with the filament of each valve. In the centre of this panel are the switches controlling the blowers and signal lamps (L) to indicate the behaviour of the blowers; at either side of these lamps are voltmeters and ammeters used in adjusting filament temperatures; above this panel are ammeters (a, a) for measuring the D.C. component of the anode current through the valves.

Further to the left is a wire grid, through which the valves, etc., in the cubicle may be observed. This portion is shown more clearly in Fig. 7. In front of this grid is the furnace itself, mounted on a table. Above the wire grid is the ammeter (*HFA*) for measuring the high-frequency current in the furnace coil, and just below it is a U-gauge to indicate the pressure within the silica tube.

OPERATION AND PERFORMANCE.

Given a particular crucible of metal to be melted, one must first select an inductor-coil of suitable size for efficient heating. For small charges not exceeding one or two kilogrammes, experience has shown that the radius of the coil should usually be about twice that of the crucible; complications occur, however, in certain instances, especially in the case of relatively poor conductors (such as carbon and ferromagnetic materials), since loads of this nature impose a heavy damping on the oscillator. To some extent the undesirable effects of this very low load-impedance may be compensated by raising the position of the anode tap "*a*" (Fig. 1); but it is generally necessary to use an inductor coil of considerably greater diameter in order to obtain stable oscillations. The inductance of the coil should be made approximately the correct value to suit the oscillator, this value being decided chiefly by the frequency and the oscillatory current to be passed at a given high-tension voltage.

If a coil of small diameter is required, it usually happens that its ohmic resistance is unduly large, since it is necessary to have a large number of turns, and as it is desirable to make the ohmic resistance as low as possible, another arrangement must be adopted. The total inductance associated with the oscillatory circuit is divided between two separate coils, one of which has the radius and length necessary to give efficient heating, and the second being designed to have minimum high-frequency resistance (i.e., its diameter is about 1·5 times its length). The two coils are then connected in series, and the crucible inserted in the appropriate one.

The furnace has proved capable of meeting the demands of an investigation dealing with high melting points. For example: Kilogramme charges of palladium (M.P. about 1550°C.), nickel (M.P. about 1450°), chromium (M.P. about 1630°), and iron (M.P. about 1530°) have each been melted successfully, using about 8½ kilowatts input.

PART II.—PERFORMANCE OF HIGH-FREQUENCY INDUCTION FURNACES.

Introduction.

Several attempts have been made to analyse mathematically the problem of heating by high-frequency induction since the original account of the furnace was published by Northrup,[*] and, as has been pointed out by Burch and Davis, some of these analyses are not reliable. Burch and Davis* have themselves published a fairly rigid solution to the problem for the case in which the charge consists of a solid cylinder, and the inductor consists of a single cylindrical sheet, long compared with its radius, and complete except for an infinitesimal longitudinal gap across which the condenser is connected.

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[*] **Northrup, Trans. Am. Electrochem. Soc., Vol. XXXV, p. 69 (1919).**

The " efficiency " of a furnace may be defined as the ratio of the energy appearing as heat in the charge to the total energy in the whole system of inductor and charge. Thus, if W_C watts appear as heat in the charge, and W_H watts are lost in heating the coil, and W_R watts are lost by radiation, the efficiency of the furnace may be defined as

$$\eta_1 = W_C/(W_C + W_H + W_R)$$

This definition of efficiency takes no account of the energy inherently lost in whatever device is used for producing the necessary high-frequency current. If η_2 is the efficiency of conversion from low-frequency current to high-frequency current, the quantity $\eta_1\eta_2$ represents the ratio of energy appearing as heat in the charge to the total energy supplied to the entire system, and may be defined as the " over-all efficiency."

In the definition of η_1 given above it has been assumed that the losses in the synchronising condensers are negligibly small, and it can easily be shown that the loss by radiation (W_R) is also small compared with W_C and W_H.

The efficiency of the furnace is a function of the following variables :—

Radius of coil (R_1) ; radius of charge (R) ; resistivity of charge material (ρ) ; resistivity of coil material (ρ') ; permeability of charge material (μ) ; permeability of coil material (assumed to be unity) ; frequency (f) ; length of charge (l) ; and to determine the thermal input to the charge we shall also require the R.M.S. value of the high-frequency current I.

The following parameters also occur in the discussion

$$\beta' = \sqrt{\rho/4\pi i p\mu} \text{ (used by Burch and Davis)}$$

and

$$m = \sqrt{4\pi p\mu/\rho} = \sqrt{8\pi^2\mu f/\rho}$$

where p is the pulsatance in radians per second and $i = \sqrt{-1}$. In their analysis of the efficiency of a high-frequency induction furnace constructed as already explained, Burch and Davis find that the efficiency is independent of frequency, provided that $R/\beta' > 3$, and has a value

$$\eta_1 = lR\sqrt{\rho}/(lR\sqrt{\rho} + l'R_1\sqrt{\rho'})$$

if μ is assumed to be unity.

A similar result is obtained as a result of the following discussion, which, although not claiming the mathematical rigidity of the solution of Burch and Davis, possibly brings out more clearly the physical processes involved, and, by taking approximate values for the resistance of a coil at high frequency, corresponds perhaps a little more closely to the real case.

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* **Burch and Davis, Phil. Mag., Vol. I, p. 768 (1926).**

Thus the determination of η_1 conveniently resolves itself into a determination of (*a*) the heat generated in the charge, and (*b*) the heat generated in the inductor coil as a result of its resistance.

The problem of determining the heat developed in the charge is analogous to that of determining the losses by eddy currents in a transformer core, which problem has been solved by Heaviside[*] and by Russell[†] for the case in which the coil is indefinitely long, so that the value of the field between the coil and the boundary of the charge has everywhere a value of $4\pi nI$ (where n is the number of turns per unit length in the coil) and the alternating current varies according to a harmonic law. Using the method adopted by the latter, the core loss (which is, in this case, the heat generated in the specimen) is found to be

$$W_e = \frac{\pi R l f \mu H^2}{m} \cdot \frac{Z(mR)}{X(mR)} \cdot 10^{-7} \text{ watts} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where H is the R.M.S. value of the magnetic intensity at the boundary of the core, $Z(mR) = ber(mR)ber'(mR) + bei(mR)bei'(mR)$ and $X(mR) = ber^2(mR) + bei^2(mR)$.

If (mR) is sufficiently great, $Z(mR)/X(mR)$ approaches the limit $1/\sqrt{2}$, the exact value being only slightly less if $mR > 10$. When this value is taken we have

$$W_e = \tfrac{1}{4} \,.\, RlH^2\sqrt{\mu \rho f} \,.\, 10^{-7} \text{ watts}$$

$$= 4\pi^2 Rln^2 I^2 \sqrt{\mu \rho f} \,.\, 10^{-7} \text{ watts} \quad . \; . \; . \; . \; . \; . \quad (2)$$

These expressions give the heat generated by eddy currents in the charge if the coil is indefinitely long. To the general accuracy contemplated (say 10-15 per cent.) it can be shown that the field within the coil is sufficiently nearly uniform and of value $4\pi nI$ if the coil is only slightly longer than its diameter. Hence the energy generated in the charge can be approximately calculated for most practical cases. It will be shown later in this Paper that the agreement between this calculated value and an observed one is sufficiently good. The length of the charge must clearly not be so great that part of it lies in a field of intensity widely different from that given by $4\pi nI$.

(*b*) The coil loss may be calculated approximately from the values given by Howe[‡] for the high-frequency resistance of coils wound of wire of square or rectangular section. The corresponding problem for coils wound with wire of circular cross-section has been treated by Butterworth.[§] As the coils usually employed in induction furnaces are wound with wire of square or rectangular section, only this type is considered in detail. The relation between low-frequency resistance (R_0) and high-frequency resistance (R_e) for such coils is

$$R_e = R_0 \beta\tau/\gamma \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

where τ is the radial thickness of the wire, $\beta = 2\pi\sqrt{f\mu/\rho}$, and $\gamma = (\cosh 2\beta\tau - \cos 2\beta\tau)/(\sinh 2\beta\tau - \sin 2\beta\tau)$.

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[*] Heaviside, Electrican (1884), and Collected Researches.

[†] Russell, Alternating Current, Vol. I, p. 503.

[‡] Howe, Proc. I.E.E., Vol. LVIII, p. 152 (1920).

[§] Butterworth, Wireless World, Vol. XIX, p. 754 and 811 (1926).

Fig. 8 (reproduced from the paper by Howe) shows the relation between R_e/R_0 and $\beta\tau$. It will be seen that if $\beta\tau$ is greater than about 2, R_e/R_0 becomes very nearly directly proportional to $\beta\tau$, that is, other things being equal, directly proportional to the square root of the frequency.

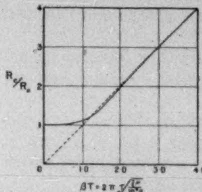

FIG. 8.—CURVE (AFTER HOWE) SHOWING RELATION BETWEEN R_e/R_0 AND $\beta\tau$, FROM WHICH R_e MAY BE FOUND FOR ANY VALUE OF $\beta\tau$.

The expression $R_e = R_0\beta\tau/\gamma$ holds for closely-wound, indefinitely long coils: if the turns are slightly separated, the flux density will be reduced. The same result would be brought about by a decrease of permeability from unity to $n\tau_1$, where τ_1 is the axial thickness of the wire. Further, the effect of the finite length of the coil may be allowed for by taking a further reduced value of the flux density. For a coil whose length is greater than its diameter, this correction does not amount to more than a few per cent.

Thus spaced winding and finite length can be allowed for by taking, instead of $\beta\tau$, a value $\beta\tau\sqrt{n\tau_1} \,.\, \sqrt{B_1/B}$. The correction factor $\sqrt{B_1/B}$ may be determined from the following values of H_1/H given by Howe. (Loc. cit.).

$l'/2R_1$	1·0	2·0	4·0
H_1/H	0·91	0·96	0·98

It is thus possible to determine the behaviour of the furnace under any given conditions to an accuracy sufficient for most purposes, when the charge consists of a solid right circular cylinder of conducting material placed coaxially within the inductor.

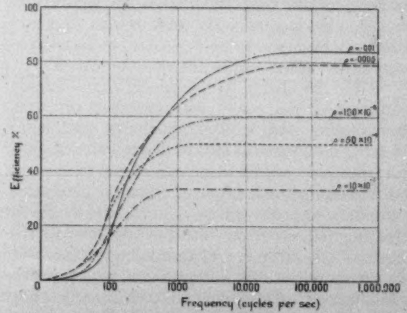

FIG. 9.—VARIATION OF EFFICIENCY WITH FREQUENCY FOR DIFFERENT VALUES OF RESISTIVITY (ρ) FOR THE PARTICULAR CASE OF A CHARGE OF RADIUS 5 CMS. AND LENGTH 10 CMS. WITHIN AN INDUCTOR OF RADIUS 10 CMS. AND LENGTH 20 CMS.

It will be seen from equations (2) and (3) that the heat generated in the cylindrical charge, and also the energy lost in heating the coil, both become proportional to the square root of the frequency after the frequency has attained a certain value. This shows that, beyond this "critical frequency," the efficiency of the furnace is independent of frequency.

Fig. 9 shows graphically the variation of efficiency with frequency for different values of the resistivity in the case of a cylindrical charge of radius 5 cms. and length 10 cms. within a coil of radius 10 cms., length 20 cms., made of 20 turns copper strip 0·5 cm. square in section.

EXPERIMENTAL RESULTS.

An investigation into the behaviour of induction furnaces was undertaken with the aid of the valve-maintained furnace described in Part I of this paper.

In this investigation the energy supplied as heat to the charge and that lost by heating the inductor were separately determined.

The inductor consisted of a water-cooled coil 7 cms. radius of 10 turns spaced 1·2 turns per centimetre, constructed of copper tubing of circular section ¼ diameter externally.

The inductor fitted closely round a silica tube, and within this was a second non-conducting refractory tube, packed round with asbestos wool, as shown in Fig. 10. The object of this second tube was to permit the specimen under investigation to be inserted and withdrawn without disturbing the lagging.

In making a determination of W_c and W_H the specimen was put in the inductor and the high-frequency current maintained at a constant value. Under these conditions the charge attained a certain steady temperature, at which the heat supplied by the furnace was equal to that lost to the surroundings. This temperature was measured by means of an optical pyrometer, and subsequently used to determine the energy input to the charge. The energy lost in heating the coil was determined by noting the temperature of the cooling water on entering and leaving the coil and measuring the flow. This was done during the early stages of the experiment, before any appreciable heat reached the coil from the charge by conduction through the lagging. By this means the coil resistance was found to be 0·34 ohm, as a mean of many observations under different conditions.

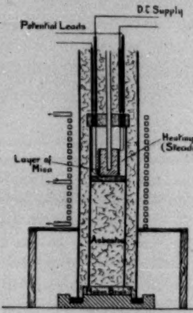

FIG. 10.—DIAGRAM OF FURNACE ARRANGED FOR MAKING OBSERVATIONS ON THE PERFORMANCE OF INDUCTION FURNACES.

After the charge had attained a final steady temperature, the power was switched off and, after cooling sufficiently, the charge was withdrawn carefully, so as not to disturb the lagging, and wrapped with a heating coil of nichrome wire, separated from the specimen by the thinnest possible layer of mica for insulation purposes. The specimen was then carefully replaced within the coil, and a steady direct current passed through the heating coil, so as to raise the specimen to the same final temperature as with the induction furnace. It was assumed that the energy necessary to raise the specimen to the steady temperature was approximately the same as that supplied by the furnace. This seems probable, since heating takes place primarily at the surface in each case, and the lagging which determines the heat loss is the same in each case. Actually it was found necessary to ascertain the temperature corresponding to values of the energy supplied by the heating coil, and determine the energy developed in the charge when heated by the induction furnace from the curve representing these observations.

The results of these observations for various specimens at a frequency of 10^6 cycles per sec. are given in Table I. The energy is given as watts per amp. current in the coil.

TABLE I.

Material.	Radius.	Length.	Resistivity.	Energy calculated.	Energy observed.
Nickel cylinder ...	2·8 cms.	7·5 cms.	4×10^{-5}	$15{\cdot}2 \times 10^{-2}$	$16{\cdot}6 \times 10^{-2}$
„ „ ...	2·5 „	3·8 „	4×10^{-5}	$7{\cdot}5 \times 10^{-2}$	$8{\cdot}0 \times 10^{-2}$
Graphite cylinder	2·5 „	5·7 „	$1{\cdot}5 \times 10^{-3}$	$58{\cdot}6 \times 10^{-2}$	$47{\cdot}4 \times 10^{-2}$
„ „ ...	4·0 „	6·0 „	$1{\cdot}5 \times 10^{-3}$	105×10^{-2}	$96{\cdot}0 \times 10^{-2}$

The value of the resistivity given in the fourth column is that at the final temperature which was adjusted to 850-1,000°C.

The agreement between observed and calculated values is better than would be expected from the nature of the problem. The fact that we are dealing with coils of finite length and charges whose length is comparable with that of the coil would lead one to expect the observed values to be considerably less than the calculated ones. That they are not less is possibly due to the presence of an appreciable amount of energy in the higher harmonics of the high-frequency current. It does appear, however, that the values calculated from the simple expression given above are of the right order.

The charges used in high-frequency furnaces are usually in the form of small fragments contained in a suitable crucible. Although it appears impossible to calculate the probable heat developed in such a charge, observations were taken on a crucible of nickel shot and one on a crucible of fragments of chromium, with the results given in Table II.

TABLE II.

Material.	Radius (of crucible).	Length (of charge).	Resistivity.	Energy observed (watts per amp.).
Nickel shot	2·5 cms.	5 cms.	4×10^{-5}	$49{\cdot}0 \times 10^{-2}$
Chromium pieces ...	2·5 „	5 „	15×10^{-5}	$67{\cdot}0 \times 10^{-2}$

The energy generated in these cases is not, as might have been expected, proportional to the square root of the resistivity, but while the specimen of nickel was in the form of shot of mean diameter, about 4-5 mms., the chromium was in the form of irregularly shaped fragments of varying sizes, and this probably accounts for the disparity.

The Author desires to acknowledge his indebtedness to the Superintendent of the Physics Department (Dr. G. W. C. Kaye, O.B.E., M.A., D.Sc.), at whose instigation the work was begun, and whose continued interest and assistance have rendered its conclusion possible, and to thank Mr. W. F. Higgins, M.Sc., for advice, and Mr. R. R. Strand, A.M.I.E.E., for advice and help.

DISCUSSION.

Mr. B. S. GOSLING said that the paper gave exactly the information which was required by those concerned. Referring to the high-frequency generating circuit shown in Fig. 1, he suggested that a push-pull arrangement might be preferable to the parallel pair of valves indicated, since transformers are inclined to perform badly with unidirectional current. It might be useful to note that stalloy stampings are not essential to the Dye type of transformer shown in Fig. 7, which will work quite well with an air core. Had the author considered the possibility of X-radiation from the apparatus, which is intended for use by non-specialists? The speaker had recently found, in testing a commercial apparatus employing valves of large power, that at a particular adjustment the anode voltage rose to double the high-tension supply. In the present case it was conceivable that X-radiation equivalent in volume to 10 mg. of radium and sufficiently hard to be dangerous, in the absence of protective screening, might be produced.

Dr. R. T. BEATTY noted with interest the author's conclusion that high frequencies, permitting the use of small condensers, are to be preferred in induction furnaces. The paper should have considerable influence on commercial practice. On what basis had the inductor coil mentioned in the penultimate paragraph of Part I been designed? It had been shown that with a prescribed diameter the optimum length of an inductance solenoid is infinite, with a prescribed volume it is zero, and with a prescribed surface it is, according to Butterworth, half the diameter, whereas in the present case the length was two-thirds of the diameter.

Dr. W. H. ECCLES said that the Author had done valuable work in attacking the problem of induction-furnace design from first principles, since high-frequency apparatus has hitherto been designed for wireless purposes, which require high voltages, whereas the present apparatus requires large currents. For instance, the Poulsen arc as designed by wireless engineers might be quite unsuited for the present purpose. What were the relative efficiencies of the Northrupp furnace and the present furnace? Valve apparatus seemed more suitable than high-frequency alternators for such furnaces, since the variation in load produced by the melting of the charge affects the circuit constants of the valve system so as to cause some degree of self-adjustment. Had the author found a change of frequency corresponding to such a change in load? The conclusion that above 10,000 cycles the frequency is immaterial to efficiency was an important one.

Dr. H. D. H. DRANE (communicated): The author's treatment of the essentials of furnace design and operation in Part I is very clearly put, and useful from a practical standpoint. I note that the arrangement shown in Fig. 1 was adopted "owing to limitations of the power supply available." Since the valves take only a half cycle supply, this arrangement would seem likely further to cramp the existing limited power supply. The back to back arrangement would not only evade the objectionable feature of half-cyclic working, but also halve the amplitude of the current supply pulses experienced with the arrangement of Fig. 1. It cannot be doubted that with industrial conditions of operation, power supply companies would object to the non-cyclic operation which the Author has adopted. A high-frequency furnace which I have had in use satisfactorily for about three years for experimental work upon ceramic materials overcomes these and other difficulties. In this two 10 kw. (anode dissipation) silica valves are arranged so that each works upon its appropriate half of the supply current cycle, and the demand upon the public power supply is thus regularised. The high-frequency side is correspondingly improved, since a sustained wave train is obtained, in place of intermittent wave trains sandwiched between idle half cycles given by the arrangement of Fig. 1. The Author would seem to sacrifice some efficiency and a measure of satisfactory operation by using the simple expedient of a grid leak and condenser only. For good and foolproof operation a permanent grid potential should be applied, and in the much larger generator to which I have referred above this is arranged by means of a specially designed grid transformer in conjunction with the grid network. With a rectified anode voltage supply this generator would give about 80 kw. output, though as at present arranged its maximum output is 15 kw.

The production of *X*-radiation from the anodes of valves during operation for furnace work, mentioned by Mr. Gosling, is not a risk to the operator under industrial conditions. With peak voltages of the order 15-20 kv. the *X*-radiation produced is not very penetrating; and with either a silica valve or with a water-cooled anode type valve, the *X*-radiation is emitted, almost exclusively, axially from the anode cylinder, which, with usual valve mountings, means away from the operator.

AUTHOR'S reply: With reference to the remarks of Mr. Gosling and Dr. Drane concerning the alternative "push pull" arrangement, I freely admit that there is every reason to believe that, in general, this would be more satisfactory from many points of view. The furnace just described, although a permanent addition to the range of furnaces in use in the department, is erected in a temporary building, in which the voltage of the available A.C. supply is far below normal; consequently it is impossible, with the existing apparatus, to obtain the 18,000 to 20,000 volts necessary for symmetrical working. The voltage could, of course, be raised initially by using a low-ratio step-up transformer; but as the furnace fulfils its immediate requirements, this step is not necessary at the moment. The modifications necessary for symmetrical working are very slight, and the values of the various capacities, inductances, etc., would probably not be appreciably altered.

No experiments have been made with regard to the production of X-rays from the valves, but tests will be made at the next opportunity. The only protection from X-rays is about 7 ft. of air, which is much more than the minimum given by the makers of the valves. As a result of the discussion following a series of Papers on various kinds of valves read before the I.E.E (*see* Inst. El. Eng. J., 65, pp. 812-822, 1927), it was considered that there was not likely to be any danger from this source. The experience of Mr. Gosling shows very clearly that no apparatus should be considered as safe until it has actually been proved to be safe by suitable measurements.

With regard to the question about coils asked by Dr. Beatty, the problem arises when it is required to heat a very small crucible of metal—say, three quarters of an inch in diameter and two inches long. The coil must fit fairly closely round the crucible, and hence must have a diameter of only about one or one and a half inches ; further, its inductance must be of the order of 12 microhenries. To obtain this inductance in a single layer coil a large number of turns is required, and the coil is unwieldy, as well as having an unnecessarily high ohmic resistance. Consequently, it is advantageous to make a coil of convenient shape and reasonably low ohmic resistance to give the required inductance, and to put this in series with the small coil of only a very few turns which serves to heat the metal. To meet the case of heating a small charge, Northrup devised what he calls the " focus inductor," which is described in his Paper in the Trans. Am. Electro-Chem. Soc. already mentioned. Efforts to utilise this in the valve-maintained furnace have not been successful.

Although no definite information is available, there is reason to believe that the efficiencies of the valve furnace and the Northrup furnace are comparable.

The interesting point regarding the change of frequency on melting also raised by the President will be investigated as soon as possible ; but I think that little change will be found when the charge consists of a solid cylindrical ingot. It is a suprising fact that there is, in general, no appreciable change in the operating constants (output current, etc.) in these circumstances. On melting, the shape of the ingot does not alter, and hence it would appear that there can be no abrupt change of resistivity on passing from the solid to the liquid state.

On melting a crucible full of pieces of metal a very definite increase of oscillatory current occurs on melting, and presumably there is a correspondingly great change of frequency.

XXVII.—THE AMPLITUDE OF SOUND WAVES IN RESONATORS.

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Abstract.

After a consideration of the relations between the " pipe " and the " Helmholtz resonator," graphs of the variation of amplitude, obtained by means of a hot-wire anemometer traversed through various types of resonator, are reproduced. By means of a calibrated manometer of the vibrating membrane type, some direct values of the impedance of orifices are obtained. With the hot wire, traverses across an orifice through which the air is vibrating in simple harmonic form are obtained, which show a tendency for the air to vibrate with greater amplitude in annuli remote from the centre of the orifice.

I. Relation between Pipe and Helmholtz Resonator.

In an earlier paper,* measurements of velocity and displacement amplitude at a number of points along a sounding organ-pipe were described. The present paper describes a continuation of the former work by similar methods to the study of the vibration in a sounding Helmholtz resonator. Interest in these two types of vibrating bodies of air seems to have been awakened lately, on the one side stimulated by the invention of the Tucker hot-wire microphone, and on the other evidenced by the results of several new measurements on the end-correction of pipes, published during the last few months, to be mentioned later.

The motion in the theoretical pipe consists of stationary waves; its longest dimension is comparable with the wave-length, and the displacement along the interior rises or falls continuously. Whereas in the more sensitive type of resonator—first devised by Sondhauss, but usually named after Helmholtz—we have a cavity of air whose dimensions are small in relation to the wave-length, and which communicates with the atmosphere by an orifice or neck whose cross-section is so restricted compared to that of the cavity that, in theory at any rate, the to-and-fro motion of the air may be regarded as confined to its neighbourhood. The classical theories of the " pipe " and the " resonator " are therefore mutually exclusive. In the former, changes of cross-section are excluded, the variation of displacement with distance along the pipe being all due to progressive phase-angle; in the latter, spatial phase-differences are excluded, the variation in displacement amplitude being due to sudden changes in cross-sectional area. In consequence of this, the law connecting the resonant frequency and the length of a cylindrical " stopped pipe " and the law for a Helmholtz resonator consisting of a cylindrical tube closed by a narrower hole at one end, are fundamentally different.

The formula for the pipe (lowest tone) is

$$4nL=c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(n is the frequency, L the " corrected " length, and c the velocity of sound).

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* Proc. Roy. Soc., 112, 521 (1926).

For the resonator,

$$\omega=2\pi n=c(\kappa/v)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(κ is the " conductivity " of the orifice, v the volume of the cylindrical cavity SL, when S is the cross-sectional area of cylinder and L its length.)

Then (1) gives nL constant, whereas (2) gives n^2L constant. Both these formulæ have been tested and found to fit the extreme conditions fairly closely, provided that with the value of L for the " pipe " is incorporated an " end-correction " amounting to 0·58 times the radius of the pipe, but few experiments seem to have been made on intermediate cases with a view to determine how a compromise is effected between the two end conditions and how one law passes over into the other.

By adapting the idea of acoustic " impedance " first introduced by Webster* it is possible, however, to obtain a formula (soluble by graphic methods) to cover all cases, as follows :—

If the displacement ξ of the air is uniform over any one section (area S)—i.e., if viscous resistance can be neglected—the velocity produced by an applied pressure difference p can be written in the form $S^{-1}(dX/dt)$, where X is of the dimensions $[L^3]$, and is a species of volume displacement. The impedance is defined as the ratio of these two quantities :—

$$Z=p/\dot{X} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

As p is alternating ($=Pe^{i\omega t}$), its average value P is taken in calculating Z.

Taking $p=Pe^{i\omega t}\cos(\omega L/c)$, with the further equation: $\rho\delta^2\xi/\delta t^2=-\delta p/\delta x$ we obtain, for the volume displacement in a " stopped pipe "

$$\dot{X}=-(SPe^{i\omega t}/i\rho c)\sin(\omega L/c),$$

the effective impedance of the " pipe " becomes

$$Z=p/\dot{X}=-\rho c/S\tan(\omega L/c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The equation of the motion in the orifice of the air of mass m is

$$(m/A^2)\ddot{X}=Pe^{i\omega t}.$$

Integrate, and $(m/A^2)\dot{X}=(P/i\omega)e^{i\omega t}$, whence $Z_0=\omega m/A^2$. But $m=Al\rho$, where l is the effective length of the orifice in the direction of vibration, and A is its area of cross-section. Put $A/l=\kappa$, then $Z_0=\omega\rho/\kappa$.

To get the total impedance of a pipe having such an orifice, the component impedances must be added in series—i.e.,

$$Z_1=Z_0+Z=\omega\rho/\kappa-\rho c/S\tan(\omega L/c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Resonance will occur when $Z_1=0$ or a minimum.

Strictly speaking, a viscous resistance F should be included in Z_1, preventing Z_1 from ever reaching zero in practice. F corresponds to " ohmic resistance " in electrical cases, and will be considered later. There is, in addition, the " radiation resistance " of the orifice to be considered if the resonator is designed to radiate sound ; but as this is a function of $(\omega/c)^2$, and (ω/c) is a small quantity, it may be neglected in these experiments.

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* Nat. Acad. Sci. Proc., 5, 275 (1919).

Otherwise, if $Z_1=0$, then

$$\tan(\omega L/c)=\kappa c/\omega S$$

or putting

$$\omega/c=k$$

$$\tan kL=\kappa/kS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For the ideal pipe, the conductivity of the orifice is infinite, giving $\tan kL=\infty$, which gives $nL=$constant. When, however, κ is comparatively small, so that $\tan kL=kL_3$ nearly, we have the resonator formula $n^2L=$constant. Our equation (6) thus embraces all cases, and the natural frequency may be found in any individual case by plotting $\tan kL$ against $(kL)^{-1}$, and finding where this curve is cut by the straight line, obtained by plotting κ/kS against the same abscissa, or, alternatively, by the use of tangent tables.

It may be noted in passing that when the impedance Z_0 consists of an orifice of equal, or nearly equal, cross-section to that of the pipe itself, the difference in the value of L from (6) corresponding to the appropriate value of κ and k, and that value given by $\tan kl=\infty$ is the end correction of the pipe.*

In order to test this formula, some resonance-tube measurements have been made in the usual fashion, one end of the tube being terminated by a water surface and the other covered with a thin metal plate pierced with a cylindrical hole. The fundamental length which resounded to various tuning forks for orifices of various diameters were determined, and are exhibited in the following table. I am indebted to Miss W. Curtis, a student, for assistance in obtaining these results.

TABLE I.

First tube : 4·3 cm. diam.

Diam. of orifice (cm.)	Resonant lengths (cm.).				
	$n=$ 1,000	$n=$ 500	$n=$ 335	$n=$ 246	$n=$ 224
0·70	1·7	2·9	11·5	17·0	20·5
1·15	2·7	9·0	15·0	25·0	29·8
1·6	3·5	11·0	18·2	28·0	32·0
2·2	4·0	12·5	20·0	29·7	34·2
2·6	4·7	13·2	21·0	30·0	35·4
3·2	5·3	14·0	22·2	31·4	36·0
4·3	6·0	15·4	23·6	32·5	36·8

Second tube : 2·2 cm. diam.

Diam. of orifice (cm.)	Resonant lengths (cm.).				
	$n=$ 1,000	$n=$ 500	$n=$ 335	$n=$ 246	$n=$ 224
0·7	4·2	11·5	19·0	27·8	32·1
1·2	5·2	14·5	21·5	30·2	35·0
1·6	5·9	15·0	22·6	32·0	35·6
1·9	6·2	15·0	22·8	32·5	36·3
2·2	7·0	16·0	24·4	32·8	37·8

* The value of the end-correction of a pipe has been worked out in connection with the stationary wave method for measuring absorption coefficients from impedance principles in a process differing from the above by E. T. Paris, Phil. Mag., 4, 907 (1927).

Taking the conductivity of a simple circular orifice to be equal to its diameter (nearly) Figs. 1 and 2 show the variation of L with κ for the various frequencies, together with the theoretical values from equation (6). If the frequency be kept constant, $\tan kL \propto \kappa/S \propto r/R^2$, where r=radius of the orifice and R of the tube. The theoretical values were obtained by calculating $\tan^{-1}(\kappa/Sk)$ in radians, giving values of kL, and hence of L. It can be seen that the theoretical and the experimental curves are similar, but that there is a lack of agreement when r is large. This, one may judge, is due to a change in the dependence of κ upon r. It is evident that some such change must take place as the "resonator" conditions are approached for then the effective length of the orifice is changed in virtue of the large pressure gradient through the orifice. It seemed better, however, to plot these graphs on the assumption that the conductivity remained equal to the diameter.

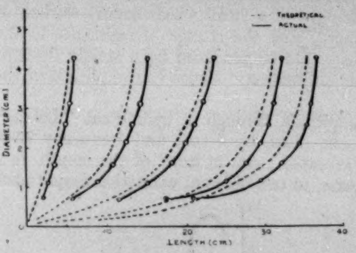

FIG. 1.—RESONANT LENGTHS: TUBE 4·3 CM. DIAMETER.

The discrepancy is greater with the wider tube, which confirms what Anderson and Ostensen* note—i.e., that the end-correction increases relatively in wider pipes. Irons† has found, in his experiments on constrictions in Kundt's tube, that the resonator formula (2) holds good if the orifice is less than 1 cm. in diameter. The present results indicate a limit of this order, but the range of orifice for which (2) holds good depends also on the frequency.

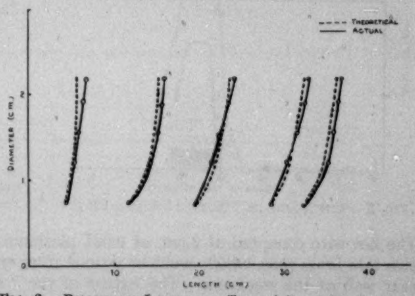

FIG. 2.—RESONANT LENGTHS: TUBE 2·2 CM. DIAMETER.

II. DISPLACEMENT AMPLITUDE THROUGH HELMHOLTZ RESONATORS.

In view of the good agreement between theory and practice in regard to the dimensions of Helmholtz resonators and their resonant frequency when the dimensions of the orifice were small compared with those of the cavity, it was expected that the assumption of Helmholtz that the motion is confined to the neck would be nearly justified. In order to test this, traverses were made from orifice to rear of Helmholtz resonators of various sizes and shapes, with the hot-wire to give measurements of the displacement amplitude in the same way as is described in the former paper for the pipe.

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* Phys. Rev., 31, 267 (1928).

† Phil. Mag., 5, 580 (1928).

The essence of this method is that the steady fall of resistance of a short length of 0·001 in. diameter wire (of platinum in some cases, of nickel in later measurements) when cooled by the oscillating air-draught, is measured on a form of Kelvin bridge. The wire is calibrated in the steady draught of a small wind-channel worked at known wind velocities. It has been shown that the change of resistance in the simple harmonic draught is the same as would be produced by a steady draught at velocity equal to the maximum in the alternation. Hence the amplitude of the aerial motion can be calculated.

Fig. 3 shows the variation of amplitude through a cylindrical "bottle" resonator (capacity 6,000 c.c., neck 3 cm. diam. by 7 cm., and frequency 75·5) excited by a tuning fork whose prongs vibrated 1 cm. in front of the mouth, in a plane perpendicular to the plane of traverse, at two different excitation amplitudes.

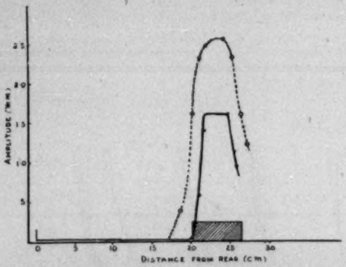

FIG. 3.—AMPLITUDE IN "BOTTLE" RESONATOR.

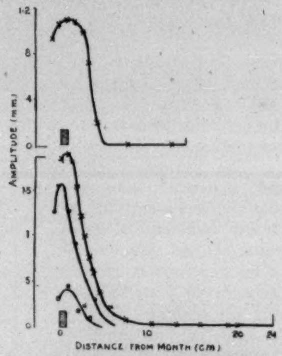

FIG. 4.—AMPLITUDE IN RESONATORS WITH CIRCULAR-HOLE ORIFICES.

The hot wire consisted of 2 cm. of 0·001 platinum stretched between the ends of two thin brass rods, which could be pushed from outside through the mouth to the rear wall of the resonator. The extent of the "neck" is shown by the shaded portion of the figures.

Fig. 4 shows similar results for two resonators of spherical type, having a circular hole as "mouth." These resonators were provided with nipples at the back for attachment to manometric capsules, enabling the pressure amplitude in the interior to be measured at the same time as the velocity in the mouth. For this purpose the manometric capsule membrane shown in Fig. 5 was employed.

The capsule widens out conically from a short cylindrical piece to a flange between which and a corresponding ring there is fixed by screws a membrane of the thinnest sheet rubber, tightly stretched in order that the natural frequency may be high. A mirror formed by silvering a tiny fragment, about 1 mm. square, of glass from a cover slip was cemented by rubber solution at a distance equal to half the radius, from the centre of the membrane. On this a beam of light was directed, so that the reflected spot was focussed on a scale 70 cm. away. Pressure variations inside the capsule caused this spot of light to be drawn out into a band. A linear relation was found between the " deflection " and the pressure amplitude over the range required when the capsule was calibrated, whether statically by applying various small steady pressures (measured by a water manometer), or dynamically, by causing a little piston to oscillate at the appropriate frequency in the cylindrical portion of the capsule. The calibration curve for the capsule is shown in Fig. 6.

Table II compares the displacement amplitude at the mouth, with the pressure amplitude in the interior for the resonator of frequency 126, at different amplitudes of the exciting fork.

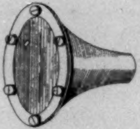

FIG. 5.—MANOMETRIC CAPSULE.

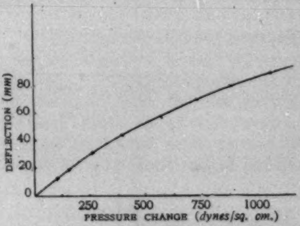

FIG. 6.—CALIBRATION CURVE OF MANOMETRIC CAPSULE.

TABLE II.

Fork amp. cm.	Amp. at mouth. a_0 cm.	Press. amp. P at rear (atmos.).	a_0/P.
0·05	0·035	0·00011	320
0·08	0·055	0·00017	335
0·12	0·075	0·00030	250
0·20	0·125	0·00042	300
0.40	0·20	0·00075	270
			Mean 295

whence the impedance of the orifice

$$Z_0=P/(\pi r^2\omega a_0)=0\cdot31$$

assuming that P, the excess pressure amplitude, is constant throughout the interior. The theoretical impedance of this orifice is $\varrho\omega/\kappa=0\cdot25$, if κ be put equal to the diameter.

Finally, a traverse was made through a Helmholtz double resonator. In this type a resonator has an orifice in the interior leading to another resonator. If the outer neck has conductivity κ_1, and the inner κ_2, the outer and inner reservoirs having capacities v_1 and v_2 respectively, then the lumped impedance of the inner orifice and vessel must be considered as in parallel with the impedance of the outer orifice. To find the total impedance of these branches we add the reciprocals of their individual impedances and invert, thus:

Inner orifice and vessel:

$$Z_2=\varrho\left(\frac{\omega}{\kappa_2}+\frac{c^2}{v_2\omega}\right)$$

This, in parallel with outer orifice:

$$\frac{1}{Z_3}=\frac{1}{\varrho}\left(\frac{\kappa_1}{\omega}+\frac{\omega v_2\kappa_2}{\omega^2 v_2+\kappa_2 c^2}\right)$$

Outer vessel:

$$Z_4=-\frac{c^2\varrho}{v_1\omega}$$

To find the resonant frequency we put $Z_3+Z_4=0$.

$$\frac{\omega^3 v_2+\kappa_2\omega c^2}{\omega^2\kappa_1 v_2+\kappa_1\kappa_2 c^2+\omega^2\kappa_2 v_2}=\frac{c^2}{v_1\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

or

$$\omega^4+\omega^2c^2\left(\frac{\kappa_2}{v_2}+\frac{\kappa_1+\kappa_2}{v_1}\right)+\frac{\kappa_1\kappa_2}{v_1 v_2}c^4=0$$

which is the equation obtained by Rayleigh,* using the " classical " methods.

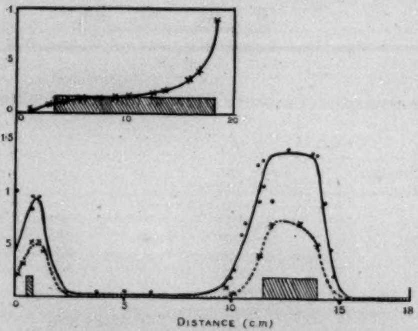

FIG. 7.—AMPLITUDE IN DOUBLE RESONATOR. INSET: DECAY OF AMPLITUDE IN LONG NECK.

The double resonator used in this experiment had an outer orifice of 3·2 cm. diameter $(=\kappa_1)$, and an outer vessel of $v_1=2{,}000$ c.c. The inner neck was 2 cm. diameter and 2·65 cm. long $(\kappa_2=1{\cdot}2$ cm.). The above equation has two roots, of which the smaller corresponds nearly to 143, the lower frequency to which the resonator responded. A traverse with the anemometer is given in Fig. 7. Note the increased amplitude in the inner neck over the outer, in spite of the former's greater impedance; this is in accordance with a formula of Rayleigh—i.e.,

$$X_1/\kappa_1+X_2/\kappa_2=0.$$

When very long necks were used, a notable decay of amplitude through the neck was noticed. This is shown (as inset to Fig. 7) for a neck 15 cm. long.

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* "Sound," §310; see also Phil. Mag., 36, 231, 1918; and Paris, Phil. Mag., 2, 756, 1926. The latter author has worked out the theory of Boys' resonator, in which the outer resonator is replaced by a stopped pipe, on those lines. We could work out this case on impedance principles by substituting $c/S \tan kl$ for $c^2/v_1\omega$ in equation (7).

III. Relation between Pressure in Single Resonator and Impedance of Mouth.

Applying equation (3) to the single resonator, if p_0 is the maximum value of pressure outside the mouth (due to the tuning fork), and p_1 that just inside, we have

$$p_1 - p_0 = Z_0 X \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

At the back of the resonator, where the pressure maximum is p_2 (this may be different from p_1), the current $\dot{X}$ is, of course, *nil*, and the impedance of the barrier is infinity. We have already tested equation (8) for the pure Helmholtz resonator. It was thought of interest to extend this part of the investigation farther by commencing with a cylindrical stopped pipe and applying the manometric membrane to a point close to the mouth, and then gradually restricting the mouth, taking corresponding readings of $p_1 - p_0$ and $\dot{\xi}$ in the mouth. The amplitude of the maintaining fork ($n=220$) was kept constant throughout the investigation, so that p_0 was preserved constant as far as possible, and the length of the cylindrical portion of the cavity (6 cm. diameter) was readjusted to best resonance with the fork each time. To this end, this part was made of two telescoping tubes. The metal plates with holes previously used, served as orifices, and some additional necks formed of 2·3 cm. diameter tubes of various lengths were also tested. Table III shows the results.

Table III.

Orifice radius, cm.	κ	$\frac{\varrho\omega}{\kappa}$	$\dot{\xi}$	$\dot{X}$	$P\left(\frac{\text{dynes}}{\text{sq. cm.}}\right)$	$P/\dot{X}$
0·75	1·5	1·1	120	210	250	1·2
0·825	1·65	0·96	27	57·5	85	1·5
1·0	2·0	0·8	27	85	85	1·0
1·25	2·5	0·64	18	90	85	0·95
1·5	3·0	0·53	22	155	70	0·45
2·3	4·6	0·35	20	83	40	0·48
Neck, 2·3 cm. diam.						
4·1 long	0·73	2·2	6	25	55	2·2
3·3 long	0·85	1·9	8	33	44	1·3
2·25 long	1·08	1·5	9	36	50	1·4

The numbers in the third column are the theoretical values of the impedance based on the usual values of the conductivities. In the last column are the corresponding values, based on the measured velocity and excess pressure amplitude at the mouth.

IV. The Impedance of an Orifice.

In the simple theory of acoustic impedance viscous drag is neglected. The velocity over the cross-section perpendicular to the direction of vibration is supposed to be uniform, so that the inertance of an orifice becomes simply the mass of vibrating air in the neck. One means of calculating the true impedance is given by the figures in the last table, but it is better to attempt to calculate the impedance from an experimental determination of the actual mean velocity configuration across such an orifice, when the air is vibrating under simple harmonic impressed displacement.

Considering a lamina, δx thick, in the plane of a circular orifice, the equation of motion is

$$\varrho A\ddot{\xi}\delta x+2\pi r\tau(1+i)\delta x=A\delta p$$

where A is the cross-sectional area, r the radius of the circle, τ the frictional force on unit area of wall, or simply the " skin friction, and δp the pressure difference over the length δx of the orifice.

Integrating over the length l of the walls of the orifice in the direction of motion (=thickness of plate), and putting $\ddot{\xi}=i\omega\dot{\xi}$

$$2\pi r\tau l+i(\varrho\omega Al\dot{\xi}+2\pi r\tau l)=A\delta p \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

In the type of motion here considered the régime is turbulent, and the idea is now generally accepted that, in such motion, the influence of viscosity may be considered as being confined to a thin layer in contact with the solid surface—the " boundary layer "—if the coefficient of friction μ is small. For simplicity we will consider that the velocity at any instant is uniform $(=\dot{\xi})$ over the major part of the orifice, but drops rapidly and with constant gradient to 0 at the boundary, in a small distance Δ. This is the so-called " kinked " velocity profile discussed by Rayleigh* and by Prandtl.† Put

$$\tau=\mu\frac{\delta\dot{\xi}}{\delta r}=\frac{\mu\dot{\xi}}{\Delta}$$

where Δ is the thickness of boundary layer. Then

$$\frac{2\pi r\mu l}{A\Delta}+i\left(\varrho\omega l+\frac{2\pi r\mu l}{A\Delta}\right)=A\frac{\delta p}{\dot{X}}$$

Divide by A, and we have, by definition, the impedance of the orifice

$$Z_0=\frac{2\pi rl}{A^2}\cdot\frac{\mu}{\Delta}+i\left(\frac{\varrho\omega}{\kappa}+\frac{2\pi rl}{A^2}\cdot\frac{\mu}{\Delta}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

where κ has been put for the conductivity in the second term. The first term here is a pure resistance, but the same term is also involved in the reactance component.

Now, for simple harmonic motion it can be shown that Δ is proportional to $(\nu/n)^{\frac{1}{2}}$.‡ In fact, this quantity has the dimensions of a length, and we may consider the factor of proportionality ε^2 (which involves the nature and smoothness of the surface, among other things) as latent in the coefficient of viscosity, and we may put $\nu'=\varepsilon^2\nu$, so that

$$\Delta\equiv\sqrt{\frac{\nu'}{n}}=\sqrt{\frac{2\pi\nu'}{\omega}}\ :\ \nu=\frac{\mu}{\rho}.$$

The thickness of the boundary layer diminishes as the frequency increases, and the latter factor is, in fact, involved in all the terms of (10). Equation (10) becomes, with this substitution

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* Lond. Math. Soc. Proc., 19, 67 (1887).

† Phys. Zeits., 23, 19 (1922).

‡ *Cf.*, Proc. Roy. Soc., 112, 536 (1926).

$$Z_0 = \frac{\sqrt{2\nu'\omega\pi}}{A^2} l\varrho r + i\varrho\left(\frac{\omega}{\kappa} + \frac{\sqrt{2\nu'\omega\pi}}{A^2} lr\right) \quad . \; . \; . \; . \; . \; . \quad (11)$$[*]

From an estimate of Δ from experimental values of the velocity at different points across the orifice, and the other known dimensions of the orifice, (10) serves to calculate the impedance when viscosity is included. The viscous terms give, in fact, the value of the resistance F, referred to in the first section.

The apparatus for testing the velocity in alternating flow through an orifice consisted of a reciprocating engine with long crank and adjustable stroke oscillating a light piston in one end of a cylindrical brass tube (internal diameter 6 cm.), which it just cleared. A vertical hot-wire anemometer was traversed horizontally across the section near the other end, either across the orifice formed by the mouth of the tube itself, or a hole in a plate soldered to the mouth. As the hot wire was necessarily a straight one, the sides of the orifice were distorted from the circular shape to form two opposite flats of 2 cm. long. A source of error arises from the use of such an anemometer, unfortunately, just over the region where the velocity needs to be known ; this error is due to the cooling of the wire when close to the brass solid, by conduction across the intervening thin layer of air. For this conduction loss there is available a correction which is based on some experiments made in another connection by Dr. N. A. V. Piercy and the writer. The theory and application of this correction is discussed elsewhere,[†] but it suffices to say that in order to determine this correction the hot wire under definite conditions of heating current, temperature, etc., was whirled round at known speeds and at known small distances from a large cylindrical brass surface. From the calibration curve of the anemometer, obtained in the usual way, the apparent velocities corresponding to the measured values of the resistance were found, and these subtracted from the true velocities gave the correction necessary at this particular distance from the surface. In the present experiments a sample of the same nickel wire was used at the same excess temperature, and the correction so obtained was applied to the apparent velocities when the anemometer was close to the brass surface. The resulting velocities are still approximate, but are the best that could be obtained under the difficult circumstances. The frequency was kept constant by stroboscopic means.

Table IV gives the figures, and Figs. 8 and 9 give the gradients, for a number of frequencies. Using the " hole-in-plate " orifice, a resonator was formed to respond to the tuning fork previously used ($n=220$), and a traverse across the orifice was made with the hot-wire anemometer, while the resonator was excited by means of the tuning-fork.

This result is included in Fig. 8, but the rest of the results were obtained by means of the oscillating engine at low frequencies and rather large amplitudes, as the hot wire was more sensitive and the boundary layer thicker under these con-

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[*] In this form the expression for the impedance is practically the same as that deduced by Stewart, Phys. Rev., 27, 489 (1926), save that he puts the unchanged coefficient ν, which is tantamount to putting $\Delta \equiv (\nu/n)^{\frac{1}{2}}$.

[†] In a paper submitted to the Aeronautical Research Council

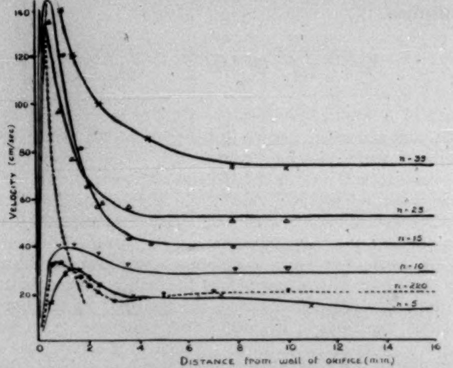

Fig. 8.—Velocity Across Orifice (1·75 cm. Radius).

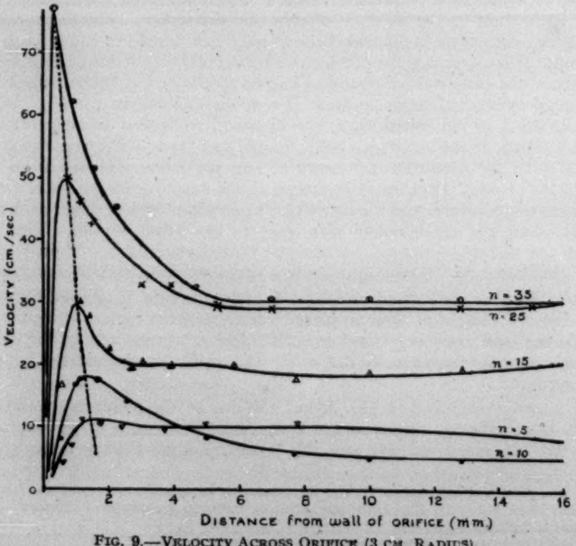

Fig. 9.—Velocity Across Orifice (3 cm. Radius).

ditions. The type of curve obtained seemed to be the same for both sets of experiments, whether the resonator or the engine was used.

TABLE IV.

(Fig. 8) $r=1{\cdot}75$ cm., l(=thickness of plate) = 2 mm.

n	Δ	$V_{\max.}/\Delta$	$\Delta\sqrt{n}$	$F(\times 10^5)$
5	0·14	180	0·31	3·0
10	0·11	350	0·35	3·8
15	0·075	2150	0·29	5·6
25	0·06	2450	0·30	7·5

(Fig. 9) $r=3$ cm.

n	Δ	$V_{\max.}/\Delta$	$\Delta\sqrt{n}$
5	0·13	85	0·29
15	0·10	290	0·38
25	0·075	715	0·35
35	0·05	1600	0·30

The table annexed to the figures shows that $\Delta \propto n^{-\frac{1}{2}}$, as predicted, reckoning Δ as the distance of the peak velocity from the boundary. But there is another curious result : $\dot{\xi}$ does not reach a nearly constant value at the edge of the boundary layer, but there is a drop of velocity before a uniform value is reached. Whereas the boundary layer gets thinner, this region of high velocity broadens out with increasing frequency. In calculating Z, therefore, it seems necessary to take an average value of $\dot{\xi}$ outside the boundary layer, which value will be somewhat greater than the value at the centre. With this modification, and with a value of l, the effective length of the orifice, the value of the frictional resistance

$$F=\frac{2\pi r l}{A^2}\,\frac{\mu}{\Delta}$$

has been calculated (Table IV). It is naturally small compared with the inertance of the air in a wide orifice, but becomes of importance when the cross-section of the orifice is attenuated. Referring back to Fig. 1, the frictional resistance seems to diminish the admittance (= the reciprocal of the impedance) when the conductivity of the orifice is small causing the experimental curves to rise above the theoretical (viscosity neglected) as the origin is approached. In other words, the length assigned to a measured value of r is really appropriate to a lesser r when viscous effects are considered.

The crowding of the motion into the outer rings of the orifice at higher frequencies reminds one of the " skin-effect " in A.C. technology. and is probably an inertia effect. The effective inertia of the air in these parts of the mouth is less in virtue of the fact that, in vibration, it can push the added mass of the outer air "round the corner," along the flange, whereas the central layers are unable to do this, so that their effective inertia is greater. It may be remarked in parenthesis that no such " skin-effect " was observed when " one-way " turbulent flow was produced in the orifice.

It seems likely that, for a given area of orifice, the conductivity may be increased by making the orifice in the form of an annular ring instead of a circle, since the vibrating air tends to crowd into the annulus. Two orifices of the pattern shown in Fig. 10 were constructed, and also two circular tube orifices of equal cross-sectional area, and applied to the cylindrical resonance tubes of Tables I and II, with the following results.

TABLE V.

Resonant lengths for tube, 4·3 cm. diameter.

Cross-section, sq. cms.		$n=$ 500	$n=$ 350	$n=$ 250	$n=$ 220	$n=$ 100
3·1	Circle ...	**11·0**	**18·3**	**28·0**	**33·3**	**42·5**
	Ring ...	**12·6**	**19·5**	**29·5**	**34·5**	**44·0**
3·55	Circle ...	**13·2**	**21·0**	**30·0**	**35·5**	**44·6**
	Ring ...	**14·2**	**21·7**	**30·8**	**36·4**	**46·5**

It appears from this table that the resonant lengths for the annular orifice are somewhat longer than those for the circular orifices of equal size, thus confirming the prediction of a less impedance for the annular orifice. This fact might be made use of in constructing hot-wire microphones, as the sensitivity of response of the hot wire should be greater if placed near the circumference of a hole, or in an annular space like that of Fig. 10.

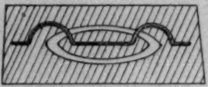

FIG. 10.—ANNULAR ORIFICE FOR RESONATOR.

The peculiar case of alternating airflow in a tube or orifice has evidently other interests apart from the present sound problems, and will be treated in further experiments.

CONCLUSION.

It is hoped that this paper demonstrates the advantage of impedance methods in application to pipes and resonators both in theory and practice. A number of physicists have been adapting the concept of acoustic impedance in connection with the construction of acoustic filters; in particular, Stewart* has tested the efficiency of such filters against theory, and has found certain deviations. The present Paper describes attempts at a *direct* measurement of impedance, and the non-uniform flow across an orifice under certain circumstances which is herein demonstrated may explain the deviations from theory which are observed when such orifices are used in filters. The fact, which several workers on end-corrections have noted†—and which is confirmed by Figs. 1 and 2—that the value of the end-correction of a pipe varies a little with frequency, may in part be due to the same cause, since the extent of the non-uniformity in flow across the section of an orifice when the air is vibrating in it is a function of the frequency of such vibration.

This work was carried out in the Carey Foster Laboratory at University College, London, and I am indebted to Prof. A. W. Porter, D.Sc., F.R.S., for facilities to carry it out, and for encouragement during its course.

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* **Phys. Rev., 27, 818 (1926).**

† *Cf., e.g.,* **Anderson and Ostensen,** *loc. cit.*

DISCUSSION.

Dr. A. H. **Davis** suggested that a more detailed analogy between the " annular effect " (viz., the increase in particle velocity which occurs close to the edges of the orifice of a vibrating resonator) and the skin effect obtained with high frequency electric currents might conceivably be worked out by comparing the vibration of air at small amplitudes (the air being treated as an incompressible fluid) with high-frequency current flow in a mass of copper divided into two parts by an embedded slab of insulation with one circular conducting portion. The author had shown the value of the impedance method as applied to acoustics, but his derivation of equation (7) really depended on Lord Rayleigh's method, since it took the value of the " conductivity " for granted.

Mr. A. G. **Warren** said that the " annular effect " was analogous to an effect which occurs in the moving-coil loud-speaker. He had calculated the pressure distribution over a disc vibrating harmonically in an infinite plane wall, not by the Rayleigh-Maxwell method of finding the common potential, but by integrating first circumferentially and then radially, the disc being supposed complete throughout. This method, though laborious, gives the radial distribution of pressure. The pressure has components in phase respectively with the velocity and with the acceleration, and these components are uniform over the disc at low frequencies. As the frequency increases the pressure at the edge first falls, particularly in the case of the component in phase with the acceleration, and then reverses, and at high frequencies there is a Bessel wave distribution of pressure over the disc. A similar method might be applied to the study of the Author's " annular effect " ; and since in the case of the loud-speaker, where the velocity is uniform over the disc, the pressure diminishes at the edge, in the case of the resonator, where the pressure is more or less uniform, it is to be expected that the velocity will increase towards the edge. This conclusion is qualitatively consistent with the author's results, but the latter give a surprisingly large increase in the velocity near the edge of the orifice.

Dr. J. H. **Vincent** said it would be interesting to know the absolute amplitude of movement in the orifice of the resonator. Would it not be possible to observe the movements of smoke particles in that region ?

Dr. W. H. **Eccles** expressed interest in the fact that whereas alternating current theory had originally been based on Lord Rayleigh's acoustical conception of the complex exponential, the conception of impedance which had been added by electrical theory, was now being transplanted back into acoustics. What were the relative sensibilities of the hot-wire anemometer and the human ear ?

Dr. E. T. **Paris** (communicated) : Dr. Richardson's formula (6) for the frequency of a stopped pipe was given by Rayleigh in 1871, though the fact that it could be used for pipes of lengths small compared with the wave-length of the fundamental tone (that is, cylindrical Helmholtz resonators) was not actually stated. One advantage of a formula of the type (6) over the usual frequency-formula (2) for a Helmholtz resonator is that the former gives the frequencies of overtones, whereas the latter gives only the frequency of the fundamental. It is to be noted, however, that (6) is strictly applicable only to the case of resonators with cylindrical bodies.

A more general formula for resonators with cylindrical parts was published by Mr. J. A. Aldis in 1924 (" Nature," Vol. 114, p. 309), who claimed to have used it as long ago as 1867. It refers to resonators (or organ-pipes) made with two pipes of unequal diameter joined together with a flange from the outer edge of the smaller pipe to the inner edge of the larger stopped pipe (making a " bottle-pipe "). The formula for the resonance frequencies is $\tan kL_1 \tan kL_2 = \sigma_1/\sigma_2$, where L_1 and L_2 are the lengths and σ_1, σ_2 the cross-sectional areas of the smaller and larger pipes respectively, and $k=2\pi/\lambda$. As I pointed out at the time (" Nature," Vol. 114, p. 465), by taking L_1 and L_2 both small compared with λ Rayleigh's formula for the frequency of a Helmholtz resonator can be obtained as a limiting case. Also if L_1, but not L_2, is supposed small compared with λ we get the formula applicable to a stopped pipe with restricted mouth of conductance $c=\sigma_1/L_1$; if L_2, but not L_1, is supposed small, we get the formula given by Rayleigh for a long tube connected with a reservoir of small volume.

Mr. E. J. **Irons** (communicated) : For some time past I have been engaged on the problem of the vibration of air in a partially stopped tube, and I take this opportunity of offering my congratulations to the author for his contribution to the subject.

Whereas Dr. Richardson has made a frontal attack on the orifice with a tuning fork, in my work—chiefly concerned with the determination of end corrections—I have deemed it advisable, for reasons already stated (Phil. Mag. 5, 580, 1928), to adopt the method of approaching the orifice from the rear. Results obtained by a Kundt's tube method have already been described (Phil. Mag., loc. cit.), and a new apparatus, consisting essentially of a hot wire detector of nodes and antinodes, in a tube having at one end an orifice of known diameter, and at the other a valve maintained source of vibration, is at present in use.

I note that Dr. Richardson has replaced the method for determining the resistance of the hot wire which he described in his Royal Society Paper by a form of Kelvin bridge. I suggest that experimental procedure would be simplified by including the balancing and sliding resistances of his first arrangement in the same arm as the hot wire.

AUTHOR'S reply: In considering the analogy between the acoustical and the electrical skin-effect, I have found it difficult to define an acoustical analogue of the inductance to which the electrical skin-effect is due. It is difficult to think of any experimental method for measuring oscillatory pressure; the Pitot tube would show nothing. I have observed the distribution of velocity in the case of direct flow of air (analogous to direct current), but no annular accumulation of flow was indicated by the hot-wire. I am grateful to Dr. Vincent for his suggestion to observe smoke particles in the orifice. Table III shows the amplitude $(=\xi/2\pi n)$ in the orifice of a resonator to be of the order of a millimetre.

The impedance method does not, of course, supersede the classical method; it is another aspect of the same physical phenomena, but might be better suited for teaching purposes, as it avoids the use of the conception of velocity potential. In calculating Tables I, II and III I have assumed Rayleigh's formula for the conductivity of a circular hole (=the diameter), but I do not think I have taken the conductivity for granted in equation (7). In the course of deriving equations (6) and (7) I have merely put k as a symbol for A/l. I thank Dr. Paris for the reference to his letter in Nature, which I had overlooked. Equation (6) is implicit in all the formulæ for end-corrections (Rayleigh, Helmholtz, etc.), but has not, I believe, been correlated with the impedance of the end of the pipe. Mr. Aldis mentions tests on his double pipes, but there were, I think, no actual data on partially stopped pipes, except those of Mr. Irons, published in March of this year, which were obtained with another object. I found the Kelvin bridge arrangement (due to L. V. King) better suited to measurement of velocities than the one I formerly used, in so far as change of zero due to room temperature merely shifts the whole set of readings up or down the bridge-wire by a constant amount.

In reply to Dr. Eccles, I understand that the hot-wire will detect sounds inaudible to the unassisted ear if the hot-wire be combined to a tuned resonator, as in Dr. Tucker's instrument.

XXVIII.—THE FOCUS OF A GAS-FILLED X-RAY TUBE.

By R. E. CLAY, *M.Sc.*, Davy Faraday Research Laboratory, Royal Institution.

(*Communicated by* PROF. SIR WILLIAM BRAGG, *K.B.E.*, *F.R.S.*)

Received March 20, 1928.

ABSTRACT.

The importance of obtaining a sharp focus in an X-ray tube used for crystal analysis is emphasised. Pinhole photographs of the focus obtained with various radii of curvature of the cathode and various distances from the anticathode are discussed and the conclusion drawn that with the tubes of the type considered, a radius of about 2 cms. and a distance of 3 or 4 cms. are the best conditions.

FOR crystal analysis it is very often necessary to confine the X-rays incident on the crystal to a fine pencil; this is usually effected by passing them through a narrow channel or through two pinholes some distance apart as indicated roughly in Fig. 1.

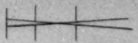

FIG. 1.

These two holes, which correspond to the collimator of an ordinary spectrometer define a cone which intersects the surface of the anticathode in a small area. In order to get the maximum efficiency from the X-ray tube, the focus where the X-rays are produced has to be within this area or preferably to cover it completely yet without being larger.

Any X-rays produced outside this area are wasted. This important fact is sometimes overlooked by X-ray workers with the result that much time is wasted in making long exposures.

In sealed-off X-ray tubes the focus is usually sufficiently small to prevent this loss, but for crystal analysis the ionic type of X-ray tube working in conjunction with a pump is very often used nowadays. In these tubes the size of the focus depends not only on the shape, size and relative position of the cathode and anticathode, but also upon the gas pressure in the tube.

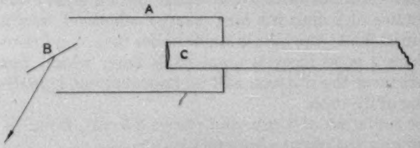

FIG. 2.

Very little systematic work on the modern type of gas-filled tube has been published, and it was therefore thought that a study of the conditions for maximum efficiency with regard to the size of the focus would be of interest.

The work to be described was carried out with one of the "standard" tubes in use in this laboratory which was developed by Shearer and is described by Bragg.* Fig. 2 shows the important parts; these are the cathode *C*—an aluminium rod of diameter 1·4 cms. encased in brass tube—the end of which is turned to a portion of a sphere; the anode sleeve *A* which envelops the end of the cathode (the diameter of this was 4 cms.) and the anticathode *B*. *A* and *B* were connected to earth and *C* to the high potential terminal of the induction coil.

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* "X-Rays and Crystal Structure," pages 34 and 295.

The end of *C* was turned in a lathe to a series of different radii—in the case of the larger radii (5 cms. and upward) by means of a pivoted tool, and in the case of the smaller radii by trial to a template.

The focus spot was photographed by means of a pinhole camera. A piece of lead with a fine hole in it was placed about 15 cms. from the tube and a photographic plate contained in a dark slide of which the front had been cut away and replaced with black paper was placed to receive an image of the focus spot. Any desired magnification can be obtained by altering the distance from pinhole to plate: in this work a magnification of 2 : 1 was used, that is to say, the plate was placed 30 cms. from the pinhole.

In most of the experiments the tube was run on an induction coil with a Wehnelt interrupter; it was connected to the pump throughout the experiments and the pressure was adjusted by a screw pinch-cock which controlled the rate at which air could leak in from the fore vacuum; the pressure in the fore vacuum was maintained constant in a similar way by keeping the backing pump running and allowing air to leak in from the atmosphere—in this way a stream of air is continuously circulated through the whole apparatus.

The pressure was measured with a McLeod gauge having a compression ratio of $10^4 : 1$.

A series of photographs was taken for various radii of curvature and various distances with a cathode of the standard diameter (1·4 cms.). Fig. 3 shows the results obtained.

The horizontal scale indicates the distance in cms. from the anticathode, while the vertical scale indicates the radius of curvature in cms. of the cathode surface. The actual focus in the X-ray tube was of $\frac{2}{3}$ the linear dimensions of the spots in the figure.

The smallest focus is obtained with the 1 cm. radius at a distance of 1·5 cms. but the tube was very unstable and the pressure adjustment very critical. The same is to be said of the next radius and the small distance at which again a good focus is obtained. In the case of a radius of 2 cms. the best focus is obtained over a range from 3 to 4 cms. from the anticathode, the tube is stable under these conditions and the pressure range over which a good focus is obtained is fairly wide. For larger radii the focus is larger whatever the distance and no improvement is made in the stability and general running of the tube.

A cathode having a radius of curvature of 2 cms. and placed 3·5 cms. from the anticathode appears, therefore, to give the most satisfactory results.

The gas pressure depends upon the distance; the figures in the right-hand column give in mms. of mercury the pressure for the distance at which the focus was a minimum. The current through the tube was of the order 1-2 milliamperes throughout and the alternative spark gap was usually about 12 mms. between spheres of 25 mms. diameter.

The effect of pressure variation is shown in Fig. 4 for two cathodes, one the standard, 1·4 cms. in diameter, and the other 2·4 cms. in diameter; the radius of curvature was 2 cms. in each case. A group of three photos was taken for each of the distances 3·0, 3·5, 4·0 cms., and in each group the pressure was adjusted to

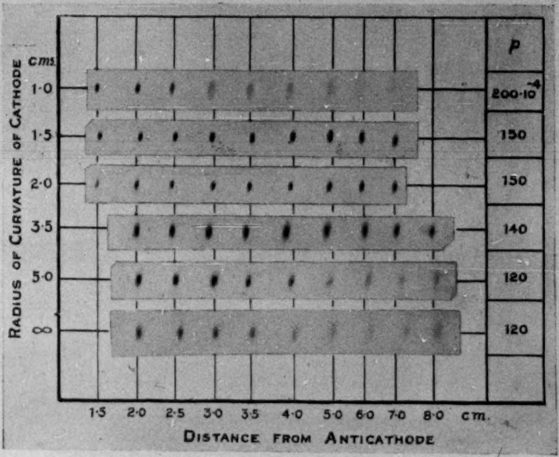
cms.
Radius of Curvature of Cathode
1·0
1·5
2·0
3·5
5·0
∞
P
200·10⁻⁴
150
150
140
120
120
1·5
2·0
2·5
3·0
3·5
4·0
5·0
6·0
7·0
8·0
cm.
Distance from Anticathode

Fig. 3.

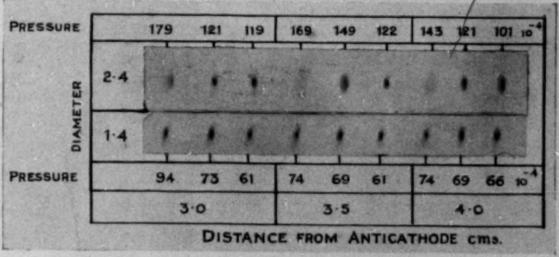
Pressure
179 121 119 169 149 122 143 121 101 10⁻⁴
Diameter
2·4
1·4
Pressure
94 73 61 74 69 61 74 69 66 10⁻⁴
3·0
3·5
4·0
Distance from Anticathode cms.

Fig. 4.

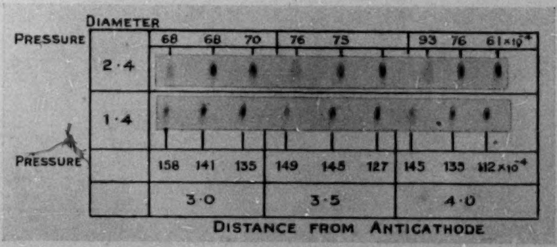
Diameter
Pressure
68 68 70 76 75 93 76 61×10⁻⁴
2·4
1·4
Pressure
158 141 135 149 145 127 145 135 112×10⁻⁴
3·0
3·5
4·0
Distance from Anticathode

Fig. 5.

To face page 222]

three values—the maximum at which X-rays could be obtained, that at which the strongest beam was obtained and the minimum at which X-rays could be obtained.

The effects are most marked in the case of the larger cathode and it is probable that some radius other than 2 cms. would prove to be more suitable for a cathode of this diameter.

The behaviour of the tube is very much the same when it is run on a high-tension transformer from A.C. mains and the same conditions for the best focus hold; the variation of focus spot with gas pressure is a little different owing, no doubt, to the voltage being a different function of the current. Fig. 5 is identical with Fig. 4 except that a transformer was used in place of the induction coil.

DISCUSSION.

Major C. E. S. **Phillips** referred to some experiments by Campbell Swinton on an early form of gas X-ray tube (with a carbon target) used in electro-therapy. In these experiments, and in the speaker's experience, the cathode stream always appeared to be hollow, so that the focus was surrounded by concentric rings. He was surprised that changes in gas-pressure should have so little effect. In his experience the length of the parallel part of the cathode stream varied considerably with pressure, and this circumstance would affect the size of the focus. Campbell Swinton's tubes were not highly evacuated, however. What was the pressure in the tubes to which the paper referred?

Mr. A. G. **Tarrant** suggested that the shape of the beam under various conditions could be reconstructed from the cross-sections obtained by the author.

Dr. L. **Simons** said that as the author had arranged the photographic plate at right angles to the X-rays, whereas the target was oblique to the rays, the photographic spot differed in shape from the focus on the target. If the plate and target were equally inclined to the X-rays the true shape of the focus would be recorded.

Mr. A. G. **Warren**: I would suggest that the Author might profitably include in the Paper an explanation of focussing. In recent years a number of curious ideas have been current, such as that the focus at normal voltage is appreciably affected by mutual repulsion of the constituents of the cathode stream. I suggest that the focus can be sufficiently explained by the direction and magnitude of the electron speed acquired in the neighbourhood of the cathode, and that subsequent deviations are small. The presence of gas is chiefly of importance in influencing the potential gradient at the surface of the cathode.

Dr. A. **Muller** said that the object of the Paper was purely practical. It had been found that the effective exposure with the Shearer tube is very uncertain, but in the light of the present investigation into the best working conditions excellent results had been obtained.

Dr. W. H. **Eccles** asked what was the composition of the residual gas in the Shearer tube.

The **Author**, in reply, pointed out several of his photographs which indicated a hollow structure in the cathode stream, and said that the focus, for various positions of the anticathode, could scarcely be regarded as a section of the same invariable cathode stream, since the field, and therefore the form of the cathode stream, would be influenced by the position of the anticathode. The object of the paper was to study the size of the focus as viewed from the crystal, but the distribution of intensity in the focus itself could be inferred by geometrical reasoning if required. The residual gas in the tube was atmospheric, though Dr. Müller had used tubes containing carbon dioxide.

XXIX.—A NOTE ON THE ELASTIC IMPACT OF THE PIANOFORTE HAMMER.

By R. N. Ghosh.

Abstract.

The present paper comprises a critical discussion of a previous paper by Mr. P. Das.

IN a recent paper* on the elastic impact of the pianoforte hammer, Mr. P. Das has brought out certain new features on theoretical grounds which deserve further consideration in the light of experimental work. Starting with Kaufmaun's case when the impact takes place near one end of the string, he obtains a formula for the pressure

$$P=\frac{VE}{c\,q}\left[\frac{1}{q}\left(e^{\xi ct}\cos\,\eta ct-e^{qct}\right)-\frac{1}{a\eta}\,e^{\xi ct}\sin\eta ct\right]\quad .\ .\ .\ .\ .\ (1)$$

V being the initial velocity of hammer.

Whatever be the value of the elasticity of the hammer, (1) shows that $P=0$ at $t=0$. On this basis Mr. Das questions the validity of the author's formula† for the pressure P

$$P=\frac{VT_0}{q}e^{-\frac{1}{2}kt}.\left[\frac{\sin qt}{a}+\frac{R}{c}\cos\,(qt+\varphi)\right].\quad .\ .\ .\ .\ .\ .\ .\ .\ (2)$$

which gives a finite value for the pressure at the beginning of impact. Formula (2) has been derived on the secure foundation of experimental results—viz., that the velocity of the striking point of the string differs little from that of the initial velocity of the hammer. This is shown by the numerous photographic records of the simultaneous motion of the hammer and the string taken both by the present author and Dr. W. G. George. The pressure is actually given by

$$P=T_0\left[\frac{T_0}{a}+\frac{1}{c}\ \frac{dy_0}{dt}\right]$$

and hence, if $T_0=0$ and $dy_0/dt=V$, we get $p=V_0T_0/c$ at the beginning of impact. Mr. Das supposes $dy_0/dt=0$ initially. Herein lies the error (*see* Fig. 1).

It is not difficult to show that this pressure initially arises from the loss of momentum which is communicated to the string. Such a type of velocity imparted to a finite portion of the string will approximately produce a motion similar to that

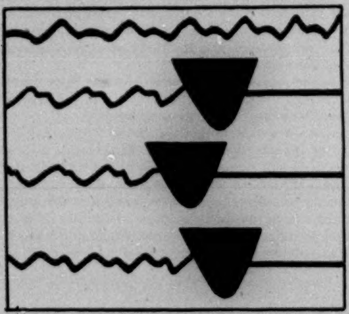

Fig. 1.

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* Proc. Phys. Soc., Vol. 40, p.29 (1927).

† Proc. Phys. Soc., Vol. 40, p. 31 (1927). q in (1) and (2) has different meanings, original symbols being preserved.

of the bowed string. This is found to be the fact in many cases.* Equation (6) of Mr. Das seems to be in error—the factor m_0 occurs with y'' and not with y'''. The term in y''' is small and of order 10^{-3} in comparison with others in the case of felt hammers contemplated by the author. As pointed out above, this term does not affect the initial value of the pressure.

Experimental observations on the pressure of impact have been made by Prof. Raman and Dr. W. George in the case of hard hammers. Their results definitely show that the pressure is finite at the beginning of impact: (1) shows, however, that, even if elasticity tends to large values, still pressure must always be zero initially; while (2) agrees with the experimental facts as the hammers become hard. Thus it is clear that (2) has wider practical application than (1). Probably in the case of light and very elastic hammers, like old tennis balls impinging upon the string, (1) may hold; but it remains an open question for experiment to decide.

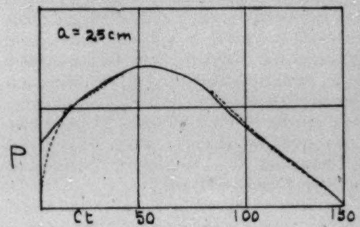

FIG. 2.

Fig. 2 shows the pressure variation during impact as calculated from author's formula (2), the dotted curve being that due to (1). The difference in the initial stages is, as pointed before, due to the assumption $\dot{y}=0$. Since $\dot{\zeta}=\dot{y}_0+\dot{u}$ where $\dot{\zeta}$=velocity of hammer, u=compression, it is evident that as the hammer increases in hardness $\dot{y}_0$ will certainly approach $\dot{\zeta}$. And in practice it has been found that in the case of felt hammers, as mentioned before, there is very little difference between $\dot{y}_0$ and $\dot{\zeta}_0$ initially; therefore (2) applies directly to the case of pianoforte hammers.

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* Proc. Roy. Soc., Vol. 108, p. 294, plate 9. The full equation of motion of the string is given in Phys. Rev., Vol. 24, p. 457 (1924).

DEMONSTRATION OF "DIE RASTERMETHODE" (DETECTION OF SPHERICAL ABERRATION BY MEANS OF SHADOWS IN ASTIGMATIC BEAMS).

By J. E. CALTHROP, *M.A.*, *M.Sc.*

THIS method, which has been applied recently by Jentzsch (Physik. Zeits., No. 750, pp. 66-72, January, 1928) and Schulz (Ann. d. Physik., p. 66, XIX, 1928) to the measurement of the spherical aberration of lenses, consists in forming a real image of a small source, such as pointolite or carbon-arc lamp, and inserting a grating in the neighbourhood of the image. The effect is viewed on a distant screen, and owing to the fact that the rays come from a caustic curve they produce a shadow which is a distorted image of the grating. From measurements of the distortion it is possible to find the spherical aberration of the lens.

The effects for a single line object were known to Prof. Silvanus Thompson in 1903, and a theoretical treatment was also given by Bennett (Proc. Phys. Soc., p. 205, XIX, 1904) in a Paper entitled "Notes on Non-homocentric Pencils and Shadows Produced in Them. I: The Standard Astigmatic Pencil."

On the present occasion the curious shadows produced by lines, gratings and gauzes were demonstrated, and the differences between a corrected and uncorrected lens were illustrated.

DEMONSTRATION OF THE BLOWING OF SELENIUM BUBBLES.

By Major C. E. S. Phillips, *F.R.S.E.*

THE object in view is to obtain thin films of selenium for observation under a microscope. The selenium is heated in a beaker, and a blow-pipe having a slightly flared end is warmed to the same temperature by immersion in it. The blow-pipe is connected through a length of rubber tubing to the mouth of the operator, who withdraws the blow-pipe from the melt, and, testing the consistency of the adhering selenium by alternate air pressure and suction, selects the right moment for blowing the bubble. The method is uncertain, but with practice good results can be obtained, and two large sausage-shaped bubbles were blown in the course of the demonstration. Pieces of the selenium film thus produced have the property of seizing or cold-welding when brought into quite light contact with one another.

DEMONSTRATION OF A PORTABLE ELECTRIC HARMONIC ANALYSER, SHOWING THE MEASUREMENT OF HARMONICS IN VOLTAGE AND CURRENT WAVES.

By R. THORNTON COE, *B.A.*, *M.Sc.Tech.*, *A.M.I.E.E.*

THE instrument is used by the British Thomson-Houston Co., Ltd., for measuring small harmonics in voltage or current waves. The determination of each harmonic is made separately from steady readings on two instruments, and only takes a few minutes. The amplitudes of small harmonics in voltage waves are found correct to at least 1/20th of 1 per cent. of the fundamental.

The method embodies the mathematical principle of obtaining the *n*th harmonic in a wave by multiplying by sin *npt* and integrating over a complete period.

The current to be analysed is passed through the moving coil of a dynamometer, while through the fixed coil is passed a sinusoidal " analysing current " of exactly the frequency of the harmonic under investigation; this is obtained by the use of a tuned circuit, valves and a synchronously driven contact disc. The phase of the analysing current is changed to give a maximum dynamometer reading $D_{\text{max.}}$, where

$$D_{\text{max.}} = kI_aI_n.$$

The analysing current I_a is measured on a thermal ammeter, and since k, the dynamometer D.C. calibration constant, is known, the required harmonic of current I_n can readily be calculated. A large current is analysed by connecting a suitable shunt across the dynamometer moving coil, while a voltage is analysed by connecting a fixed condenser in series. The apparatus was shown in operation upon the voltage wave-form of current taken by a small auto-transformer supplying the synchronous motor of the analyser. The most prominent harmonics in the voltage wave were the 3rd, 5th and 7th, and also the 17th and 19th. The former (of amounts 2·1 per cent., 3·9 per cent. and 1·25 per cent. respectively on a fundamental of 112 volts) were due to the somewhat rectangular wave-form of flux distribution round the armature, while the latter (of amounts 3·0 per cent. and 1·5 per cent. respectively) were due to the ripple of flux caused by the slots in which the armature conductors are placed.

The transformer current wave showed large 3rd and 5th harmonics of amount 13·1 per cent. and 4·6 per cent. respectively on a fundamental of 5·2 amps. These large harmonics were due to the well-known distortion of the magnetising current wave of a transformer when the maximum flux-density is sufficient to saturate the iron core.

XXX.—SOME NEW METHODS OF LINKING MECHANICAL AND ELECTRICAL VIBRATIONS.

By W. H. Eccles, *F.R.S.*, and W. A. Leyshon, *Ph.D.*

Received May 11, 1928.

ABSTRACT.

The paper describes new arrangements for linking the vibrations of a tuning fork or quartz plate to the oscillations in an electric circuit. For this purpose use is made of various crystal contacts, of a neon tube, or of a pointolite lamp.

In a previous paper(1) an account has been given of a method of sustaining a tuning fork or quartz plate in oscillation by means of a neon tube flashing circuit, and in that paper it was shown that the mechanically vibrating body reacted upon the flashing circuit to such an extent as to keep the frequency of flashing constant over a wide range of the circuit conditions.

This method of maintaining tuning forks, quartz plates and the like in continuous oscillation is quite distinct in many respects from the well-known method employing a triode (or tetrode)(2); and the steadying influence of the mechanical vibrator upon the concomitant electrical oscillations, which have made the thermionic method of importance in the laboratory and in engineering, is even more marked in the neon tube method than in the older method employing a triode (or tetrode). That this is to be expected is shown by B. Van der Pol in his papers(3) on "Relaxation Oscillations." The matter has also been discussed by one of the present authors in a recent paper.(4)

The present paper is concerned with a number of new methods of producing linked mechanical and electrical oscillations, some of which owe their properties to electrical conditions due to "relaxation" phenomena, but are often broadly described as due to "negative resistance."

CRYSTAL CONTACT METHOD.

In 1910 one of the present authors demonstrated(5) to the Physical Society of London a crystal contact generating oscillations in an inductance coil and condenser connected across the contact. Almost any of the crystal combinations used as detectors in wireless receiving circuits could be used for generating oscillations over a wide range of frequencies, those of acoustic frequency being sufficiently energetic to be easily audible in a telephone. The frequency of the oscillations was found to depend principally upon the reciprocal of the square root of the product of the inductance and the capacity as was expected, but it varied somewhat erratically with the voltage applied and the condition of the contact. In 1924 these contact-generated oscillations were employed by Lossev(6) for receiving signals by the heterodyne method.

In 1926 and 1927 we attempted to apply contact-generated oscillations to the task of sustaining the natural vibrations of tuning forks or quartz plates. Fig. 1 shows one of the circuits arrived at after much experiment. Forks of frequency 512 and 2,000 have been successfully used in this circuit. In each case the capacity of the condenser *C* was chosen so as to co-operate with the resultant inductance of its circuit to make the electrical oscillations generated by the contact of about the same frequency as the natural frequency of the free fork. It was found that the fork could be maintained in steady oscillation for several hours, and that the electrical oscillations, as perceived by aid of the telephones *T*, remained equal in frequency to that of the fork even when circuit alterations were made that would ordinarily take the circuit far out of tune. When the circuit was altered just too far, however, beats became audible in the telephones and the fork gradually came to rest. The steadiness of the arrangement may be illustrated by the fact that the whole experimental board may be carried from one room to another without stopping the vibrations.

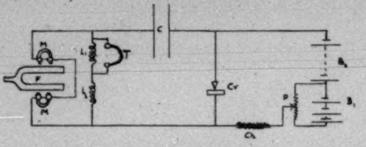


FIG. 1.

Cr. zincite crystal with steel cat-whisker—or needle-point.
C condenser.
L_1, L_2 inductances.
T telephone.
M, *M* telephone magnets.
F tuning-fork. *P* potential divider.
B_1, B_2 batteries. *Ch* choke coil.

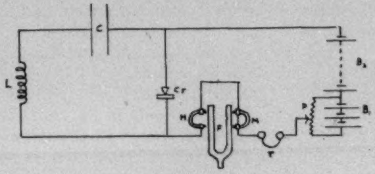


FIG. 2.

Another circuit which works well is shown in Fig. 2; in this the electromagnets of the fork and the telephones replace the choke coils of Fig. 1. When the capacity and inductance in the circuit of Fig. 1 are made sufficiently small, electrical oscillations of radio frequency are generated by the contact. The circuit can then be applied to maintaining the vibrations of a quartz plate, by which the tuning fork and its electromagnets are replaced. The circuit shown in Fig. 3 was used at first. The condenser C_1 was taken small enough to give the desired high frequency oscillations and the condenser C_2 was large enough to give an audible note in the telephones. Keeping

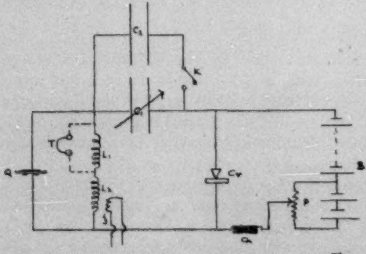


FIG. 3.

Cr. zincite crystal and steel needle-point.
Q quartz "oscillator" crystal.
C_1, C_2 condensers.
K switch-key.
L_1, L_2 inductances.
L_3 coil coupling circuit to heterodyne wave-meter.
T telephones.
B battery.
P potential divider.
Ch. choke coil.

C_2 in circuit, a good working contact was found by trial while listening in the telephones; then C_2 was switched out of circuit. The high-frequency oscillations now being generated were detected and measured by aid of the heterodyne principle, an auxiliary oscillating wave-meter with its own telephones being employed. After gaining experience, it was found possible to dispense with the preliminary audio-frequency oscillations. In order to maintain a quartz plate in vibration it was placed at Q (Fig. 3) between electrodes connected to the circuit as shown. By suitable adjustment of the condenser C_1 the plate could be set in vibration. Once it started it reacted upon the circuit so as to force its natural frequency upon the whole circuit, and the heterodyne note heard in the wave-meter changed exceedingly little even when the capacity of the condenser was varied widely within limits. On passing either of these limits an abrupt change of frequency took place which was reproduced in the heterodyne note, and thereafter the latter followed continuously the variations in capacity. The quartz plate in use had a natural frequency corresponding to a wave-length of 2,320 metres; it could be satisfactorily sustained in vibration when the contact-generated oscillation was of wave-length 4,640 metres—an example of harmonic synchronisation.

In another example, with a different zincite crystal (now mounted in a crystal holder, with vertical steel needle substituted for the usual spiral cat-whisker) strong oscillations were obtained with B about 66 volts. In one region of this particular crystal, oscillations of wave-length 1,000 metres (300 kilocycles) could be obtained. A high resistance in series with the battery was found advantageous; the resistance actually used was a continuously variable "anode resistance" of the type used in resistance-capacity coupled amplifiers (0 to 100,000 ohms).

The quartz crystal (wave-length 2,320 m.) was placed either in the position shown in Fig. 3 or in parallel with the condenser C_1. In one experiment, with the crystal in the latter position, C_1 was altered until the quartz responded; it was then found that on continually increasing C_1 the heterodyne note heard in the wave-meter changed very little in pitch and then suddenly disappeared altogether. At this point, on leaving the crystal circuit unaltered and readjusting the setting of the wave-meter the heterodyne notes reappeared, a "silence point" being reached at about 2,500 metres. This gives some indication of the great range of control of frequency by the quartz.

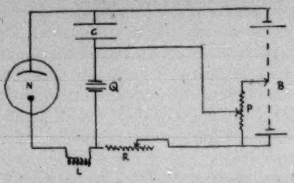
NEON TUBE METHOD.

The method of maintaining linked mechanical and electrical oscillations by aid of a neon tube flashing circuit has already been described in the case when the vibrating body was a tuning fork. Here we may describe the application of the same method to maintaining the high-frequency vibrations of a quartz plate. A suitable circuit is shown in Fig. 4. As it is difficult to cause a neon tube to flash faster than, say, 30,000 times per second, the electrical circuits were chosen so that the rate of flashing was one-sixth the natural frequency of the quartz plate. In this case, as might be expected, the permissible range of variation of the circuit conditions is smaller than when the natural frequency of the mechanical vibrator is the same as that of the flashing phenomenon. As in the case of the tuning-fork controlled neon tube circuit, the control was most effective when the battery voltage was as low as possible and the series resistance of the critical value for flashing. It was noticeable that adding inductance to the circuit allowed the resistance to be reduced below its normal critical value.

OTHER CIRCUITS.

(*a*) It has been found possible to obtain oscillations of audible frequency using the circuit shown in Fig. 5.

A battery voltage of 66 volts was used. A high note was heard in the telephones, increasing in pitch when C or R was diminished.* There was some indication that this note was controllable in frequency by an associated quartz crystal.



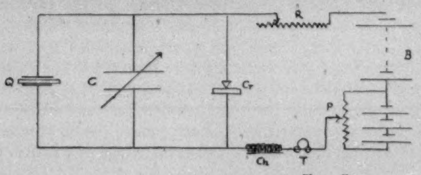
N neon lamp.
B battery.
P potential divider.
R high resistance.
Q quartz " oscillator " crystal.
L coupling coil.
C blocking condenser.

FIG. 4.

(*b*) The circuit of Fig. 1 is with the omission of the tuning fork and its coils, the well-known circuit for sustaining electrical oscillations by means of an electric arc—the latter having been replaced by the zincite steel combination.

Using a " pointolite " lamp in place of the crystal combination, and with a suitable battery voltage and electric oscillatory circuit, it was found possible to maintain a tuning fork in oscillation.

(*c*) Using a tuning-fork controlled flashing neon lamp circuit, it has been found possible to coerce another circuit flashing at a frequency as much as seven times greater than the first by introducing into the circuit a voltage derived (through a transformer) from the first.



C variable condenser
Cr. zincite-steel contact.
Q quartz " oscillator " crystal
R high resistance (→ $100,000_{\omega}$.).
B battery.
P potential divider.
Ch. choke coil.
T telephones.

FIG. 5

REFERENCES.

(1) W. H. Eccles and W. A. Leyshon, The Electrician, p. 65 (July 16, 1926).

(2) W. H. Eccles and F. W. Jordan, The Electrician, p. 704 (June 20, 1919).

(3) B. van der Pol, Jahrbuch der drahtlösen Teleg. and Teleph., p. 178, Band 28 (1926).

(4) W. A. Leyshon, Phil. Mag., p. 305, Vol. 4 (1927).

(5) W. H. Eccles, The Electrician, p. 384 (Dec. 16, 1910). *See* also H. J. Round, Wireless World, p. 217 (Aug. 29, 1925).

(6) Lossev, Wireless World (Oct. 22, 1924).

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* *See* also F. Seidl.

DISCUSSION.

Mr. T. G. **Hodgkinson** expressed great interest in the circuit arrangements which had been described. The mastery of the tuning fork over the natural frequency of the associated electric circuits must be attributed to the fact that the energy stored in the fork was very much greater than that stored electrically.

Dr. E. H. **Rayner** suggested that if a copper oxide rectifier were used instead of a crystal it might be possible to maintain electric oscillations of considerable power.

Dr. W. H. **Eccles** said that the most interesting feature of the circuits was one which it was difficult to demonstrate effectively to an audience—namely, the ability of the fork to impose its frequency on circuits of widely different natural frequency. The voltage-time curve in a neon lamp, arranged in association with a condenser to give intermittent illumination, tapers off in the neighbourhood of the cut-off, and the voltage decrements due to the fork are consequently able to introduce considerable changes into the time of discharge of the condenser.

XXXI.—CONTACT POTENTIAL IN THE DOLEZALEK ELECTROMETER CONNECTED IDIOSTATICALLY.

By L. F. Richardson, *F.R.S.*, and Students of Westminster Training College.

Received May 11, 1928.

Abstract.

It was found that the deflection x was related to the voltage V by the formula

$$x=k\left\{\tfrac{1}{2}V^2+\eta V\right\},$$

k being constant from month to month, but η varying from 0·3 to 1·2 volts on different days. If it be desired to measure V it is therefore essential to reverse the polarity in order to eliminate η

§ 1. The Main Asymmetry.

It seems not to be generally mentioned in standard text-books that the original formula* for Kelvin's quadrant electrometer—namely,

$$\theta=k_1(A-B)\left(C-\frac{A+B}{2}\right)$$

is a bad misfit to the behaviour of a sensitive Dolezalek instrument when used idiostatically.† As θ is unchanged if A, B, C are each altered by the same addition, we may write this classical formula in its simplest form if we state that

The potential applied to the vane is the conventional zero . . . . (1)

For then
$$\theta=\tfrac{1}{2}k_1(A^2-B^2) \qquad (2)$$

in which θ is the angle turned by the mirror, A and B are the potentials of the quadrants, and k_1 is an instrumental constant. For the idiostatic connection, either $A=0$, $B=V$, or else $B=0$, $A=V$, and in either event (2) becomes

$$\theta=\tfrac{1}{2}k_1V^2 \qquad (3)$$

The following set of observations shows that the classical formula (3) is quite wrong in so far as it does not describe the important shift on reversing the sign of V.

Electrometer by Pye, No. 8646; quadrants lacquered black inside. Vane by Cambridge Instrument Co., looking like aluminium. Idiostatic connection.

Object connected.	Reversing switch.	Deflection from short-circuited reading.	Mean deflection. M	Shift on reversal. $\triangle$
		Cm.	Cm.	Cm.
Battery P ...	↑	9·22	6·46	5·52
...	↓	3·70		
Battery Q ...	↓	2·10	4·42	4·64
...	↑	6·74		

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* Clerk-Maxwell, Electricity and Magnetism. Third edn., p. 339.

† There is a hint of it in Pidduck's Treatise on Electricity. Second edn., p. 127.

Values of k_1, calculated from other observations by formula (3), were found to vary widely, even changing sign at a certain voltage.

An obvious way of modifying (2) is to suppose that there is a contact potential η, so that we must write

$$\theta=\tfrac{1}{2}k_1\{(B+\eta)^2-(A+\eta)^2\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

The scale deflection x is so nearly proportional to θ that by a change of constant from k_1 to k

$$x=\tfrac{1}{2}k\{(B+\eta)^2-(A+\eta)^2\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (4\text{A})$$

And as for the idiostatic connection, we have $A=0$, $B=V$; therefore

$$x=k\{\tfrac{1}{2}V^2+V\eta\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

On reversing, V becomes $-V$ and x becomes x', say, so that

$$x'=k\{\tfrac{1}{2}V^2-V\eta\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (6)$$

Hence,

the mean deflection, say $M, =\tfrac{1}{2}(x+x')=\tfrac{1}{2}kV^2$ . . . . (7)

and the shift on reversal, say $\Delta, =x+x'=2k\eta V$ . . . . (8)

From (7) $$\frac{\text{E.M.F. of battery } P}{\text{E.M.F. of battery } Q}=\left(\frac{M_P}{M_Q}\right)^{\frac{1}{2}}=\left(\frac{6{\cdot}46}{4{\cdot}42}\right)^{\frac{1}{2}}=1{\cdot}20_9.$$

Measured by an electromagnetic voltmeter (Paul Unipivot) the E.M.F.'s were: P, 4·06 volts; Q, 3·36 volts; ratio, $1{\cdot}20_8$. Thus by reversing V and using means we obtain good agreement. To find η we eliminate k between (7) and (8), obtaining

$$\eta=\frac{V\Delta}{4M} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (9)$$

And combining this formula with the observations it is found that $\eta=0{\cdot}87$ volts from P, $\eta=0{\cdot}88$ volts from Q, the results agreeing as accurately as could be expected from voltmeter observations.

A better test of equation (7) is obtained by plotting M against V^2 in Fig. 1. Similarly, equation (8) is tested by plotting Δ against V in Fig. 2. The observed points lie close to straight lines through the origin, thus confirming the equations. The voltages for these diagrams were measured by means of a Rayleigh potentiometer and Weston cell, both of which had been standardised by the N.P.L. The electrometer was not that previously mentioned, but one by Griffin, having brass quadrants unlacquered inside; and the vane, by the Cambridge Instrument Co., looked like aluminium.

The most convenient way to find the contact potential η from these diagrams is to take the slopes of the lines. The slope of Fig. 1 gives $M/V^2=1{\cdot}68_8$ cm. volt^{-2}. The slope of Fig. 2 gives $\Delta/V=1{\cdot}98$ cm. volt^{-1}. Then, from (7) and (8)

$$\eta=\tfrac{1}{4}\,\frac{\Delta}{V}\cdot\frac{V^2}{M}=\tfrac{1}{4}\,\frac{1{\cdot}98}{1{\cdot}68_8}=0{\cdot}29_3 \text{ volt}$$

Observations of the type shown in Figs. 1 and 2 have been obtained on six occasions by students, and the graphs have always been found to be straight lines within the uncertainty of observation. On one occasion the (V^2, M) graph missed the origin, otherwise they all went through it.

§ 2. OTHER ASYMMETRIES AND ABERRATIONS.

The importance of η is in the idiostatic connection. In the heterostatic connection η is usually small relative to A or B, and different asymmetries are conspicuous. Let us see whether any of these have complications with η. In heterostatic experiments, A/B is nearly unity, so that AB can scarcely be distinguished from $\frac{1}{2}(A^2+B^2)$ when they occur as factors of small correcting terms. The formula of A. H. and K. T. Compton* is not expressed in terms of *differences* of potential. Differences alone are actually effective. There is some doubt as to how it should be translated into the present difference-notation, but the following is a fairly inclusive form

$$\theta=\tfrac{1}{2}k_1(B^2-A^2)-AB(\lambda+\mu\theta+\nu\theta^3)-\tfrac{1}{2}(A^2+B^2)(\lambda'+\mu'\theta+\nu'\theta^3) \quad . \quad . \quad . \quad (10)$$

where λ, μ, ν, λ', μ', ν' are instrumental constants.

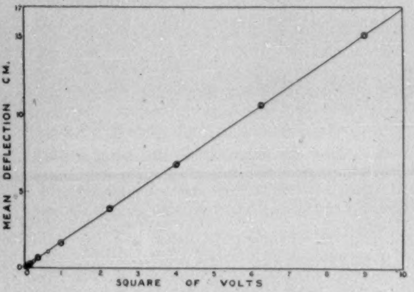

FIG. 1.†

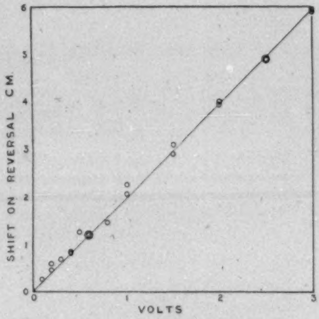

FIG. 2.†

Now to be general, let A and B have separate contact potentials α and β. Then (10) should be changed to

$$\theta=\tfrac{1}{2}k_1\{(B+\beta)^2-(A+\alpha)^2\}-(A+\alpha)(B+\beta)\{\lambda+\mu\theta+\nu\theta^3\}$$
$$-\tfrac{1}{2}\{(A+\alpha)^2+(B+\beta)^2\}\{\lambda'+\mu'\theta+\nu'\theta^3\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Experiment, however, shows that α is nearly equal to β, and that λ, μ, ν, λ', μ', ν' are not large enough seriously to disturb the measurement of η in the way already described. For the Pye electrometer aforesaid, this was seen, firstly, by joining the quadrants and applying various voltages, $A=B=V$. The deflection was nearly proportional to V^2, so that μ and ν were unimportant when $V<90$ volts. Also at 4 volts the deflection would be only 0·03 cm. when the quadrants were all joined. Secondly, to find whether it made much difference which pair of quadrants were at the same potential as the vane, the following observations were obtained on the Pye quadrants and Cambridge vane as before. The voltage was 4·072 throughout, being applied by four Weston cells in series.

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* A. H. and K. T. Compton, Phys. Rev., XIV, p. 85 (1919).

† The centre of a circle, regardless of its radius, marks an observation.

Date: 1928, May 5, 11h. to 13h.

Which quadrant terminal to vane.	Temperature near case.	M Cm.	Δ Cm.
Back	18·1	6·28	5·50
Front	18·5	6·44	5·51
Front	18·7	$6{\cdot}45_5$	5·51
Back	18·8	$6{\cdot}28_5$	5·53

It is seen that there is a slight asymmetry of M, but that it is only 1·3 per cent. of Δ.

Changing the potential of the case by 90 volts did not alter the deflection by 1 part in 1000.

Similar remarks apply to the Griffin electrometer.

The geometrical relation of x to θ is perhaps most conveniently included with internal aberrations due to slight imperfections in the form of quadrants and vane.

§ 3. CHANGE OF CONTACT E.M.F. FROM DAY TO DAY.

The following Table shows that, whereas the sensitivity measured by M/V^2 remained steady to a few per cent. when the suspension was the same, yet η sometimes changed in a ratio as large as two in a few days. Change of humidity is suspected to be a cause of this.

Which quadrant terminal to vane.	Date. 1928.	MV^{-2} cm. volt^{-2}	η volt.	Room temp. °C.	Percentage humidity in room.	Observers.
Griffin quadrants, bare brass inside. Cambridge Inst. Co. Vane.						
Front ...	Jan. 23 ...	$1{\cdot}68_8$	$0{\cdot}29_8$	7·7 to 10·6	Room colder for previous days	L. F. Richardson Figs. 1 & 2
	Jan. 25 ...	1·67	0·67	...	...	E. R. Collins and J. Dowson
	Jan. 26 ...	1·70	0·49	11·0	60	H. M. Gill and B. Ramsell
Front ...	May 3 ...	1·70	0·58	18·0	...	J. A. Cowell and A. H. Harper
Front Back	May 8	$1{\cdot}64_8$ $1{\cdot}65_7$	$1{\cdot}18_8$ $1{\cdot}19_5$	15·0	End of spell of hot, dry weather	L.F. Richardson
(No. 1646) *Pye quadrants lacquered black inside. Camb. Inst. Co. Vane.*						
	Mar. 12 ...	$0{\cdot}106_6$	$0{\cdot}86_7$	...	...	W. Toyne
	Mar. 27 ...	new suspension.				
	Mar. 29 ...	$0{\cdot}39_3$	$0{\cdot}87_5$	...	...	L. F. Richardson Table I.
	April 27 ...	0·388	0·906	18·9	...	F. E. Blamey.
Front Back	May 5	0·390 0·379	0·871 0·894	19·0	...	L. F. Richardson
Front ...	May 21 ...	0·39	0·84	$13{\cdot}0_5$	...	F.H. Brownridge and T. B. Cotton
Front ...	June 4 ...	$0{\cdot}40_3$	$0{\cdot}86_8$	$19{\cdot}0_9$	93	S. O. Myers and D. J. Proctor.

The observations by students were in part supervised by Mr. R. S. Maxwell, M.A., B.Sc.

§ 4. CONCLUSION.

When a Dolezalek electrometer is used idiostatically it is most important to reverse the voltage and take the mean of the two deflections. Formula (5) is a good description, and (11), though presumably better, is perhaps unnecessarily elaborate. The contact E.M.F. η must be eliminated at the time, because it cannot be relied upon to remain constant for days. The mean is recommended, not on the rough principle "when in doubt take the mean," but as a clean elimination.

DISCUSSION.

Dr. **EZER GRIFFITHS**: I should like to ask the author whether he has made experiments to determine what factors may account for the changes observed in the values of the contact potential. In some experiments on the ionisation current for various applied potential differences between two aluminium electrodes cut from the same slab of metal I had trouble with contact potential effects, and found that heating one of the plates would produce marked changes. If one of the electrodes was heated and allowed to cool slowly a smooth curve relationship could be obtained between the contact potential and the temperature. The curve was not always reproducible, and after the electrode had cooled down to air temperature the value of the contact potential changed at a slow rate. Similar effects were observed with electrodes of other metals, such as zinc. In these experiments the electrodes were in an enclosure containing air dried with calcium chloride. Although the variations in the value of the contact potential were fractions of a volt, they were readily measured with the Compton electrometer. In view of the complexity of the phenomena, it would be useful to have the experiences of other workers who have encountered similar troubles.

Mr. R. W. **PAUL** said that he had found it possible to eliminate contact potentials almost completely in the electrometer by making all the plates from the same piece of aluminium and baking them in a moist atmosphere.

Mr. G. L. **ADDENBROOKE** said that he had avoided contact potentials by using heavily gilded quadrants and a heavily gilded mica needle, aluminium not being employed. He had also designed an electrometer in which the conductors were made from the same piece of metal and brought to the same condition by heating and annealing. A double needle and eight quadrants were used astatically, and the deflection due to contact potential was thus reduced to 2 mm. or 3 mm. It depended on humidity, and was subject to long-period changes.

Dr. E. H. **RAYNER** said that in his laboratory it was the practice to calibrate electrostatic voltmeters with alternating current. A Kelvin instrument with aluminium needle and brass quadrants was found to give a contact potential of about 0·36 volts, and the substitution of aluminium for the brass made matters worse. By careful attention to the condition of the surfaces the contact potential had been reduced to 0·05 or 0·15 volts, but it varied with the weather. It might conveniently be eliminated by the application of a correcting voltage to the quadrants.

Mr. A. P. **CASTELLAIN** said that at East London College it had been found that asymmetry of readings could be almost entirely eliminated by ensuring coincidence of the electrical zero (determined by joining the quadrants and applying a high voltage to the needle) with the mechanical zero. The asymmetry in a particular case amounted originally to 0·03 cm.

Mr. W. E. **BENHAM** said that as the electrostatic voltmeter was more often used for A.C. than for D.C., it was important to know to what extent A.C. readings would be affected by the errors discussed in the paper. The accuracy varied with frequency, the resistance and inductance of a silvered quartz suspension filament having appreciable effects at radio frequencies.

Dr. D. **OWEN**: The Author's Paper serves a useful purpose in calling attention to the existence of the contact potential effect in quadrant electrometers used idiostatically. References to the effect and the necessity of allowing for it in steady voltage measurements are to be found in text-books, e.g., in Laws' "Electrical Measurements," and in Moullin's "Radio-Frequency Measurements." It will also be found that in all precision work in which the quadrant electrometer is to be used idiostatically workers are well aware of the existence of contact potential differences, and that instruments for use in alternate current measurements must be calibrated with alternating voltages (*vide* Paterson and Rayner, Journ. I.E.E., 1913). Nevertheless, it appears to be true that the existence of this effect is not generally realised by electrical workers. These contact potential differences are puzzlingly indefinite, as may be seen by reference to a famous report by Sir Oliver Lodge to the British Association of 1884, entitled "On the Seat of the Volta Contact Electromotive Force in the Voltaic Cell." In some recent experiments on a new gold leaf electrometer one of my students, Mr. S. Ray, has observed a contact E.M.F. which varies from time to time, and which averages about 1·5 volts. These discrepancies vanish if the instruments are used for measuring alternating voltages, and it is preferable when small direct voltages are to be measured idiostatically to avoid the use of electrostatic instruments.

Dr. L. F. RICHARDSON : I communicated the Paper with some diffidence, expecting to be told that it was not new. The references mentioned by Dr. D. Owen will be useful. In reply to Dr. Ezer Griffiths, I suspect humidity to be a cause of the variation of η. Prof. Porter has chosen the Volta effect as the subject of his Presidential Address to Section A of the British Association, and we may expect further enlightenment on this question then. In reply to Dr. Addenbrook, Dr. Rayner and Mr. Paul, who have described processes for making η small, it is satisfactory to know that, even if they do not quite abolish η, yet η could be simply and accurately eliminated by reversing the polarity. In reply to Mr. Castellain, I would point out that 0·03 cm. is small in comparison with the 6 cm. mean deflection produced by the same voltage applied idiostatically. In reply to several speakers who were chiefly interested in alternating currents, the second member of equation (5) is proportional to the electrostatic couple when the vane is in equilibrium. When subject to an alternating E.M.F. the vane is not in equilibrium, except at two instants per cycle. But the classical theory, from which (2) is derived, when pursued further shows that the electrostatic couple is independent of the position of the vane ; and although this theory fails when A and B are large, yet it has been shown in §2 to be satisfactory for the idiostatic connection. Hence the equation of motion of the vane is

$$\theta = k_1\{\tfrac{1}{2}V^2 + V\eta\} + I d^2\theta/dt^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

where t is the time and I is a constant times the moment of inertia. Integrating over a period of oscillation T, we have

$$\frac{1}{T}\int_0^T \theta dt = \frac{k_1}{2}\frac{1}{T}\int_0^T V^2 dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

the term in η vanishing provided $\int_0^T V dt = 0$ . . . . (14)

That is to say : Whatever the contact E.M.F. may be, the mean of the deflections for V and $-V$ applied steadily is the same as the mean deflection for an alternating E.M.F. of effective value V, provided that there is no resultant direct E.M.F. applied. Here (with reference to Mr. Benham's remarks) V is the potential at the foot of the suspension.

XXXII—SOME EXPERIMENTS ON THE LIGHT-SENSITIVITY OF COMMERCIAL SELENIUM CELLS.

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(*Communicated by* DR. J. W. T. WALSH.)

ABSTRACT.

Part I.—The Relation Connecting the Change in Conductance of Selenium Cells with Illumination.

As the result of an experimental investigation, it is shown that the change in conductance C due to a given intensity of illumination I is proportional to some power of the illumination I—i.e., $C \propto I^x$. The index value x varies from cell to cell, and is probably dependent on the construction of the cell. A sensitivity curve is given.

Part II.—The reaction of Selenium to Various Spectral Regions.

Some filter experiments are described in Part II, from which it appears that the change in conductance of selenium cells is dependent, not on the number of foot-candles incident on the cell, but rather on the amount of radiant energy received. Contrary to general belief, it is shown that, for the same amount of energy received, the action of the infra-red is relatively much weaker than that of some shorter wave portion of the spectrum.

Some experiments on the decay of conductance of selenium after exposure to radiation from various portions of the visible spectrum indicate that, throughout a large portion of the visible spectrum, the internal state of the selenium, as determined by the change in conductance, is independent of the wave-length of the exciting radiation.

A brief note on the nature of the light-action in selenium cells is added.

INTRODUCTION.

SINCE the discovery of the light-sensitive property of selenium in 1873,(1) selenium has been the subject of approximately 1600 articles, as well as of numerous patents.(2) Further, about 150 scientific papers have been published giving experimental data as to the light-sensitive properties of selenium. The results have been so contradictory and so marked with uncertainty that, so far, it has been impossible to establish by their means any theory of the action of light on selenium.

Empirical formulæ for the relation between the increase of conductance and the intensity of the illumination, purporting to represent the observations of different observers, range from the linear relation to the fourth-root law.

If C=the change in conductance due to illumination of intensity I, then we have

(1) $C=aI$ (Stebbins), where a is a constant.
(2) $C=aI^{\frac{1}{2}}$ (Rosse, Adams, Berndt, Fournier d'Albe),(3) where a is a constant.
(3) $C=aI^{\frac{1}{3}}$ (Hopius).(4)
(4) $C=aI^{\frac{1}{4}}$ (Rankine).(5)

In addition, we have the following empirical relations :—

(1) $R_a/R_b=(a/b)^n$ (Ruhmer),(6) where R_a and R_b are the values of the resistance under the smaller and brighter illuminations a and b respectively. The value of n was found to vary between 0·25 and 0·35.

(2) $C(C-b)=aI$ (Athanasiadis),(7) where I is the intensity of illumination, C the conductance and a, b are constants.

The wide differences in the empirical formulæ may doubtless be explained by differences in the time of exposure, range of intensity, colour temperature of the source; but the author has come to the conclusion that the discrepancies are, in a large measure, due to inattention to the conditions for accuracy in the measurement of conductivity and intensity, rather than to true physical differences in the quality of the cells exposed, arising from difference of construction.

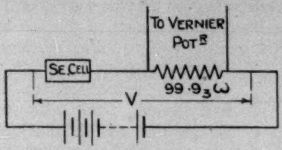

FIG. 1.—TEST CIRCUIT.

In the experiments here described great care was taken to attain as high a degree of accuracy as possible in the measurements, and it is suggested that this accounts for the consistency of the results now obtained. The measurements described in the paper have been repeated at intervals of several months, and are found to be quantitatively reproducible.

PART I.

Experimental Details.

The change in conductivity *C* of a number of selenium cells due to a given intensity of illumination was determined over the range 0-100 foot-candles. The source was a standard tungsten filament lamp mounted on a two-metre photometer bench. The colour temperature of the lamp was approximately 2290°K. By means of an electrostatic voltmeter, 100±0·01 volts were maintained across the lamp. The candle-power was then known to within 1/400. The cell under test formed part of the circuit shown in Fig. 1. The resistance of the standard shunt used for current measurement was 99·93 ohms; the fall of potential across this resistance was measured by a vernier potentiometer, so that the current flowing in the circuit was ascertained to within 2/10,000. The voltage across the selenium cell and the standard shunt was maintained at exactly 10 volts, as indicated by a calibrated voltmeter which could be read to within 1/1,000.

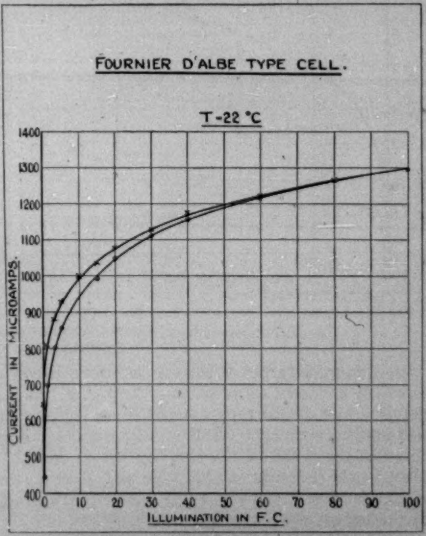

FIG. 2.—CURRENT ILLUMINATION CURVE FOR A FOURNIER D'ALBE TYPE OF CELL.

The "dark" current through the cell was first measured, thus giving the "dark" conductance $(1/R_0)$. A series of measurements was then taken with the cell exposed to illumination over the range 0-100 f.c. increasing, similar measurements being repeated with decreasing illumination. The measure of the ultimate change of conductance due to a given intensity of illumination may be conveniently defined as the total change of current resulting from the exposure of the cell to illumination. At each reading time was allowed (usually two to three minutes) for the current to attain a constant value, for, as is well known, the rise to maximum conductance of selenium when exposed to light occurs much more rapidly than the reduction to minimum conductance after the light is cut off. The change in conductance $(1/R_I - 1/R_0)$ due to a given illumination I was plotted against I.

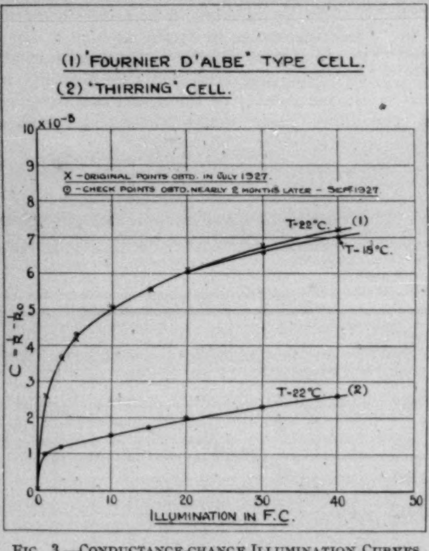

FIG. 3.—CONDUCTANCE-CHANGE ILLUMINATION CURVES.

The temperature of the cell was frequently noted. The necessity for this became evident in the course of later experiments on the temperature coefficient of resistance of selenium cells between 10-40°C., which showed that both the "light" and "dark" conductance increased linearly with temperature, whilst the change in conductance $(1/R_I - 1/R_0)$ due to a constant intensity of illumination I decreased linearly with temperature.

RESULTS.

As representative of the results, three curves are shown relating to two cells of widely different construction. In Fig. 2, representing observations on a cell of the Fournier d'Albe type, current is plotted against intensity of illumination; the well-known remanence effects of selenium are clearly evident. A similar curve was obtained for a cell of the Thirring type. In Fig. 3 the change in conductance C, of two cells (Fournier d'Albe type and Thirring type) is plotted against intensity of illumination I. In Fig. 4, log C is plotted against log I. The following formulæ fit the curves:—

(i.) Cell of Thirring type

$$C = 1{\cdot}63 \times 10^{-5} \times I^{0.390},$$

where I is measured in foot-candles and C in amperes $\times 10^{-5}$;

or

$$C^{2.56} \propto I.$$

(ii.) Cell of Fournier d'Albe type

$$C = 2{\cdot}80 \times 10^{-5} \times I^{0.252}$$

I and C measured as above ;

or $$C^{3.97} \propto I.$$

In Fig. 5 the sensitivity, defined as dC/dI, is plotted against I.

Experiments with different types of cells—i.e., cells of different make, have led the author to the conclusion that the change in conductance, under the conditions specified, can in every case be represented by a formula of the type

$$C = (1/R_I - 1/R_0) = AI^x,$$

where A is some constant, x being determined by a variety of conditions which at present can only be approximately estimated. Although each cell has its own particular index value, this value, for the same cell, is found to persist even after a lapse of several months. It is to be noted that, even amongst cells of the same type, small variations of the index value occur. In general, no two cells of any one type have the same quantitative reaction towards a given intensity of illumination.

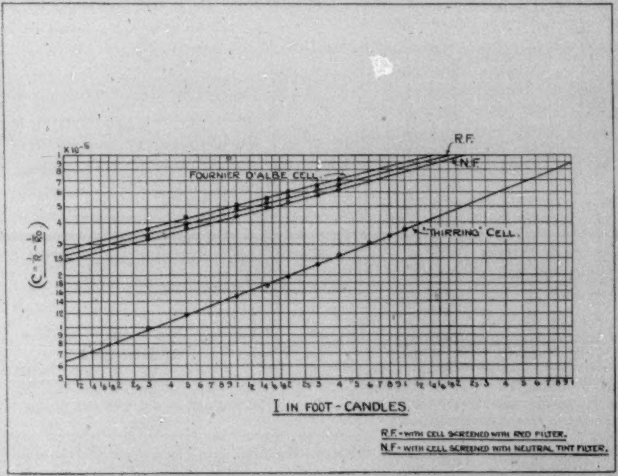

FIG. 4.—LOGARITHMIC CONDUCTANCE-CHANGE ILLUMINATION CURVES.

PART II.

The experiments described on the change of conductance with intensity of illumination were repeated on a cell of the Fournier d'Albe type with the cell screened (1) with a neutral tint filter, and (2) with a Corning red filter, the spectral transmission curve of which shows a fairly sharp "cut-off" at 6250 Å., with a mean transmission of 78·2 per cent. between 6400 Å. and 7100 Å. The results are shown in Fig. 6. Plotting log *C* against log *I*, we see that *C* is still proportional to $I^{0.252}$ in both cases, although when the red filter was interposed the radiation incident on the cell was restricted to a very limited portion of the visible spectrum.

Considering the value of *C* corresponding to any particular value of *I* for the cell unscreened, we find that the effect of interposition of the neutral tint filter is equivalent to reducing the intensity of illumination by a mean value of 38·6 per cent. of the original intensity. Thus, according to the selenium cell, the transmission factor of this filter=61·4 per cent. Ordinary photometric measurements, however, gave the transmission factor as =12·0 per cent. This proves that the response of the selenium cell is not simply dependent on the number of foot-candles transmitted by the filter.

Similarly, ordinary photometric measurements gave the transmission factor of the red filter as 11·45 per cent., whereas the effect on the selenium cell was that which would have corresponded to a mean transmission factor of 84·0 per cent.

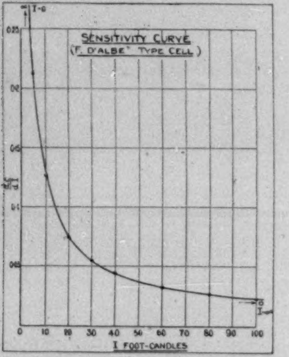

FIG. 5.—SENSITIVITY (dC/dI) ILLUMINATION CURVE FOR FOURNIER D'ALBE TYPE OF CELL.

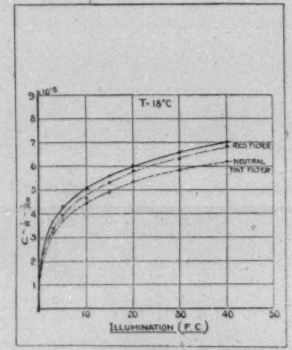

FIG. 6.—CONDUCTANCE - CHANGE ILLUMINATION CURVES FOR SCREEN CELL.

A test was made with an infra-red filter which absorbed all radiation of a wavelength below 6800 Å. It was found that the effect on the conductance corresponded to a reduction of the intensity of illumination by an amount equal to 75 per cent. The spectral distribution of the source being practically that of a black body, Wien's radiation formula can be applied. It is found upon integration between the limits 7000-20,000 Å. and 4000-20,000 Å., and allowng for the infra-red energy absorption of the filter (approx. 10 per cent.), that the interposition of the infra-red filter only reduced the total radiant energy incident on the cell by a small amount. Thus the action of the infra-red upon the change in conductance of selenium is relatively much weaker than that of some shorter wave portion of the spectrum.

In the course of some experiments on the decay of the conductance of selenium after exposure to light the following tests were made :—

The cell was screened with a red filter and the tungsten lamp adjusted on the bench until the steady value of C was equal, to, say, a. The illumination was then cut off and the change of C was plotted against the time t in seconds.

Exactly similar experiments were performed with the cell screened with blue, green and orange filters successively; at each test the tungsten lamp was adjusted to give the steady value of $C=a$. The results are shown plotted in Fig. 7. It will be seen that, within the limits of experimental error, there is remarkable agreement between these colour-decay curves. Thus, although the steady value of $C=a$ was obtained by exciting the cell with radiation from different spectral regions distributed throughout the visible spectrum, it appears that the effective result on the cell was the same in each case. The most natural inference to be drawn is that the internal state of the selenium was the same in each case.

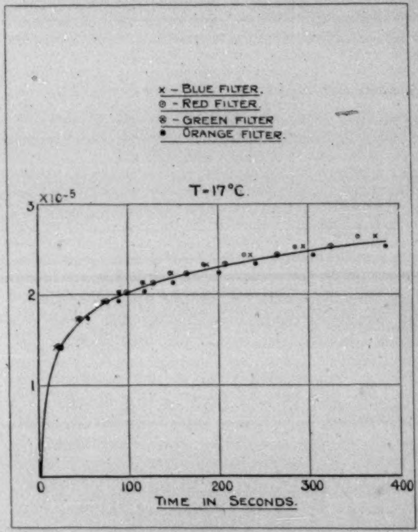

Fig. 7.—Conductance-change Time Curves for Screened Cell.

These results appear to conform with the general character of a curve recently obtained by Fournier d'Albe.(8)

Note on the Nature of the Light-action in Selenium.

Many theories have been proposed to account for the effect of light on selenium. On the whole, it cannot be said that our insight into the matter is very complete. Most recent investigators incline to the internal photo-electric theory in some form or other, the view being that light simply acts by liberating electrons. They endeavour to explain the inertia by assuming finite velocity of migration of these electrons. It is evident from the work of Gudden and Pohl(9) that the number of electrons which could be liberated by photo-electric action of the absorbed light according to the quantum equations are insufficient to account for the increases in conductance observed in selenium cells. Of course, it could be assumed that each primary electron liberated by photo-electric action of the absorbed light creates a sufficient number of secondary electrons; but experimental support is wanting for such an assumption. Moreover, any complete theory must not only account for the effects observed in light-positive selenium (increased illumination giving increased conductance), but also the inverse effects observed in light-negative selenium (i.e., increased illumination giving decreased conductance).(10) The author is inclined to believe that the effects are due to crystalline structure. Brown⁽¹¹⁾ has shown that the sensitiveness of a selenium crystal varies upon application of mechanical pressure to the crystal. E. O. Dieterich⁽¹²⁾ has shown how the resistance of selenium depends upon different annealing temperatures. Further investigators have demonstrated how the resistance of selenium cells depend on the voltage applied.⁽¹³⁾ G. W. White⁽¹⁴⁾ has shown that the resistance of selenium cells depend on the nature and pressure of the electrodes. It is therefore possible that there exists in the surface layers of the selenium a large number of bounding crystal surfaces in loose contact. The complete theory of the action of light on selenium is possibly intimately connected with the theory of coherer-action.

It is hoped later to continue these experiments with the object of assisting towards the elucidation of the nature of light-action on selenium.

ACKNOWLEDGMENT.

The author desires to express his very sincere appreciation of the kind encouragement and help of Dr. J. W. T. Walsh, and to thank Dr. D. Owen and Dr. M. A. F. Barnett.

APPENDIX.

REFERENCES.

(1) W. Smith, Soc. Teleg. Eng., p. 81 (1873).

(2) Selenium, a list of references. M. F. Doty, New York Public Library (1927).

(3) E. E. Fournier d'Albe, Proc. Roy. Soc., Series A, Vol. 89, pp. 75-90 (1913).

(4) E. Hopius, Abhängigkeit der elektrischen Leitfähigkeit des Selens von dem Beleuchtungsgrad. Beiblätter zu den Annalen der Physik und Chemie. Bd. 28, p. 723 (1904).

(5) A. O. Rankine, Phil. Mag., Vol. 39, pp. 482-8 (1920).

(6) E. Ruhmer, Elektrotechnische Zeitschrift, Vol. XXV, p. 1021 (1904).

(7) G. Athanasiadis, Annalen der Physik und Chemie, Bd. 27, pp. 890-896 (1908).

(8) E. E. Fournier d'Albe and E. O. Symonds, Proc. Opt. Conv., Part ii, p. 884 (1926).

(9) B. Gudden and R. Pohl, Zeitschrift für Physik, Bd. 35, pp. 243-259 (1925).

(10) L. B. Crum, Phys. Rev., Vol. 33, pp. 538-48 (1911); F. C. Brown, Phys. Rev., Series 2, Vol. 1, p. 237 (1913).

(11) F. C. Brown, Phys. Rev., Vol. 25, pp. 501-2 (1907).

(12) E. O. Dieterich, Phys. Rev., Series 2, Vol. 4, pp. 467-476 (1914).

(13) E. E. Fournier d'Albe, Proc. Roy. Soc., Series A, Vol. 86, pp. 452-61 (1912).

(14) G. W. White, Phil. Mag., Series 6, Vol. 27, pp. 370-81 (1914).

DISCUSSION.

Prof. A. O. RANKINE suggested that Fig. 3 was in danger of being misread in a sense unfavourable to the Thirring cell, since it contrasts the "light" current from such a cell with that from a Fournier d'Albe cell, whereas the proper test for the sensitivity of a cell is the ratio of "light" current to "dark" current. He was surprised at the reproducibility of the results in view of a factor not mentioned in the Paper—namely, the "voltage effect": if a steady E.M.F. greater than 1 volt be applied to a darkened cell, the resistance changes continuously with time. An even more important factor than precision of measurement in such investigations was the regulation of temperature, since the cell constant has a temperature coefficient of 5 per cent. per degree. How had the author regulated and measured the temperature? Mr. Avery and the speaker had shown that the sensitivity is much increased if the cell be desiccated, and Prof. Thirring was now sealing his cells hermetically. The author was to be congratulated on his study (Fig. 6) of the dependence of the response on the quality of the stimulus.

Dr. D. **Owen**, who had suggested the subject of selenium cells to the author, said that the latter was the first student member to read a paper to the Society, and offered him good wishes for his future work.

Dr J. W. T. **Walsh** said that the author's work was somewhat reassuring from the practical point of view of photometry and electrical engineering. Selenium cells had always attracted inventors, but their irregular behaviour was disconcerting and it was important to establish reproducible conditions.

Mr. T. **Smith** asked whether the curves in Figs. 5, 6 and 7 had been drawn through observational points or derived by calculation.

Major C. E. S. **Phillips** said that the most promising method of obtaining consistent results was to use selenium crystals of a definite character. An ordinary cell contains crystals soluble in water and its reliability can be much improved by boiling in this liquid. He had been able to make selenium crystals several mm. long; they could be spread into a thin layer without loss of light-sensitiveness. It was a curious fact that contact of metallic conductors with the crystals could be improved by working under oil.

Author's reply: I should like to express my very sincere appreciation of Prof. Rankine's remarks. In reply, I desire to point out that Fig. 3 does not contrast the "light" current from a Thirring cell with that from a Fourner d'Albe cell in the sense implied by his statement. Fig. 3 contrasts the *extra* current through the two cells resulting from their illumination. It was not originally intended that Fig. 3 should give any information as to the ratio of "light" current to "dark" current for a given illumination for these cells, for the reason that this ratio has not been found to be a constant quantity for any one cell. At the commencement of the investigations it was thought that the "dark" resistance R_0 of any cell, measured under the conditions specified in the Paper, would not vary sensibly with age from the original "dark" value. This, however, was not so, since it was found that R_0 varied from day to day. Moreover, the "light" resistance R_l, corresponding to any given intensity of illumination, was by no means constant. But it was found that, provided the conditions of test were reproduced, the one constant quantity was the difference of the conductances $1/R_l - 1/R_0$. This is quantitatively reproducible for any of the cells examined, and is, as shown in Figs. 3 and 4, quite a definite function of the illumination. For example, the limits of variation of the "dark" resistance R_0 of the Fournier d'Albe cell, mentioned in the Paper, over the period of tests was $19{,}500^\omega - 22{,}500^\omega$, From the point of view of general interest, I give here values of R_0 and the corresponding values of R_I (for the cells exposed to 40 f.c.) for the 2 cells mentioned in the Paper:—

(1) Thirring cell: $R_0 = 366{,}800^\omega$, $R_l = 34{,}830^\omega$, $R_0/R_l = 10.5_3$.

(2) Fournier d'Albe cell: $R_0 = 22{,}420^\omega$, $R_I = 8{,}535^\omega$, $R_0/R_l = 2.6_3$.

The voltage effect mentioned by Prof. Rankine was duly considered. Prior to commencing the tests detailed in the Paper, an experiment was made to determine the variation of current through a cell, exposed to a constant illumination of 10 f.c., with the steady E.M.F. across the standard shunt and the selenium cell (Fig. 1) maintained at exactly 10 volts. In the extreme case, curiously enough, for the Thirring cell mentioned in the paper, the increase of current through the cell after 30 minutes was not greater than 1·0 per cent. of the value of the current flowing through the cell after 1 minute, a further 70 minutes giving a further increase of 3·4 per cent. Since the results expressed in Figs. 3 and 6 did not take more than 20 minutes to obtain, this effect was not considered to be serious. Moreover, part of the initial increase of current can be attributed to the extra current, flowing through the cell due to illumination, rising to a steady value. It was assumed that any error thereby introduced would appear in each test and be included in the cell constants. The consistency of the results appears to offer some justification for this assumption.

A separate investigation was carried out in order to determine the temperature coefficient of the cells examined, and in no case was a temperature coefficient observed as high as that stated by Prof. Rankine. The cells were placed in an electrical oven so arranged that they could be exposed to illumination, the source of which was also contained inside the oven. The surface of the inside walls, etc., of the oven was blackened. Any temperature desired between the range 10°−40°C. was easily obtained and maintained for 2 hours to ensure that the cell was at the temperature of the enclosure. As typical of the results we may take the case of the Fournier d'Albe cell. The temperature coefficient of the "dark" conductance $1/R_0$ was $+2{\cdot}2$ per cent. per degree centigrade over the range 10°−40°C., the temperature coefficient of the "light" conductance $1/R_I$ (with the cell exposed to a constant illumination of 58·6 f.c.) was $+0{\cdot}5$ per cent. per degree centigrade, but the temperature coefficient of the change of conductance $1/R_I-1/R_0$ due to a constant intensity of illumination was $-0{\cdot}5$ per cent. per degree centigrade. Since I have expressed the results in terms of the change of conductance of the cells due to illumination, I had to contend with a possible error due to changes of temperature owing to the temperature coefficient of the change of conductance stated. It was not considered necessary to take any special precautions to regulate the temperature of the cell under test. Before testing, the cell was left in a darkened photometer room for 5-6 hours in order to ensure that the temperature of the cell was that of the room. During the period of the test the temperature of the room was not found to vary more than 0·2°C. The room-temperature was measured by a mercury-in-glass thermometer correct to 0·1°C. over the range used. There was not considered to be any serious heating of the cell due to the illumination during the time occupied by the test.

The cells examined were not hermetically sealed. Only in the case of the Thirring type of cell was any irregular behaviour observed, attributable to the presence of moisture films as suggested by Rankine and Avery[*] in their recent paper. It seems possible that Thirring cells are, from their construction, specially susceptible to trouble due to humidity, and in this connection it is interesting to note that Prof. Thirring is now sealing his cells hermetically.

I should like to record my appreciation of the remarks of Dr. D. Owen and Dr. J. W. T. Walsh.

In reply to Mr. T. Smith, the curve in Fig. 5 was derived by calculation, while the curves in Figs. 6 and 7 were drawn through observational points.

In thanking Major C. E. S. Phillips for his remarks, I should like to add that, in continuing the work with a view to assisting towards the elucidation of the nature of the light-action in selenium, I feel that observations should be made of the electrical, optical and mechanical behaviour of selenium crystals rather than of commercial selenium cells.

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[*] Proc Phys. Soc., Vol. 39, p. 187.

XXXIII.—THE VAPOUR PRESSURE OF WATER OVER SULPHURIC ACID-WATER MIXTURES AT 25°C., AND ITS MEASUREMENT BY AN IMPROVED DEW-POINT APPARATUS.

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ABSTRACT.

A critical study has been made of data previously used by Wilson in the construction of a mean curve for the vapour pressure of water over sulphuric acid-water mixtures at 25°C. The observations of Sorel (employed by Wilson over the concentration range 44-82 per cent. sulphuric acid) have been shown to be probably inaccurate, by calculations based on thermodynamics, and by determinations at 25°C., using an improved dew-point apparatus. The mean curve has accordingly been reconstructed, making use of the latest available data, and a set of most probable values for the vapour pressure has been compiled, taking into account the relative claims to accuracy of all investigators.

INTRODUCTION.

THE vapour pressure of water over sulphuric acid-water mixtures appears to be regarded as more accurately known than that over any other aqueous solution. The values at 25°C. are of especial importance, on account of the use of this temperature as a standard for purposes of comparison, e.g., in physico-chemical investigations concerned with ionic activity in solution; it is, therefore, a matter of some importance that the most accurate data at this temperature may be available.

Wilson(1) has constructed a mean curve for the vapour pressure of water over sulphuric acid-water mixtures at 25°C., from the measurements of Regnault, Sorel, Helmholtz and Brönsted at temperatures in the neighbourhood of 25°C.; he has also incorporated values obtained by means of a thermodynamic calculation, from the determinations of Burt, Sorel, Dieterici and Briggs, at other temperatures. He states that the mean curve thus obtained is probably accurate to within 0·1 mm. of mercury. The plan employed by Wilson consisted in plotting on a large scale, on the same graph, the results of the investigators before mentioned, and drawing a smooth curve to pass through the points so obtained. Mean values for the vapour pressure were read off at concentration intervals of 5 per cent., so that the curve might be reconstructed by other workers.

The author has made a critical study of the data and methods employed in this compilation and has arrived at the conclusion that Wilson's mean curve does not exhibit the most accurate values available. This view applies particularly to that portion of the curve relating to 40-60 per cent. sulphuric acid, over which range Wilson finds the graphical results in surprisingly good concordance. The author's criticism is based on the fact that Wilson makes extensive use of the data of Sorel, more especially over the range mentioned, whereas the work of subsequent investigators renders it probable that Sorel's figures are in error.

It is worthy of mention that the work of Smits and of Hacker, prior to 1921, has apparently been overlooked by Wilson. Since 1921, Daudt, Grollmann and Fraser, and McHaffie have made further determinations, which serve to locate more accurately the two extremes of the curve, viz., for high and low concentrations respectively. An examination of Wilson's mean curve over these ranges reveals the fact that serious divergencies exist between the results of the separate investigators from which his data is drawn.

It appears to the author that exception may be taken to Wilson's method of compilation, on the score that he has given equal weight to the work of all investigators, irrespective of the relative degrees of accuracy attained or claimed in their work, and of probable errors due to defects in the method or apparatus used. In accordance with these views, a modified method of compilation has been adopted in this paper, by means of which a series of most probable values for the vapour pressure at 25°C. has been obtained.

CRITICAL DISCUSSION OF RESULTS OBTAINED BY EARLIER WORKERS.

In this section of the paper are given the results of an examination of the experimental methods employed by earlier investigators, in chronological order, and of criticisms made by later workers.

The method of Regnault(2) (1845) consisted in direct cathetometrical determination of the difference in level between the mercury in a normal barometer and that in one connected to a globe of 500 c.c. capacity containing the sulphuric acid-water mixture used. The difference in pressure so obtained was corrected for the pressure of air remaining in the globe (corresponding to approximately 0·1-0·2 mms. mercury). The temperature of the observations varied from 0-35°C. According to Tables given in the paper, the maximum error in the observations appeared to be of the order 0·3 mm. of mercury. Regnault's method has been criticised by Helmholtz, and later by Dieterici and by Daudt, who point out that the influence of the air remaining in the vapour space on the vapour pressure of water is ignored. Further, the author finds that when Regnault's figures, interpolated to 25°C., are plotted on a large scale against the concentration of sulphuric acid, the curve so produced is not continuous if the point corresponding to the mixture of concentration 52 13 per cent. sulphuric acid is included. A similar result obtains when the vapour pressures at 20° and 30°C. respectively are plotted against the concentration. The existence of this point renders the results of Regnault over the range 44-58 per cent. sulphuric acid of questionable value. If the point is disregarded and the curve made continuous, the values interpolated at concentration intervals of 5 per cent. are in moderately good agreement with those of other investigators, with the exception of the lower range (60-80 per cent. sulphuric acid), where the figures are undoubtedly high. (Table I, column 6.)

Helmholtz(3) (1886) determined the vapour pressure of water over three sulphuric acid-water mixtures, at temperatures between 0° and 50°C., by allowing air containing water vapour in equilibrium with the sulphuric acid-water mixture employed to expand adiabatically from a known pressure, in excess of atmospheric down to atmospheric pressure. The excess pressure employed was adjusted until a mist formed on lowering the pressure to atmospheric. The temperature depression was calculated from the initial and final pressures, the required vapour pressure of water being read off from vapour pressure tables for water. It may be noted that the method is closely analogous in principle to the dew-point method, as described by the author. The accuracy of the results obtained is stated to be within 1 per cent. The author has not found it possible to make use of Helmholtz's data, as the figures obtained are too widely spaced for graphical interpolation. The results are in good agreement with those of later workers.

The observations of Sorel(4) (1889) were obtained by a dynamical method. A measured volume of dry air was passed through the sulphuric acid-water mixture used, and the weight of water vapour carried over was determined gravimetrically, applying certain correction factors. The observations covered a concentration range of 44-82 per cent. sulphuric acid, up to 95°C. The figures given in Sorel's table are quoted to an accuracy of 0·1 mm. only. Burt(9) has pointed out an important source of error in Sorel's method. Owing to the small quantity of acid employed, and the necessarily long duration of the experiment, the concentration of the acid-water mixture used becomes appreciably altered at the close of each determination. Further. Porter(5) has shown by thermodynamic methods similar to those of Wilson, and concerned with the calculation of the latent heat of evaporation of sulphuric acid-water mixtures, that Sorel's results are probably inaccurate. A comparison of the figures interpolated at 5 per cent. intervals from a large scale plot of Sorel's data with those of later investigators (Table I, column 7) shows that Sorel's results are consistently high.

Dieterici(6) (1893 and 1897) has carried out careful determinations of the vapour pressure of water over aqueous solutions at 0°C. ; he has pointed out two important sources of error in this type of work :—(1) The manometers previously employed were not sufficiently accurate for the measurement of low pressures ; (2) the pressure of air in the manometer may influence the results obtained. He, therefore, employed a differential tensimetric method, the manometer being closely analogous to that employed in the aneroid barometer. Differences in pressure were read off by means of a telescope and scale from the deflections of a small mirror attached to the corrugated metal box of the manometer. The accuracy of the observations is stated to be of the order 0·005-0·01 mm. of mercury. Wilson has recalculated Dieterici's results from 0°C. to 25°C., employing a thermodynamic method based on the approximate form of the Clausius-Clapeyron equation, the actual formula employed being :—

$$\text{Log}\,\frac{r_2}{r_1} = \frac{\Delta H_2}{2{\cdot}3.R}\cdot\frac{T_2 - T_1}{T_2 T_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where r_1 and r_2 are the relative vapour pressures at the temperatures T_1 and T_2, and ΔH_2 the heat absorbed when one mole of water is removed from the solution without change in concentration. Values read off at 5 per cent. concentration intervals from a large scale plot of Dieterici's results, recalculated from 0°C. to 25°C., are given in Table I, column 9. They are in good agreement with those of later workers.

Smits(7) (1898 and 1902) has carried out very accurate determinations of the lowering of vapour pressure of water over sulphuric acid-water mixtures of low concentration (0-18 per cent. sulphuric acid) at 0°C. A micromanometer of special pattern, employing aniline and water, and similar in principle to the McLeod gauge, was used in these determinations. The results have been recalculated by the author from 0°C. to 25°C., making use of the thermodynamic method due to Wilson, and are given in Table I, column 8. They are in excellent agreement with the direct measurements at 25°C. of Grollmann and Fraser.

Briggs(8) (1903) extended Sorel's observations, using the same experimental method as that employed by Sorel, up to 200°C. for concentrations up to 91 per cent. sulphuric acid. His results are open to the same objections as those of Sorel; they have, however, been recalculated by the author from 100°C. down to 25°C., employing Wilson's formula, and are quoted in Table I, column 12. They are in good agreement with the direct measurements of McHaffie.

Burt(9) (1904) measured the actual pressure under which the sulphuric acid-water mixture boiled at different temperatures, ranging from 55°C. upwards. Porter found that Burt's data gave very regular results when employed for calculation of the latent of evaporation, and regarded them as much more reliable than those of Sorel. The author has recalculated Burt's figures for 75° and 100°C. down to 25°C., using formula 1; the results, when recalculated from each temperature, are in fair agreement amongst themselves and with those of other workers (Table I, column 11).

Brönsted(10) (1910) determined directly the vapour pressure of water over sulphuric acid-water mixtures at 20°C. and at 30°C. (*see* Table I, column 5). His results appear to have been obtained with great care, a modified tensimeter being employed in the determinations. Later workers (e.g., Grollmann and Fraser, Bennett) appear to regard the observations of Brönsted as the only earlier determinations for sulphuric acid-water mixtures which merit attention. Further, Grollmann and Fraser state that their results indicate the accuracy of Brönsted's measurements over the range 0-22 per cent. sulphuric acid to extend only to the first decimal place, and not to the third, as quoted.

Hacker(11) (1912) has carried out determinations of the vapour pressure of water over sulphuric acid-water mixtures, with the object of showing that the heat of dilution is variable with the temperature. A modified differential tensimetric method was employed, similar in principle to the Fortin barometer. The temperature at which observations were made ranged from 30-80°C. The results at 50°C. have been recalculated by the author to 25°C.; the values given in Table I, column 13, have been obtained by interpolation at 5 per cent. concentration intervals from the recalculated figures. They are in fair agreement with those of other workers, except at concentrations 45 per cent. and 50 per cent. respectively, where they appear to be high.

The more recent determinations, subsequent to Wilson's Paper, include those of Daudt, Grollmann and Fraser, and McHaffie.

The measurements of Daudt(12) (1923) refer to sulphuric acid-water mixtures of high concentration (65-85 per cent.). A differential method for the measurement of small pressures (of the order 1 mm. or less), based on the variation of thermal conductivity with pressure, was employed. The maximum probable error is stated to be ±4 per cent. (for pressures of the order 0·01 mm.). The author has recalculated Daudt's figures up to 25°C., employing formula 1 (Table I, column 11). They are in good agreement with the determinations of McHaffie over the same range.

Grollmann and Fraser(13) (1925) have made direct determinations at 25°C. of the difference between the vapour pressure of water over sulphuric acid-water mixtures of low concentration (0-22 per cent. sulphuric acid) and that over pure

TABLE I.—*Vapour Pressure of Water over Sulphuric Acid-Water Mixtures at 25°C.*

Grams Sulphuric acid per 100 gms. solution.	§ 1. From determinations actually carried out at 25°C.			§ 2. By interpolation from determinations over a temperature range including 25°C.			§ 3. Recalculated, using Wilson's thermodynamic method, from determinations at :—						Most probable values.	Wilson's mean values.
	Grollmann and Fraser.	Author.	McHaffie.	Brönsted.	Regnault.	Sorel.	0°C.			75 and 100°C.	100°C.	50°C.		
							Smits.	Dieterici.	Daudt.	Burt.	Briggs.	Hacker.		
5	23·32	...	...	23·53	23·30	...	23·32	23·40	...	23·08	...	...	23·32	23·40
10	22·72	...	...	23·05	22·60	...	22·73	22·90	...	22·23	...	...	22·72	22·78
15	21·98	...	...	22·50	21·68	...	22·03	22·17	...	21·30	...	...	21·98	22·07
20	20·92	...	...	20·92	20·58	...	21·11	21·20	...	20·24	...	20·72	20·92	21·03
25	19·60	...	...	19·41	19·18	...	...	19·83	...	19·00	...	19·64	19·60	19·70
30	...	...	...	17·68	17·80	...	...	18·08	...	17·58	...	18·08	17·78	17·96
35	...	...	...	15·72	15·90	...	...	16·00	...	15·80	...	16·00	15·84	15·87
40	...	...	...	13·30	13·32	...	...	13·42	...	13·44	...	13·54	13·39	13·49
45	...	10·80	...	10·80	10·78	11·0	...	10·90	...	10·90	...	11·26	10·85	11·35
50	...	8.41	...	8·37	8·32	8·8	...	8·39	...	8·32	...	8·92	8·36	8·74
55	...	5·90	...	6·08	6·16	6·5	...	6·12	...	6·00	...	...	6·05	6·37
60	...	...	...	4·10	4·22	4·3	...	4·03	...	3·72	...	...	3·95	4·09
65	...	...	2·05	2·32	2·88	2·8	...	2·06	2·30	1·98	...	...	2·14	2·33
70	...	...	1·00	0·97	1·70	1·8	...	0·80	1·00	0·66	...	...	0·88	1·21
75	...	...	0·39	0·49	0·88	1·1	...	...	0·36	...	0·40	...	0·41	0·55
80	...	...	0·14	0·14	0·40	0·6	...	...	0·13	...	0·12	...	0·13	0·19
Col. No. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

water. A very elaborate experimental method was adopted by these investigators, making use of the Rayleigh manometer. Special precautions were taken to avoid previously recorded sources of error (e.g., presence of air in the apparatus). The authors state that they believe the measurements to differ from the true values by not more than 0·001 mm. Their results are recorded in Table I, column 2.

McHaffie's[14] (1927) determinations at 25°C. cover a concentration range of 65-80 per cent. sulphuric acid. A dynamical method was employed, according to which the water vapour contained in a measured volume of air, in equilibrium with the mixture, was condensed in a tube of known volume, immersed in liquid air. The condensed water was vaporised by raising the temperature to atmospheric and the pressure read off. The equilibrium pressure of water vapour at 25°C. over the mixture employed was calculated from the data so obtained. The results (Table I, column 4) agree closely with those of Daudt and of Briggs.

METHOD OF COMPILING MOST PROBABLE VALUES.

The results of all investigators are given in Table I. They have been classified as follows : Section 1 (Grollmann and Fraser, Author, and McHaffie) contains results of direct determinations at 25°C. Section 2 (Brönsted, Regnault, Sorel) gives figures obtained by interpolation from determinations over temperature ranges including 25°C. Section 3 (Smits, Dieterici, Daudt, Burt, Briggs, Hacker) contains figures recalculated from determinations at other temperatures. It seems probable that the figures of Section 1 represent the most reliable data, since, unlike the other figures, they are not liable to errors introduced through interpolation (Section 2) or extrapolation (Section 3). Of the figures of Section 2, those of Regnault and Sorel are probably not reliable, while those of Brönsted are regarded as reliable. Hence the author has not included the results of either Regnault or Sorel in compiling the most probable values for the vapour pressure. The figures of Section 3, being obtained by extrapolation, cannot be regarded as of the highest order of accuracy. However, as Wilson has shown, the factors which have to be applied in recalculating the relative vapour pressure from 0°C. to 25°C. are not so large as those for recalculating from 100°C. to 25°C. and therefore the error introduced by the extrapolation is probably smaller in the former case. This conclusion is confirmed by the agreement of the figures obtained with the direct determinations quoted in Section 1.

The results of Burt over the range 30-55 per cent. sulphuric acid are, however, in good agreement with those of other workers, and have been taken into account in obtaining the most probable figures over this concentration range, while those of Briggs are in good agreement with the figures of McHaffie and of Daudt.

The mean figures (Grollmann, and Fraser, Brönsted, Smits, Dieterici) of the vapour pressure for 0-25 per cent. sulphuric acid were found to be in close agreement with those of Grollmann and Fraser. Further, as already pointed out, the very careful observations of Smits and of Grollmann and Fraser appear to represent the most reliable figures over this range. The author has therefore adopted Grollmann and Fraser's figures exclusively as the most probable values for 0-25 per cent. sulphuric acid. The most probable values over the concentration range 30-60 per cent. sulphuric acid have been obtained by averaging the results of the author, Brönsted, Dieterici and Burt, which are in good agreement, while those for the concentration range 65-80 per cent. have been compiled from the data of McHaffie, Daudt and Briggs, whose observations agree very closely.

The most probable values so obtained are set out in Table I, column 14, and are compared with the figures given by Wilson (Table I, column 15). The mean curve, constructed on a large scale from the most probable values obtained by this means, is found to be continuous, a circumstance which tends to confirm the accuracy of the figures. The divergencies between the author's figures and those of Wilson are not great until concentration 45 per cent. sulphuric acid is reached. At concentration 45 per cent. sulphuric acid, Wilson's figure is 0·50 higher than the most probable value; at concentration 50 per cent. sulphuric acid, the divergency is 0·38 mm., and at 55 per cent. 0·32 mm. Further divergencies occur over the higher range of concentration—viz., from 65-80 per cent.; Wilson's figure for 65 per cent. sulphuric acid is 0·19 mm. higher than the most probable value; for 70 per cent., 0·33 mm., and for 75 per cent. 0·14 mm.

The most serious divergencies between Wilson's mean values and those deduced by the author from the results of Brönsted, Dieterici and Burt were found to occur over the range 45-55 per cent. sulphuric acid. A fresh determination of the vapour pressures of water over sulphuric acid-water mixtures over this concentration range, therefore, appeared desirable, in order to locate accurately this part of the curve. It further appeared desirable to carry out the actual experiments at 25°C, in order to obviate errors introduced by interpolation. The author has accordingly carried out determinations at 25°C. of the vapour pressure of nine mixtures over this concentration range, using an improved dew-point apparatus. Details of the determinations are given in the experimental section. The results obtained are in good agreement with those of Brönsted, Dieterici and Burt. Table II shows the vapour pressures of the mixtures employed by the author, from the experimental results, and by interpolation from the figures of Brönsted and Sorel. The results of Sorel are in every case higher than those of the author, the maximum difference being 0·5 mm.

EXPERIMENTAL.

The dew-point method employed in the determinations was originated by Lescoeur,(15) and improved by Cumming(16). (*See* also McBain and Salmon(17), Bennett(18).)

In preliminary experiments with Cumming's apparatus, difficulties were experienced through leakage. The liability of the rubber stopper to retain moisture introduced a further source of error. It was therefore decided to employ an apparatus made wholly of glass, with the exception of the dew-point tube. This apparatus (Fig. 1) consisted of a spherical bulb *A*, of 100 c.c. capacity, with two side tubes at the neck. The tube on the right led to a manometer *B*, via a cylindrical glass bulb, which facilitated filling and cleaning. The tube on the left connected with the vacuum tap *C*. This tap was designed so that the vapour space, including the tap bore, could be completely immersed in the thermostat tank, thus preventing condensation of vapour. The right-angle bend in the tap bore also minimised gas leakage through the tap. The neck (3 cm. in diameter) of the bulb was ground to fit the tube *D*, which was open at both ends. A silver thimble, identical with that used in Regnault's dew-point hygrometer, was cemented at the base of this tube.

The tube was fitted with a double-bored stopper carrying the thermometer *E* (graduated to 1/5°C.) and the tube *F*, which was designed as a combined gas bubbler and stirrer. All joints were lubricated with rubber-vaseline mixture of moderate viscosity. The apparatus was supported in a retort stand by a special clamp, which enabled it to be lowered into the thermostat tank and held firmly during experiments.

In order to surmount leakage difficulties in the first apparatus due to the cement used, a second apparatus was designed, consisting wholly of glass. This apparatus was brought into use before it was found possible to prevent leakage in the first glass apparatus. The essential feature of the second apparatus consisted in the substitution of a silvered glass tube for the silver thimble used in the first apparatus. Although Regnault, in his description of the dew-point hygrometer, points out the errors involved in Daniell's hygrometer, in which the dew formed on a glass bulb, he does not definitely state that the silver thimble employed in his apparatus is essential to the accuracy of the estimation. It therefore occurred to the author that a silvered glass surface (i.e., a surface on which a thin film of metallic silver has been deposited) might function equally well, provided that the thickness of the glass were not so great as to create a temperature lag from the inner to the outer surface. Thus, in Dines' hygrometer, a thin plate of glass or mica is employed to detect the formation of dew.

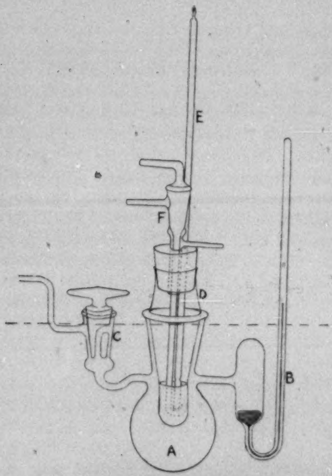

FIG. 1.—FIRST IMPROVED DEW-POINT APPARATUS.

In order to put this idea to the test, experiments were initiated on silvering glass test-tubes, which were substituted for the silver tube in Cumming's apparatus. After a number of trials, a suitable formula was found in which formalin is employed to reduce an ammoniacal solution of silver oxide. The formula quoted(19) was used, omitting the methyl violet specified. Each tube was thoroughly cleaned before silvering by immersion in chromic acid mixture for at least twenty-four hours. The results obtained with soft glass tubes were very satisfactory. The silver deposit took a high polish, superior, in fact, to that obtained with the tube of metallic silver, when rubbed up with chamois leather. The thickness of the glass tubes employed was 0·5 mm.

A form of Cumming's apparatus employed by the author, in which the tube of metallic silver was replaced by a silvered glass tube, was now used to determine the dew-point with a given solution of sulphuric acid. The result obtained, using a pressure of 3 cm. of mercury in the vapour space, was identical (within 0·05°C.) with that obtained using the tube of metallic silver. It appears, therefore, that a silvered glass tube, provided that it is sufficiently thin, may be employed in the place of the silver tube of Regnault's dew-point hygrometer.

The second glass apparatus was similar in design to the first. The manometer (*B* of Fig. 1) was omitted, being replaced by a small U-tube manometer placed loosely inside the cylindrical bulb before evacuation. This arrangement allowed the vacuum to be tested, and at the same time considerably facilitated the cleaning of the apparatus after each determination, besides eliminating the possibility of mercury from the manometer entering the bulb. A cylindrical bulb (capacity 300 c.c.) enabled the silvered tube to be brought close to the observer. This was of some importance, since the apparent size of the silvered tube fell rapidly as the apparatus was moved away from the glass window in the side of the thermostat tank. The other parts of the apparatus were similar to those of the first, except that the section of the silvered tube was enlarged near the ground glass joint; the width of the neck was also increased. These precautions were taken to prevent the silvered tube from becoming contaminated, during removal between each experiment, through touching the greased side of the joint. The movable tap-joint of the bubbler tube (Fig. 1) was replaced by a sealed-in joint. This apparatus was successfully employed in two of the determinations recorded in Table II. The results obtained by its use were in agreement with those obtained using the first glass apparatus.

THE VAPOUR PRESSURES OF SULPHURIC ACID-WATER MIXTURES AT 25°C.

The sulphuric acid-water mixtures employed in these determinations were prepared in each case by admixture of pure sulphuric acid and water, and were standardised by titration against normal sodium carbonate solution. The standardization was carried out by determinations in triplicate, agreement being obtained within a limit of 0·05 per cent. This corresponds, in general, to a vapour pressure difference of 0·03 mm. of mercury.

The two improved forms of apparatus, previously described, were employed, the greater number of the observations being made with the first apparatus. Observation No. 7 (Table II) was made with both forms of apparatus, observation No. 3 with the second apparatus only.

TABLE II.

Solution No.	Percentage of sulphuric acid.	Dew-point °C.	Vapour pressure at 25°C. (mms. of mercury).		
			Author.	Brönsted.	Sorel.
1	41·81	14·50	12·38	12·34	...
2	43·79	13·35	11·49	11·33	11·7
3	45·04	12·40	10·80	10·71	11·0
4	45·59	11·95	10·49	10·38	10·7
5	47·46	10·40	9·46	9·48	9·8
6	49·45	8·70	8·44	8·57	9·0
7	51·45	7·30	7·67	[illegible]	8·2
8	53·24	5·35	6·71	6·85	7·3
9	55·23	3·25	5·79	5·97	6·4

Before use, the apparatus was cleaned by successive treatment with chromic acid, distilled water and alcohol, and finally dried by means of a current of hot air.

5 c.c. of the sulphuric acid-water mixture were run into the bulb, the tube affixed and the apparatus connected to a suitable pump. The pressure in the vapour space was reduced to that of 2 cm. of mercury, and no difficulty was experienced, using the rubber-vaseline lubricant, in maintaining this pressure over a period of 24 hours or more. The evacuated apparatus was supported in a constant temperature tank, which had a capacity of 30-40 litres, and was fitted with a large glass window. It was allowed to remain until equilibrium was reached ; in the case of the sulphuric acid-water mixtures a period of 30 minutes was found to be sufficient for this purpose.

The dew-point tube was half-filled with ether, and air drawn through by means of an aspirator. This method of cooling the tube was found to be very much superior to the hand-blower method recommended by Cumming ; it was also the method originally employed by Regnault in the dew-point hygrometer. The temperature was steadily lowered until a deposit of dew formed which could be clearly seen. The current of air was then caused to be less rapid, so that the temperature as indicated by the thermometer placed in the tube rose slowly. It was noticed in such preliminary observations that super-cooling occurred before deposition of dew, the first dew-point observed being lower than the following ones ; consequently, in all experiments a clearly observable deposit of dew was produced, and allowed to disappear, before taking observations of temperature. Experiment showed that, provided the temperature was not lowered to a point more than 0·5°C. below the dew-point, the point of disappearance of dew, with slow rise of temperature, was the same, whether the dew was deposited at the dew-point or at a temperature slightly below the dew-point. Using a highly-polished tube, it was found possible, within 0·05°C., to obtain the point of appearance of dew with falling temperature in agreement with the point of disappearance of dew with rising temperature. However, the first appearance of dew was difficult to observe, whereas the point of disappearance of dew was more readily perceived, and probably corresponded more closely to the condition that the vapour pressure of the dew on the cooled surface was equal to that of the vapour above the aqueous solution. Consequently, in all experiments, the point of disappearance of dew is quoted as the dew-point.

McBain and Salmon recommend a preliminary immersion of the lower portion of the silver surface in boiling water ; no dew subsequently deposits upon this portion, and the appearance of a line of division upon the bright surface is claimed to facilitate the observation of dew-point. These authors consider that the procedure mentioned is essential to the accuracy of determinations by the dew-point method. For the reasons given above, however, such preliminary treatment was judged to be unnecessary, and was therefore omitted in the author's experiments.

In every determination the observation of dew-point was carried out at least three times, allowing a period of 15 minutes to elapse between each observation for equilibrium to be re-established, and the determination was not accepted unless the three results agreed within 0·05°C. The results are shown in Table II. They have previously been discussed (page 255).

THE VAPOUR PRESSURE OF WATER OVER SATURATED SOLUTIONS OF SODIUM CHLORIDE.

An independent comparison of results yielded by the dew-point and tensimetric methods appeared to be desirable ; the author has therefore examined that part of Landolt-Börnstein's tables which refers to aqueous solutions, with the object of finding a modern reliable determination of the vapour pressure of water over a salt solution which might readily be reproduced. The work of Speranski,(20) on the vapour pressure of water over saturated solutions of salts appears to attain a high degreee of accuracy in tensimetric measurements. The author has interpolated Speranski's figures for the vapour pressures of water over saturated solutions of sodium chloride, at 20°C., 25°C. and 30°C., and has redetermined these figures, using the first modified dew-point apparatus previously described. The results are given in Table III.

TABLE III.

Temperature °C.	Dew-point °C.	Vapour pressure in mms. of mercury.	
		Author.	Speranski.
20	15·65	13·34	13·28
25	20·20	17·75	17·86
30	25·10	23·90	23·88

They indicate that the dew-point method is capable of yielding results in close agreement with those obtained by tensimetric measurements.

The glass apparatus employed was made by Messrs. Gallenkamp. The author desires to thank Dr. T. J. Drakeley, Head of the Department of Chemistry and Rubber Technology, Northern Polytechnic, for facilities provided.

REFERENCES.

(1) Wilson, J. Ind. Eng. Chem., 13, 326 (1921).
(2) Regnault, Ann. Chim. Phys., (iii), 15, 179 (1845).
(3) Helmholtz, Wied. Ann., 27, 532 (1886).
(4) Sorel, Zeit. Angew. Chem., 272 (1889).
(5) Porter, Trans. Faraday Soc., 13, 373 (1918).
(6) Dieterici, Wied. Ann., 50, 60 (1893) ; 62, 616 (1897).
(7) Smits, Arch. Neerl. (ii), 1, 97 (1898) ; Zeit. Phys. Chem., 39, 385 (1902).
(8) Briggs, Journ. Soc. Chem. Ind., 22, 1275 (1903).
(9) Burt, Journ. Chem. Soc., 85, 1339 (1904).
(10) Brönsted, Zeit. Phys. Chem., 68, 708 (1910).
(11) Hacker, Wied. Ann. (iv), 39, 1342 (1912).
(12) Daudt, Zeit. Phys. Chem., 106, 255 (1923).
(13) Grollmann and Fraser, Journ. Amer. Chem. Soc., 47, 712 (1925).
(14) McHaffie, Journ. Chem. Soc., 112 (1927).
(15) Lescoeur, Ann. Chim. Phys. (vi), 16, 378 (1889) ; 19, 533 (1890).
(16) Cumming, Journ. Chem. Soc., 95, 1772 (1909).
(17) McBain and Salmon, Proc. Roy. Soc., 97, A44 (1920).
(18) Bennett, Journ. Phys. Chem., 31, 890 (1927).
(19) Brit. Journ. Phot., March 28, 1919.
(20) Speranski, Zeit. Phys. Chem., 70, 521 (1910).

DISCUSSION

Dr. L. F Richardson asked whether the percentages given in the paper represented the ratio of solute to solvent or of solute to solution.

Mr. J. H. Coste said that the tables given by the author were most valuable since a good deal of industrial research is being conducted on the effects of moisture on various substances, and it is important to know to what surface tension they are being exposed.

The Author replied that a good deal of confusion existed in the literature between the two percentages referred to by Dr. Richardson. The percentages in the paper represented the ratio of the mass of sulphuric acid to mass of solution.

Capt. G. I. Finch said that the definition of percentage used by the author represented the best chemical practice and it was important that it should be generally adopted.

XXXIV.—THE STABILITY OF A VALVE AMPLIFIER WITH TUNED CIRCUITS AND INTERNAL REACTION.

By R. T. Beatty, *M.A.*, *B.E.*, *D.Sc.*

Received May 19, 1928.

Abstract.

The paper gives algebraic and graphical methods for determining the conditions of stability for a tuned multi-valve amplifier.

I. Introduction.

The stability of a valve amplifier is considered in this paper. The input, output, and intervalve circuits are tuned; the only mutual action between the circuits is that due to the capacity between the grid and the plate electrodes of each valve. It is found that the input grid admittance can be represented as a continued fraction with complex numbers from which a geometrical construction is derived which gives the input admittance at each stage. From this construction the *minimum* input conductance is derived in the form of a continued fraction with real numbers which can be summed stage by stage. In the special case of a *repeating amplifier*, the summation can be effected in general terms, giving the stability in terms of the valve constants and the repeating circuits in relation to the output load. When the output load is identical with the repeating circuits the formula is identical with that due to K. Posthumus,* but differs from that given by Hull† in that the phase angle of the repeating circuits at the limit of stability is found to be —45° instead of zero.

List of Symbols in Practical Units.

ω	$2\pi\times$frequency in cycles/sec.		
C_0	Grid-plate capacity of valve.		
a_x	Reciprocal of valve differential resistance.		
a_0	Grid-plate admittance of valve.		
a_1	Output admittance.		
a_n	Admittance of circuit between valves n and $n-1$.		
$a_{g\,.\,n+1}$	Total grid input admittance at valve n.		
g	Mutual conductance of valve.		
$\sigma_{g\,.\,n+1}$	Total grid input conductance at valve n.		
σ_{n+1} t_{n+1}	conductance of	tangent of phase angle of	Equivalent input circuit.
σ_n t_n			Equivalent circuit between values n and $n-1$.
σ_1 t_1			Equivalent output circuit.
σ_0 —			Grid-plate capacity.
T_n	$1+j\,.\,t_n$		$(n+1)$ . . . 1
R	H.F. resistance of a coil.		
L	Inductance of a coil.		
A	$a_0(g-a_0)$.		
$a_{n+1\,.\,n}$	$C_0\omega : g/2\sigma_{n+1}\cdot\sigma_n$.		
a_n	$C_0\omega\,.\,g/2\sigma^2{}_n$.		
β	$\sqrt{1-4a}$.		
η	$\text{Tan}^{-1}\,\sigma_0/C_0\omega$.		

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* Posthumus, Tydschrift van het Nederlandsch. Radiogenootschap, 3, pp. 106-112.

† Hull, Phys. Rev., 27, pp. 432-454 (1926).

II. The Input Admittance as an Operational Continued Fraction.

In a multi-stage amplifier (Fig. 1) each valve is represented as a generator with an admittance a_x in series; a_0 is the grid-plate admittance; a_1 is the output admittance, and includes the plate-filament admittance of the valve; a_2 a_3 . . . are the admittances of the networks between valves 1 and 2, 2 and 3 . . ., and include the grid filament admittance in each case.

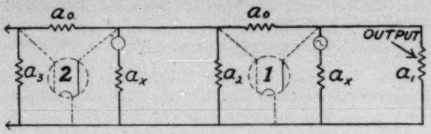

Fig. 1.

The plate load a_1 on valve 1 can be expressed as an equivalent grid input load on valve 1 by the method due to Miller* in the form†

$$a_0+A/(a_1+a_0+a_x),$$

where

$$A=a_0(g-a_0).$$

Hence the total grid load on valve 1 is

$$a_2+a_0+A/(a_1+a_0+a_x),$$

and may also be considered as the plate load on valve 2; accordingly the equivalent grid input load on valve 2 is

$$a_0+\frac{A}{(a_2+2a_0+a_x)+}\quad\frac{A}{(a_1+a_0+a_x)}.$$

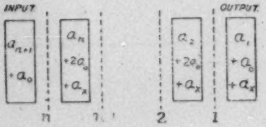

Fig. 2.

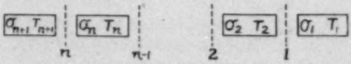

Fig. 3.

Hence the total grid input load at the grid of valve n is

$$a_{g.n+1}=(a_{n+1}+a_0)+$$

$$\frac{A}{(a_n+2a_0+a_x)+}\quad\frac{A}{(a_{n-1}+2a_0+a_x)+}\cdots\frac{A}{(a_2+2a_0+a_x)+}\quad\frac{A}{(a_1+a_0+a_x)}\;. \qquad (1)$$

where a_{n+1} is the admittance of the network prefixed to valve n, and includes the grid filament admittance. The scheme in Fig. 1 may now be replaced by a set of equivalent circuits,† shown in Fig. 2, from which the equivalent total grid input at any valve can be found by the operations indicated in equation (1).

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* **Miller, Bull. Bur. Stand., 15, p. 367 (1919).**
† **Beatty, Phil. Mag., IV., p. 1085 (1927).**

Since an admittance may be written as $\sigma[1+j\,.\,t]=\sigma T$, where σ is the conductance, t is the tangent of the phase angle, and T is evidently an operator, we may put

$$\begin{aligned} a_{n+1}+a_0&=\sigma_{n+1}\,.\,T_{n+1}\\ a_n+2a_0+a_x&=\sigma_n\,.\,T_n\\ \vdots\quad&\quad\vdots\\ a_2+2a_0+a_x&=\sigma_2\,.\,T_2\\ a_1+a_0+a_x&=\sigma_1\,.\,T_1, \end{aligned}$$

which gives a simpler notation for the equivalent circuits (Fig. 3). Equation (1) now becomes

$$\sigma_{n+1}\,.\,T_{n+1}+\frac{A}{\sigma_n\,.\,T_n+}\quad\frac{A}{\sigma_{n-1}\,.\,T_{n-1}+}\cdots\frac{A}{\sigma_2\,.\,T_2+}\quad\frac{A}{\sigma_1\,.\,T_1}.\quad.\quad.\quad(2)$$

If we neglect a_0 in comparison with g, and assume that the grid-plate condenser has zero power factor we can write the approximate expression $A=j\,.\,C_0\omega\,.\,g$. On putting $2a_{21}=C_0\omega\,.\,g/\sigma_2\,.\,\sigma_1\,.\,.\,.$ we may rewrite equation (2) in the form

$$a_{g\,.\,n+1}/\sigma_{n+1}=T_{n+1}+\frac{j\,.\,2a_{n+1\,.\,n}}{T_n+}\quad\frac{j\,.\,2a_{n\,.\,n-1}}{T_{n-1}+}\cdots\frac{j\,.\,2a_{32}}{T_2+}\quad\frac{j\,.\,2a_{21}}{T_1}\quad.\quad(3)$$

III. GRAPHICAL REPRESENTATION OF THE INPUT ADMITTANCE.

T_1 is represented as a vector with one end moving along the vertical line in Fig. 4; the operation $j\,.\,2a_{21}/T_1$ is equivalent to $j\,.\,2a_{21}\,.\cos\theta_1\,.\,e^{-j\theta_1}$, where $\theta_1=\tan^{-1}t_1$, and this gives the thick line vector in Fig. 5, whose end moves on a circle of radius a_{21}. The operation is equivalent to inverting the vertical line t_1, and reflecting the circle so obtained by a line making an angle of 45° with the horizontal.

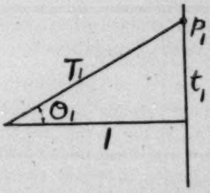

FIG. 4.

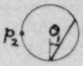

FIG. 5.

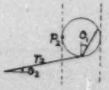

FIG. 6.

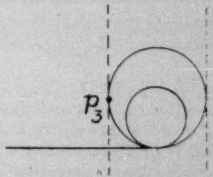

FIG. 7.

The vectorial addition of T_2 is shown in Fig. 6. It is seen that $T_2+j\,.\,2a_{21}/T_1$ is represented by a vector with one end fixed, while the other moves on the circle. As $T_2=1+jt_2$ varies, owing to variation of t_2, the circle suffers a vertical displacement, so that a doubly infinite set of vectors exists; each such vector terminates in the domain enclosed between the vertical dotted lines in Fig. 6.* The operation

$$\frac{j\,.\,2a_{32}}{T_2+}\quad\frac{j\,.\,2a_{21}}{T_1}$$

is performed in the same way as Fig. 5 is derived from Fig. 4, except that since the

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*** This is only true for small variations of ω, such as take place with tuned circuits, otherwise the diameter of the circle will vary owing to the occurrence of the quantity $C_0\omega$ in the expression for α.**

domain between the dotted lines in Fig. 6 may be considered as composed of an infinite number of vertical lines each giving rise to a circle on inversion, an infinite number of coaxial circles will result. The limiting circles of the system are shown in Fig. 7, the radii are $a_{32}/(1-a_{21})$ and $a_{32}/(1+a_{21})$.

The geometrical interpretation of the subsequent stages is now obvious, and the stages are shown in Fig. 8. If the half breadth of any domain should exceed unity the input load may become a negative conductance, self-oscillation would ensue, and the construction would fail beyond this point. The minimum input conductance at the end of the chain is σ_{n+1} times the horizontal thick line in Fig. 8,

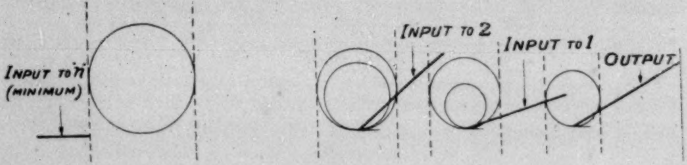

FIG. 8.

whose length is unity minus the half breadth of domain n—i.e.,

$$\sigma_{g\,.\,n+1}/\sigma_{n+1}=1-\frac{a_{n+1\,.\,n}}{1-}\;\frac{a_{n\,.\,n-1}}{1-}\;\cdots\;\frac{a_{32}}{(1-a_{21})}, \qquad (4)$$

where all the terms are real. The value of $\sigma_{g\,.\,n+1}$ can thus be obtained by simple arithmetic in any particular case, and the amplifier is stable if $\sigma_{g\,.\,n+1}$ is positive.

IV. CONDITIONS FOR STABILITY WHEN THE LINKS OF THE CHAIN ARE EQUAL.

The case when all the valves and intervalve circuits are similar is of special interest; we put all the a's equal except the output term a_{21}, so that $2a=C_0\omega\,.\,g/\sigma^2$ and $2a_{21}=C_0\omega\,.\,g/\sigma\,.\,\sigma_1$ where σ is the conductance of any equivalent circuit lying between two consecutive valves, and σ_1 refers to the equivalent output circuit. Equation (4) now becomes

$$\sigma_{g\,.\,n+1}/\sigma=P/Q, \qquad (5)$$

where

$$P=\frac{(1+\beta)^{n+1}-(1-\beta)^{n+1}}{2^{n+1}\,.\,\beta}-a_{21}\left[\frac{(1+\beta)^{n}-(1-\beta)^{n}}{2^{n}\,.\,\beta}\right], \qquad (6)$$

and

$$\beta=\sqrt{1-4a}.$$

Q is of the same form as P, but with each power diminished by unity.

The relation between a and a_{21} to make $P=0$ is found by help of an auxiliary angle φ to be

$$\left.\begin{aligned}&\tanh\varphi=\sqrt{1-4a},\ \text{when } a<\tfrac{1}{4}\\&\tanh\varphi/\tanh n\varphi=2a_{21}-1.\end{aligned}\right\} \qquad (7)$$

The corresponding relation when $a>\frac{1}{4}$ is

$$\left.\begin{aligned}&\tan\varphi=\sqrt{4a-1},\\&\tan\varphi/\tan n\varphi=2a_{21}-1\end{aligned}\right\} \qquad (8)$$

The use of such terms as $\alpha_{n+1\,.\,n}$ each of which contains the product of the conductances of two consecutive circuits has simplified the notation, but physically terms of the form $\alpha_n=C_0\omega\,.\,g/\sigma_n{}^2$, each of which involves a single circuit, are more intelligible. We have $\alpha^2{}_{n+1\,.\,n}=\alpha_{n+1}\,.\,\alpha_n$, and the output term α_1 is given by $\alpha_{21}{}^2=\alpha\,.\,\alpha_1$ in the case where all the intervalve α's are equal.

Fig. 9 shows the relation between α and α_1 according to equations (7) and (8)

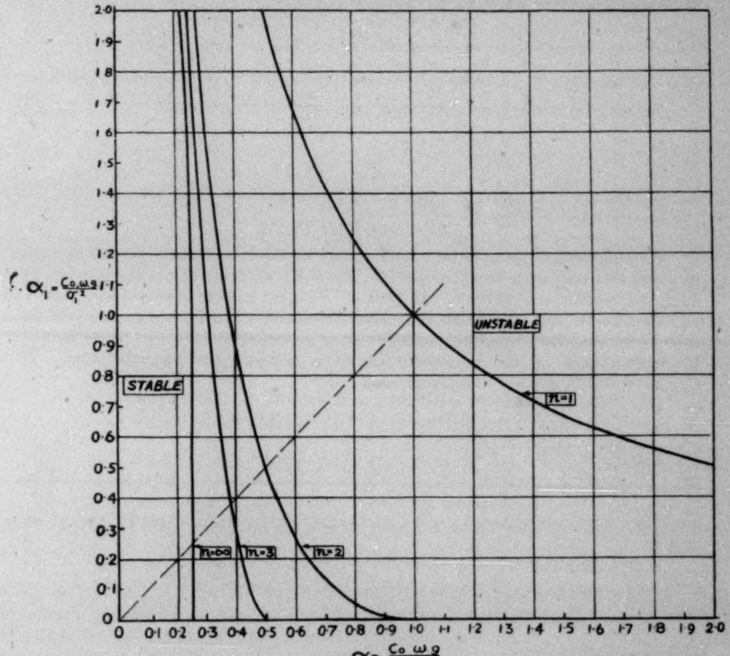

Fig. 9.

for an amplifier on the verge of instability. Curves are drawn for a one- two- three- and infinity-stage amplifier. For any given output load the critical value of α decreases rapidly at first as the number of stages increases and then more slowly. If

$$\alpha_1 < 1,\ \alpha = \tfrac{1}{4} \text{ for } n = \infty.$$

In the special case where $\alpha_1=\alpha$ Posthumus* has obtained a solution in the form

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* Loc. cit.

$4a=\sec.^2\frac{\pi}{n+2}$, a result which follows easily from equations (8) of the present paper. Posthumus starts from the difference equation connecting the three potentials at the plates of three consecutive valves and expresses the result as a continuant which is formally equivalent to the continued fraction in equation (4). The values of a for various values of n are given in the following table and correspond to the intersection of the dotted line with the curves in Fig. 9.

TABLE.

n	$a(=a_1)$
1	1
2	0·500
3	0·382
4	0·333
⋮	⋮
∞	0·250

V. DISCUSSION OF FIG. 9.

As a numerical example take an amplifier in which each tuned circuit consists of a coil and condenser tuned to $\omega=10^7$: the H.F. resistance of the coil is taken as $42\,\Omega$ and the reactance of the coil as $1000\,\Omega$. Then the coil conductance is $R/L^2\omega^2=42\times10^{-6}$ mhos, and on adding the valve differential conductance assumed to be $0{\cdot}8\times10^{-5}$ mhos (the grid-plate condenser being supposed to have zero power factor) the conductance of the equivalent circuit is $(4{\cdot}2+0{\cdot}8)10^{-5}=5\times10^{-5}$ mhos. If $C_0=10^{-13}$ farads, $g=10^{-3}$ amps/volt, then

$$a=10^{-13}\times10^7\times10^{-3}/(5\times10^{-5})^2=0{\cdot}4.$$

Fig 9 shows that if $a_1=1$ the system will be stable for one valve but unstable for two; if $a_1=0{\cdot}6$ a two-stage amplifier will be stable: if $a_1=0{\cdot}2$ three stages will be stable. Since the output circuit is necessarily more heavily damped than the intermediate circuits we may put $a_1<0{\cdot}4$, so that stability is to be expected with three stages.

If we take a better coil in which $L\omega=1000\,\Omega, L\omega/R=200$, and a valve in which $g=5\times10^{-4}$, $a_x=0{\cdot}8\times10^{-5}$ and in which C_0 has been reduced to $0{\cdot}2\times10^{-13}$ farads,

$$a=0{\cdot}2\times10^{-13}\times10^7\times5\times10^{-4}/(1{\cdot}3\times10^{-5})^2=10^{-10}/1{\cdot}69\times10^{-10}=0{\cdot}59$$

so that stability is possible for two stages only.

VI. RESULT WHEN THE POWER FACTOR OF C_0 IS APPRECIABLE.

If σ_0 is not negligible so that $a_0=\sigma_0+j\,.\,C_0\omega$ we find

$$A=j\,.\,C_0\omega\,.\,g\,.\,\sec\eta\,.\,e^{-j\eta},$$

where $\tan\eta=\sigma_0/C_0\omega$,

and consequently

$$2a_{21}=C_0\omega\,.\,g\,.\,\sec\eta\,.\,e^{-j\eta}/\sigma_2\,.\,\sigma_1.$$

The result is that any circle as in Fig. 10 is rotated clockwise through an angle η

while its diameter is multiplied by sec η and the distance pq becomes $(1-C_0\omega \cdot g \cdot \sec\eta(1-\sin\eta)/\sigma_2 \cdot \sigma_1)$. Under these conditions we must write in equation (4) $a \cdot \sec\eta(1-\sin\eta)$ instead of a. This value is always smaller than a so that the stability is increased by the introduction of σ_0.

For example, if $C_0=10^{-13}$ farad : $\omega=10^7$ radians/sec : $\sigma_0=10^{-6}$ mhos.

$$\tan\eta=\sigma_0/C_0\omega=10^{-6}/10^{-13}\times10^{7}=1$$

$$\therefore \qquad \sec\eta(1-\sin\eta)=(\sqrt{2}-1)=0{\cdot}414.$$

Hence the condition for the stability of an infinite number of stages given by the bottom number in the table becomes $=\dfrac{0{\cdot}250}{0{\cdot}414}=0{\cdot}605$. In general the introduction of σ_0 increases stability with a corresponding decrease of amplification.

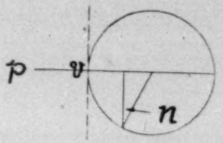

FIG. 10.

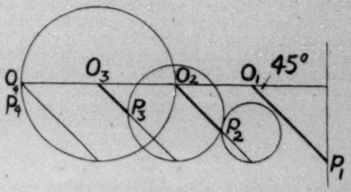

FIG. 11.

VII. PHASE ANGLE ON THE THRESHOLD OF INSTABILITY.

In order that the amplifier should be on the threshold of instability the first condition is that the half breadth of the n^{th} domain should be unity : this condition does not fix the magnitude of the phase angles of the circuits. The second condition is that no thick line vector in Fig. 8 should penetrate into its corresponding domain, for if such penetration should occur at any stage it is evident from the method of formation of these vectors as given in Section III that all following vectors including the final one would also penetrate so that the final vector could not become zero. It may be seen from Figs. 4 to 7 that the point p_2 in Figs. 5 and 6 can only be obtained from p_1, in Fig. 4 if $\theta_1=-45$ deg. and that p_3 can only be obtained from p_2 if $\theta_2=-45$deg. Hence all phase angles must be -45 deg., that is, all circuits must be tuned to be inductive to the incoming signal.

Fig. 11 represents three stages on the verge of instability. The consecutive vectors are O_1P_1, O_2P_2, O_3P_3, and O_4P_4 which is zero. The radii of the circles are 0·382, 0·618, 1, and are calculated from the assumption that $a=a_1=0{\cdot}382$.

DISCUSSION.

PROFESSOR C. L. FORTESCUE : The elements of the amplifier as shewn in Fig. 1 are the same as those assumed by other workers and it is to be expected that the result given in equation (3) should be substantially the same as that obtained by others who have worked out the algebra of the circuits. Where Dr. Beatty has advanced our knowledge particularly is in the elegant graphical treatment given in Section IV of the Paper. But in this connection it is important to notice that the method is strictly limited to tuned-circuit amplifiers where the range of frequency under consideration is small. The graphical construction gives the limiting conditions of stability and enables the frequency of the oscillations, where they break out, to be ascertained, but it does not give any indication of the voltage amplitudes at frequencies near that of the self-maintained oscillation. This is the important point in practice and it would be interesting to hear from Dr. Beatty whether the graphical construction can be adapted for this purpose with reasonable ease.

AUTHOR'S reply: No novelty is claimed for the relation given by equation (3) as regards its contents, but care has been taken to put it in a compact form which leads naturally to the subsequent graphical development; the use of admittance symbols instead of impedance symbols leads in this investigation, as in many others, to simple results. The analysis could have been performed in a different way by writing down a series of difference equations, each equation involving the potentials at the plates of three consecutive valves, and forming a continuant by elimination of these potentials; but the graphical method has the advantage of keeping the physics of the amplifier in view throughout the analysis. Moreover, it is possible that self-oscillations may occur in the output stages of an amplifier, while the early stages are stable. This condition would be indicated by the graphical method, which tests the stability step by step, but the continuant would indicate a stable system.

As regards the phase angles required for maximum voltage amplification, I doubt whether a generalised graphical construction can be given. I have given the result for two stages (Phil. Mag., IV, p. 1081), but for three stages the problem involves the maximum ratio between two lines drawn from a fixed point to a parabola, and in this form is incapable of geometrical solution.

XXXV.—THE SCATTERING OF LIGHT BY PARTICLES OF METALLIC OXIDES DISPERSED IN DRY AIR.

By T. C. Nugent, *M.Sc.*, and H. P. Walmsley, *M.Sc.*

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Abstract.

A cursory examination has been made of the time changes in the brightness of the tyndall-light from ZnO, CuO, CdO and As_2O_3 clouds. The changes found can be accounted for by supposing that the cloud particles disappear by aggregation and sedimentation. The curves obtained fluctuate in a similar way to the ionisation currents from the same cloud, as if aggregation produced unstable complex particles which periodically break up in large numbers. Evidence has been obtained that amicroscopic particles of cadmium oxide act as nuclei and collect cadmium oxide vapour during a condensation process. A series of clouds vaporised from various masses (m) of arsenic trioxide has been obtained in which the initial brightness of the tyndall-beam was approximately proportional to $m^{2/3}$. In certain clouds, the rapid fluctuations in brightness appear to form groups which succeed one another at intervals given by the formula $t=Ar^n$, where A and r are constants and n is an integer.

Theoretical.—I.

THE equations for the intensity of the light scattered by a spherical particle when placed in the path of a plane wave have been solved by Mie(1). His formulæ have been used by Shoulejkin(2) to compute numerical values for the scattering when the particle is an ideal dielectric of refractive index 1·32. Shoulejkin finds that as the diameter d of the particle increases there is a smooth continuous change from the scattering of light according to Rayleigh's law to its reflection and refraction as given by Fresnel's equations. When $d \ngtr \lambda/3$, where λ is the wavelength of the incident light, the scattered energy closely obeys Rayleigh's law, and at right angles to the incident beam the scattered light is practically plane polarised. As d increases the polarisation becomes less complete. For values of d greater than the general character of the distribution of dispersed energy does not change greatly. It approximates closely to that given by the ordinary laws of reflection and refraction. The results for $d \ngtr \lambda/3$ seem in close accord with Mecklenburg's(3) experiments with sulphur sols, particularly for $5\mu\mu < d < 100\mu\mu$.

Thus, if a quantity of matter C is dispersed into n spherical particles, and $d \ngtr \lambda/3$, the intensity I of the laterally scattered light varies as nd^6. But C varies as nd^3, whence

$$I=RCd^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

At the other limit, the total surface of the particles varies as nd^2, so, as a surface reflection effect

$$I=R'C/d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where R and R' are constants.(4)

Tolman(5) finds with silica suspensions that (2) is closely obeyed down to $d=997\mu\mu$.

It seems legitimate to assume from the above that the relation between I and d when C is constant will be represented by a smooth curve, approximately given by (1) and (2), at the limits, and possessing a single maximum roughly near $d=\lambda$. This shows that if a cloud of uniform spherical primaries coagulates, the secondaries also being spherical, the intensity of the tyndall-beam will alter with time (t). The $I-t$ curve will depend upon the law of coagulation, but it seems reasonable to suppose that when the primaries are very small it will be somewhat similar in shape to the previous curve, and that in practice we should obtain the whole or part of this curve according to the size of the initial particles.

The experimental curves to be described will be interpreted on the assumption that their theoretical $I-t$ curves, when coagulation occurs, are of this shape, although X-ray analysis has shown that the particles employed are crystalline. Since there seems nothing to hinder their formation from vapour, they are probably faceted—i.e., we are dealing with tiny crystals and crystal aggregates. It is further assumed that as coagulation proceeds the fraction of light which is polarised decreases. The initial polarisation for spherical primaries of $d \ll \lambda/3$ will be almost perfect ; but, judging by Gans [(6)] results for ellipsoidal particles, it is to be expected in our case that, even when $d \ll \lambda/3$, the polarisation will only be partial.

Anticipating the experimental results, the $I-t$ curves observed conform with these assumptions. It is found, however, that they are not smooth, but fluctuate[(7)]. In many cases the magnitude of the fluctuations seems too great to be attributed to observational error. Suppose coagulation is causing a general diminution in scattering. A subsequent increase in brightness must be due to an increase in the number of particles in the element of cloud (about 1 c.c.) on which observations are made. If the cloud is homogeneous in the sense that the element is representative of the cloud, this must arise from the disintegration of unstable aggregates. If, however, it is due to heterogeneity, resulting perhaps from imperfect mixing, little change should occur in the percentage of polarised light which is scattered. As this depends on the degree of dispersion, it should however increase if disruption of aggregates takes place. When, therefore, the brightness and the polarisation fluctuate collaterally, the effect has been interpreted as indicating that the increases in the number of particles from disruption periodically outnumber the decreases due to aggregation.

EXPERIMENTAL.—I.

The apparatus employed for dispersing the clouds and for measuring the ionization currents from them has been described previously[(10)]. The photometric measurements were made upon a sample of the cloud which was withdrawn from the 850 litre cistern, and introduced into a slightly modified form of the tyndallmeter described by Tolman and Vliet[(8)], the brightness of the scattered light being measured with a Macbeth illuminometer.

The light scattered laterally by clouds dispersed by arcing between metallic electrodes was found to be coloured. That from copper electrodes was deep blue, zinc gave a bluish green colour, whilst lead cadmium and tin gave bright green clouds. The light from a cloud produced from cadmium when examined with a spectroscope showed a continuous spectrum from the blue to the green. The yellow and red end of the spectrum seemed totally absent. Although the blue colour is similar to that found by Tyndall with clouds produced by photo-chemical reactions, the clouds behave differently on ageing. Tyndall[(9)] found that his clouds altered with time, and ultimately became white. With the clouds from the arc there was no marked *visible* change in colour with time. The light was found to be partially polarised, the plane of polarisation containing the incident ray and the line of sight. For the photometric measurements, a nicol prism was attached in front of the eyepiece of the illuminometer, and the intensities of the maximum and minimum transmissions were measured throughout the experiments (curves *A* and *B* in the graphs).

In taking the observations it was found that if, at a given time, the fields of the Lummer-Brodhun " cube " were adjusted for equality of brightness and left, the brightness of the tyndall-light fluctuated fairly rapidly about the standard. This fluctuation seems superposed upon the general change in brightness as the cloud ages. As it was evident that all the changes in brightness could not be recorded, the following procedure was adopted : Five settings for the stronger component were read alternately with five for the weaker. The times of the first and last reading of each set of ten were noted. Their mean was taken as the " time of observation," and the means of each set of five readings were taken as the values of the components at the time of observation. This method of reducing the observations tends to eliminate the more rapid changes in the light values.

Since the period (approx. 2 mins.) occupied in recording the ten settings is approximately equal to that required to make a measurement of the ionization current, it makes the results more comparable with the conductivity measurements. The point is illustrated by Fig. 5, where an attempt has been made to plot the individual readings for the stronger component of a cadmium oxide cloud. The readings are joined by straight lines. The smooth curve passes through the means of sets of five observations. The former curve more closely represents the actual changes in brightness which occurred than the latter, which gives only the general trend of the phenomena.

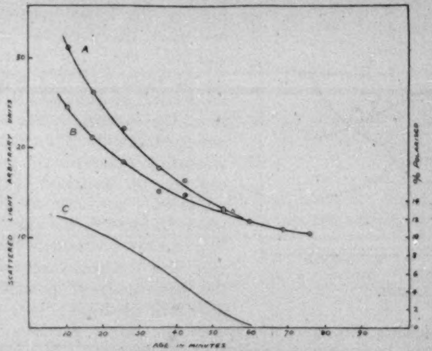

FIG. 1.—ZINC OXIDE.

The results of typical experiments on clouds dispersed from an electric arc are shown in Figs. 1 to 6. In no case was the scattered light completely plane polarised, as it should be if the scattering conformed with Rayleigh's formula. Thus the diameters of the cloud particles are either not extremely small compared with the wavelength of the incident light or they are not spherical in form. The colour of the scattered light shows that very few of the particles can have diameters greater than the wavelength of light, since they are evidently not sufficiently numerous to reflect from their surfaces the colour characteristic of the disperse phase in bulk. The colour and polarisation of the scattered light, therefore, show that the clouds consist mainly of ultramicroscopic particles.

ZINC OXIDE CLOUDS.

Fig. 1 shows the curves which were obtained from a comparatively dense cloud from a zinc arc. Since the curves ultimately unite, the polarised light ultimately disappears; showing that the average size of the particles has increased, and hence that aggregation has taken place. The decrease in *A* and *B* with age indicates, therefore, a gradual decrease in the number of particles per unit volume which must be due to loss by aggregation or sedimentation.

Fig. 2 is due to a weaker cloud of zinc oxide particles produced by a silent arc The general shape of the curves *A* and *B* is what one would expect from the coagulation of particles whose initial dimensions are small compared with the wavelength of light. The average size of the particles should be increasing whilst the scattering is passing through the maximum. This seems the case, for the percentage polarised diminishes steadily during the first 25 minutes. Sedimentation and further aggregation will account for the subsequent loss shown by *A* and *B*.

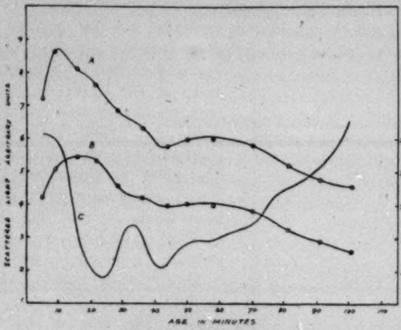

FIG. 2.—ZINC OXIDE.

Sedimentation should have no effect on the polarisation of the scattered light, unless it alters the composition of the cloud. When particles of varying sizes are present, the larger particles are more likely to be precipitated than the smaller. As the latter scatter a larger proportion of polarised light than the former, continued sedimentation in such a cloud will tend to increase the percentage of polarised light scattered. So long as the effect of growth in size by aggregation is greater than that due to sedimentation, the percentage of polarised light will diminish; but, as the rate of aggregation diminishes with age, the loss of the larger particles by sedimentation may predominate, and cause the percentage to increase. The joint effects of continuous coagulation and sedimentation will in general produce a smooth curve for the percentage possessing a minimum. Neglecting the fluctuations, this is the general shape shown by curve *C*.

COPPER OXIDE.

Fig. 3 shows the curves obtained from a cloud produced by passing a current of 5 amperes across a copper arc for 15 minutes. The percentage of polarised light is considerably greater than before. The variations with time of the total scattering and the fraction polarised are oscillatory, the maxima for each curve occurring approximately simultaneously. Apparently at these times a reversal of the process of aggregation is taking place—i.e., the aggregates in the cloud are breaking up at a rate greater than their rate of production. The high percentage of polarised light points to a high degree of dispersion in the cloud, and this seems supported by the relatively slight fall of the percentage on the whole. The much more rapid decrease in the total scattered light would indicate that the loss of particles is very largely due to sedimentation.

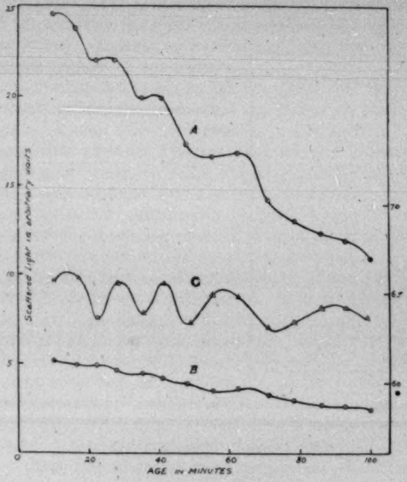

FIG. 3.—CUPRIC OXIDE.

CADMIUM OXIDE.

Most of our experiments were carried out on clouds from cadmium electrodes, and Fig. 4 gives a set of curves frequently obtained. The scattered light is green, and shows small fluctuations as its intensity diminishes with age, the most marked in this cloud occurring after 15 minutes. From the colour and high percentage of polarised light, the degree of dispersion seems fairly high. The general fall in intensity with age indicates the loss in number of the particles by coagulation and settling, and the fluctuating percentage seems to indicate that unstable aggregates are formed during the coagulation process, which break up periodically in large numbers. Fig. 5 shows for the early stages of this cloud the actual observations and the means from which Fig. 4 is drawn. The ionization current *D* was obtained with 400 volts on the collecting condenser. The rapid fall to 15 minutes is due mainly to recombination. The subsequent rise is due to the production of charges in the cloud. It will be noticed that as the percentage curve passes through maxima and minima, collateral changes occur in the current. From electrical considerations, the increases in current strength have been attributed to the

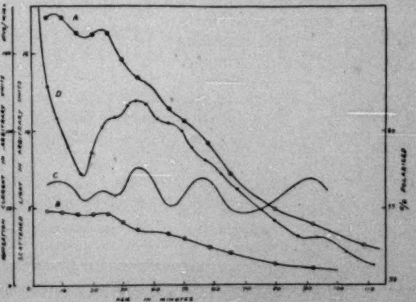

FIG. 4.—CADMIUM OXIDE.

disruption of aggregates in the cloud.(10) The explanation offered for the increases in the polarisation is the same, so the curves seem corroborative.

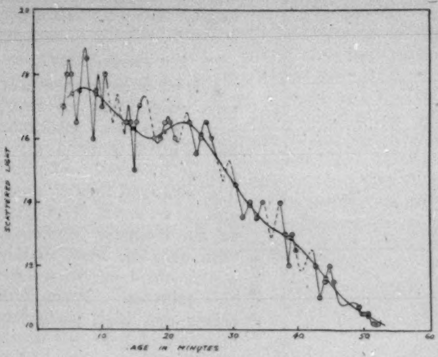

FIG. 5.—CADMIUM OXIDE.

Fig. 6 shows the behaviour of another cadmium oxide cloud. The initial distribution of number and size amongst the particles makes the total scattering and the percentage polarised roughly the same here as in Fig. 4, but the percentage, instead of remaining fairly constant, falls rapidly and vanishes after 50 minutes. At 20 minutes the total scattering is again equal for both clouds, but the percentage polarised is considerably less in Fig. 6. At this time the average size of the particles is therefore greater in Fig. 6 than in Fig. 4, so their number must be less. Again, the greater rate of fall in the polarisation in Fig. 6 shows that coalescence of the responsible particles has been more rapid than in Fig. 4—i.e., their collision frequency has been greater. From the initial similarity of the clouds, the greater average size of the particles at 20 mins. in Fig. 6 cannot be due to the aggregation of larger primary particles. It must arise from a greater initial volume density of particles. These scatter slightly less total light than the corresponding particles of Fig. 4, so their average size must be smaller. Such particles alone would scatter a higher percentage of polarised light than those in Fig. 4. Hence, in Fig. 6, we must have initially a number of much larger particles also, the less perfect polarisation from which counterbalances the more perfect polarisation from the larger number of smaller ones. The distribution of size amongst the primary particles of Fig. 6 is therefore more heterogeneous than in Fig. 4. There were more particles dispersed and the mass was distributed between a number of comparatively large particles and a much larger number of comparatively small ones.

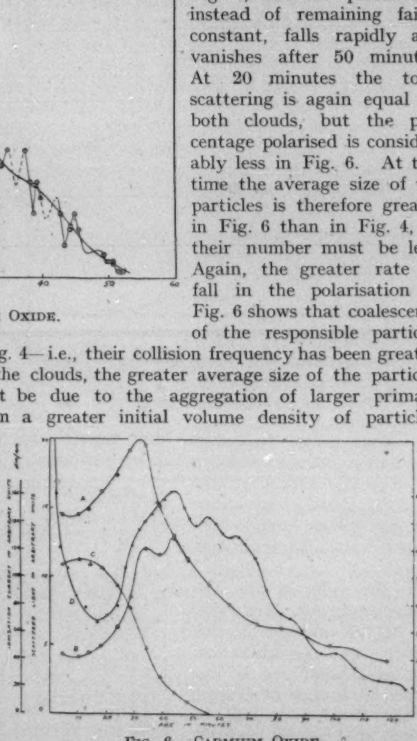

FIG. 6.—CADMIUM OXIDE.

This seems to account for the difference in behaviour. The small particles tend to aggregate with the large ones rather than with one another, and this tendency increases as their number diminishes. This accounts for the rapid loss in the polarised light. The continuous growth of the large particles at the expense of the small increases their individual scattering power; so the total brightness increases until this effect is counterbalanced by the loss due to their aggregation and sedimentation.

The curves of Fig. 6 do not show well marked fluctuations, and these only appear after the maximum. The ionization curve shows rapid fluctuations before the maximum, and these continue subsequently. Prior to the maximum the dismembered particles do not greatly affect the scattering; which conforms with the view that disruption is the exact reverse process of aggregation, the primary particles remaining as distinct entities throughout the life of the cloud.

The initial distribution of mass amongst the primary particles in Fig. 6 is interesting, because the cloud, instead of being produced in dust-free air, was dispersed in the residues from a cadmium oxide cloud dispersed three days previously. The medium therefore contained such of the fine particles of the previous cloud as had not deposited in this period. Before dispersal the medium appeared optically empty. The electricity consumed by the arc (3·0 amps. ×40 secs.) was double that used in Fig. 6. (2·0 amps. ×30 secs.). In dust-free air this would have given normally a much brighter tyndall-beam. Actually the brightness was less and the colour more white. The minute particles left in the medium appear to have acted as nuclei for the condensation process, and collected most of the products vaporized by the arc. Had all the vapour deposited on these particles we should have obtained for these systems the analogue of Zsygmondy's(11) method of depositing gold on gold nuclei in colloidal solutions.

The effect feels more familiar when we remember that the best nuclei on which to grow large crystals from supersaturated solutions are tiny crystals of the same sort. The cloud particles are cadmium oxide crystals.(12)

EXPERIMENTAL.—II.

As the weight concentrations of clouds produced from an arc were unknown observations were made on clouds dispersed by volatilizing known masses of arsenic trioxide from a porcelain boat. The boat was placed on bare nichrome wire, and the contents volatilized by heating the wire by a constant current left on for the arbitrary period of four minutes (timed − 2·0 mins. to +2·0 mins.) in each experiment. During the process a draught was maintained across the boat by means of a fan, and to ensure constancy of conditions the dispositions of the fan and the boat were never altered. The fan was kept running continuously throughout each experiment to maintain homogeneity in the cloud, although this probably accelerated the rate at which it disappeared.(4)

A "Pointolite" lamp placed outside one end of the chamber was used as a source of light. The beam, collimated and cut down to a cross-section of one sq. cm. by diaphragms, passed inside a steel tube parallel to the face of the cistern. The inner end of the tube was closed by plate glass, through which the incident beam passed into the cloud. The beam was ultimately absorbed by passing into a second cylinder whose inner surface was blackened. The tyndall-light was viewed just after entering the cloud through a hole cut in the front of the chamber. It was hoped that in this way the intensity of illumination at the point of observation would be independent of the density of the cloud.

The clouds were dispersed in dry air free from dust and carbon dioxide. In general, the scattered light was partially plane polarised and slightly green in colour. Fig. 7 shows the results obtained from the dispersal of 45 milligrams of As_2O_3 in the 850 litre cistern. *A* gives the total scattering, the sum of the components. *B* is the polarised light—the ratio of the difference between the components, to their sum expressed as a percentage, and *C* is the ionization current in arbitrary units. The same general changes occur here as with arc clouds, so need not be discussed. As *B* depends upon the difference of two quantities, both of which ultimately become small and difficult to measure, its accuracy diminishes as the cloud ages. Within the limits of accuracy of the measurements the fluctuations in *A*, *B* and *C* are practically synchronous.

Observations were taken on a series of clouds in which the masses of arsenic trioxide dispersed varied from 15 mg. to 500 mg. The ionization currents were always weak. With a constant field on the ionization chamber, the initial currents did not vary greatly throughout the series. The currents were always oscillatory, and on the whole diminished fairly steadily as the cloud aged. The reciprocal of the current strength plotted against age gave a curve convex to the axis of time in all cases. The initial number of positive charges was therefore unequal to the initial number of negative charges. A direct test showed that there were more positive charges than negative. As the process of vaporization is unlikely to produce electrification, it appears probable that the initial charges arise from a low temperature emission from the heated nichrome wire. This would account for the excess of positive charges and the lack of variation with the mass of material dispersed. A blank experiment with no material in the boat showed that the heater did actually emit ions.

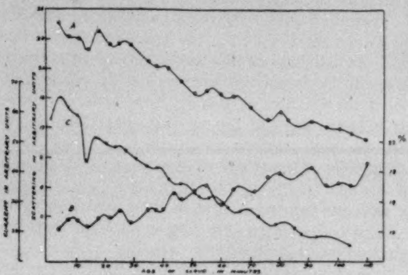

FIG. 7.—ARSENIOUS OXIDE.

CONCENTRATION AND TYNDALLMETER READINGS.

In each cloud of the series the intensity rapidly increased to a maximum, which occurred close to the arbitrary zero of time. Where sufficient observations were obtained, the readings before the maximum fell on straight lines, which cut the time axis approximately one minute after making contact in the heating circuit. This seems to indicate that the arrangements for producing dispersal worked fairly consistently throughout the series. With increasing mass dispersed the maximum brightness of the tyndall-beam increased, and in general the percentage of polarised light present decreased. When 400 mg. and 500 mg. were dispersed, there was no appreciable difference between the values of the two components. Fig. 8 shows the variation of intensity with the mass dispersed. If logarithms of the mass dispersed are plotted against logarithms of the maximum brightness, the points lie about a straight line of slope 2/3. Hence the brightness of the tyndall-beam is given closely by the equation

$$I = A \cdot m^{2/3}$$

where A is a constant; m the mass of material dispersed. This is the equation to the smooth curve. The clouds behave, therefore, as if they contain a constant number of particles which reflect light from their surfaces. Under these conditions the masses of the individual particles in different clouds become proportional to the mass dispersed and their surfaces to the quantity $m^{2/3}$. The growth in size of the primary particles with increasing weight concentration is consistent with the general decrease in the polarisation shown as the mass dispersed increased.

It has already been mentioned that the nichrome wire heater emits ions during the dispersal of the cloud. It also emits nuclei (particles), as can be demonstrated by passing the air from its neighbourhood into a Wilson expansion apparatus, where they are caught with a very small expansion. If we may assume that their number remains as constant in each experiment as the number of ions emitted, the above results become explicable if condensation takes place upon them, for many must be produced before the boat is hot enough to vaporize its contents. From a super-saturated solution crystal growth occurs less readily spontaneously than upon foreign nuclei.

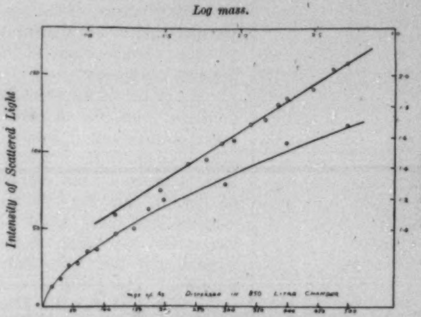

FIG. 8.—ARSENIOUS OXIDE.

The other limiting case in which the number of condensation centres increases proportionally with the mass dispersed seems to have been discovered by Tolman(18) with ammonium chloride smokes. Here the average *size* of the particles apparently remained constant, and the brightness of the tyndall-beam was directly proportional to the concentration. Unless special precautions are taken, it is to be expected that usually both the number and size of the particles will increase with increasing mass dispersed. Even when the *same* mass of material was dispersed it was found with our arrangements that the maximum brightness and the percentage polarised were never *exactly* the same, which is perhaps not unnatural.

THE FLUCTUATIONS IN BRIGHTNESS.

The stronger components for clouds of various concentrations are shown in Fig. 9. The curves are plotted from the "mean" values, and are drawn to include each observation. As drawn, several of the fluctuations show a rough tendency to synchronise, which would have appeared more marked by allowing some of the points to fall off the curves. If the curves prove similar when measured under more ideal conditions, the time of occurrence of a given fluctuation would be independent of the mass of material dispersed and of the total surface area of the particles, since their number is the same in each cloud. This suggests that the fluctuations arise from some effect of the gaseous medium on the particles, and that they appear after the surfaces of the particles have suffered a definite number of molecular impacts. On this view, the departures from synchronism in Fig. 9 would be attributed partly to the method of reducing the readings and partly to variations in temperature and pressure. Each cloud of Fig. 9 was dispersed on a different day, and, owing to the bulky nature of the apparatus, had to be measured at the temperature and pressure of the day.

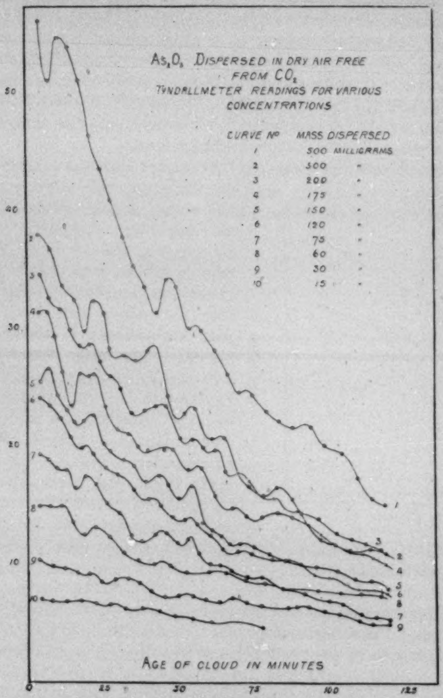

FIG. 9.

As in the case of clouds produced from the arc, the actual fluctuations in brightness occur much more rapidly than is shown by the curves of Fig. 9. In Fig. 10 an attempt has been made to record the individual observations for the two components. The cloud is the same as that of Fig. 7. The observed points are joined by straight lines. The diagram gives a more accurate picture of the rapidity of the fluctuations, and illustrates very well the extremely complex systems with which we are dealing. The percentage curve which indicates changes in the degree of dispersion is calculated from the components as drawn. Although it may not record all the variations which occurred, an attempt has been made to analyse it. The origin of time is taken as one minute after making contact in the heating circuit. It is found that the times of occurrence of a large number of the minima are given by the formula

$$t = Ar^n$$

where n is an integer and A and r are constants. A comparison of the observed times and those calculated from the formula with $A=3{\cdot}664$ and $r=1{\cdot}097$ is given in the Table. The deviations of the calculated values from the observed are well within the limits of experimental error.

TABLE.—*Comparison of Observed Times of Occurrence of Minima with the Times Calculated from the Formula* $t=Ar^n$.

n	Calculated times.	Observed times.
1	4′ 01″	...
2	4′ 25″	4′ 30″
3	4′ 50″	...
4	5′ 12″	...
5	5′ 50″	6′ 00″
6	6′ 24″	...
7	7′ 01″	7′ 00″
8	7′ 42″	...
9	8′ 28″	8′ 30″
10	9′ 18″	...
11	10′ 12″	10′ 15″
12	11′ 12″	...
13	12′ 16″	12′ 00″
14	13′ 29″	13′ 30″
15	14′ 47″	14′ 36″
16	16′ 14″	...
17	17′ 48″	17′ 15″
18	19′ 32″	20′ 00″
19	21′ 27″	21′ 30″
20	23′ 33″	23′ 30″
21	25′ 50″	25′ 30″
22	28′ 22″	28′ 30″
23	31′ 07″	31′ 00″
24	34′ 10″	34′ 00″
25	37′ 30″	37′ 15″
26	41′ 08″	40′ 45″
27	45′ 09″	44′ 36″
28	49′ 33″	49′ 30″
29	54′ 23″	54′ 30″
30	59′ 41″	59′ 45″
31	65′ 30″	65′ 30″
32	71′ 54″	72′ 00″
33	78′ 55″	...
34	86′ 36″	86′ 36″
35	95′ 02″	95′ 15″
36	104′ 18″	105′ 30″

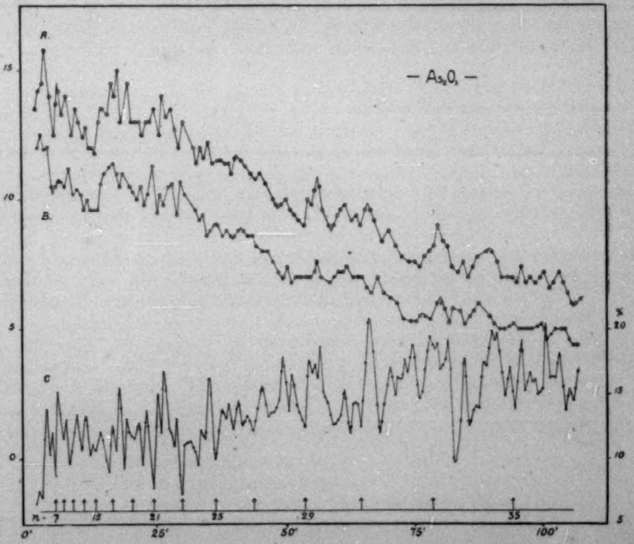

FIG. 10.

The calculated times for some odd values of n are shown by arrows in the figure. These seem to divide the fluctuations into groups, as if the phenomena involved were periodic. As the cloud ages the time required to describe a group of fluctuations lengthens exponentially. A similar relation was found to hold for the rapid fluctuations with cadmium oxide. The investigations along these lines were not carried far, for it is obvious that for further progress mechanical registration of the fluctuations is necessary.

REFERENCES.

(1) Mie, Ann. d. Phys. (4), 25, p. 429 (1908).
(2) Shoulejkin, Phil. Mag., 48, p. 307 (1924).
(3) Mecklenburg, Koll. Zeit., 14, p. 172 ; 15, p. 149 (1914) ; 16, p. 97 (1915).
(4) Tolman and others, J. Am. C. Soc., 41, p. 304 (1919).
(5) Tolman and others, J. Am. C. Soc., 41, p. 575 (1919).
(6) Gans, Ann. d. Phys. (4), 37, p. 883 (1912).
(7) Cf. Tolman and others, l.c., p. 304.
(8) Tolman and Vliet, J. Am. Chem. Soc., 41, p. 297 (1919).
(9) Tyndall, Phil. Mag., 37, p. 384 (1867).
(10) Walmsley, Phil. Mag. (7), 1, p. 1266 (1926) ; 3, p. 587 (1927).
(11) Zsygmondy, Zeit. Phys. Chem., 56, p. 65 (1906) ; Westgren, Zeit. anorg. Chem., 93, p. 151 (1915).
(12) Walmsley, Proc. Phys. Soc., 40, p. 7 (1927).
(13) Tolman and others, J. Am. C. S., 41, p. 300 (1919).

XXXVI.—NOTE ON THE TRANSMISSION OF CATHODE RAYS THROUGH THIN FILMS.

By B. L. Worsnop, *B.Sc.*, *Ph.D.*, University of London, King's College.

Received June 12, 1928.

Abstract.

The results of the experiments of G. P. Thomson on the transmission of electrons through thin films are discussed. It is suggested that the explanation in terms of the Debye-Scherrer diffraction is untenable in certain cases. In particular for the case of celluloid films, and "amorphous" films generally, it is suggested that scattering ofm the echanical waves at centres might account for the observed results.

Calculation shows that the distance apart of the scattering centres is of the order of the distance between atoms. An analogy between this case and that of X-rays (Keesom and Smedt) is made, and it is shown that the test between a scattering and a diffraction origin of the rings is the relative diameters of the rings.

A brief announcement of another "diffraction" experiment is made.

Recent theoretical discussions on the new quantum theory have shown an analogy between electrons and waves of high frequency, the frequency ν being given by $\nu=mc^2/h$. This has led to experiments designed to show that electrons behave as if they were waves. Evidence of this kind is to be found in the scattering of electrons in gases (Dymond*), by crystal faces (Davisson and Germer† and Davisson and Kunsman‡), and more recently in the experiments of G. P. Thomson.§

In the experiments of Thomson a narrow pencil of electrons was directed normally on to a very thin film, and the result examined on a photograhic plate, as in the corresponding case of the production of patterns by the Debye-Scherrer powder method in X-rays. In these experiments it was found that, in addition to the direct transmitted beam, rings were present on the plate when thin films of aluminium, gold, celluloid and an unknown substance were used. To account for these results the view was taken that the electron had a definite wavelength, given by $\lambda=h/mv$, and that the mechanical waves were diffracted in some way in the films.

In a discussion on the results it is stated that if the rings are of the nature of a diffraction pattern, the diameter should be proportional to the wavelength, or that D/λ=a constant, where D is the diameter of the ring.

When v is small we may write $\lambda=h/m_0v$, where m_0 is the rest mass of the electron; also $\frac{1}{2}mv^2=Pe/300$, where P is the applied potential measured in volts—i.e., $\lambda=(h/m)\{150/(Pe/m_0)\}^{\frac{1}{2}}$, so that for this approximation we have $D/\lambda=DP^{\frac{1}{2}}(m/h)(e/150m_0)^{\frac{1}{2}}$=a constant. This means that if the applied potential be varied to produce a variation in λ, then $D\sqrt{P}$ should be constant to be consistent with a diffraction origin of the rings. This constancy was found to exist approximately in the cases tested, and better agreement was found when correction was made for the mass of the electron in terms of its velocity. The tabulated values which agree to within 4 per cent., including the correction factor, were for $D\sqrt{P}(1+Pe/1200\, m_0c^2)$, instead of $D\sqrt{P}$.

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* Nature, **118**, 336-7 (1926).

† Phys. Rev., **30**, 705 (1927).

‡ Science, **64**, 522 (1921). Phys. Rev., **22**, 242 (1923).

§ Proc. Roy. Soc., Vol. 117, p. 600 (1927).

In the case of the metal films, it was suggested that the rings were produced in the same way as the "rings" of the Debye-Scherrer powder method in X-rays. By suitable selection of atom layers, Thomson was able to calculate the lattice constants of the minute crystals involved, and the result of the calculation came to be within about 6½ per cent. of the accepted values obtained by X-ray methods.

It is not so easy to see, however, how such effects may be used to explain the production of the rings in the case of celluloid. As far as one can gather, the cause in this case was attributed to diffraction at minute centres in the film, as in the production of haloes.

Now it appeared to the writer that the observed rings are very similar in appearance to the concentric rings which were found in the scattering experiments of Keesom and Smedt.* These observers investigated the scattering of a very narrow pencil of X-rays in liquid oxygen and nitrogen, also in water and carbon bi-sulphide. They used a method very similar to that of Thomson, but the incident beam was the filtered radiation from a copper target X-ray tube. For these liquids it is clear that no explanation in terms of a random crystal distribution meets the case, and the authors were led to explain their results in terms of a theoretical investigation of Debye and Ehrenfest. This theoretical investigation deals with the problem of the scattering of radiation by two centres separated by a distance s, the line joining the centres inclined at all possible directions to the incident radiation. A deduction of the result for the case of two electrons is to be found in Compton's "X-rays and Electrons," Appendix IV, page 384. It is shown from considerations of scattering on classical principles that the intensity of the scattered beam in any direction φ is given by the expression

$$I_\varphi=\frac{Ie^4(1+\cos^2\varphi)}{r^2m^2c^4}\left\{1+\frac{\sin x}{x}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where $x=(4\pi s/\lambda)\,.\sin(\varphi/2)$, and I is the incident intensity and λ is the wavelength of the incident radiation—i.e.,

$$I_\varphi=2I_e(1+\sin x/x) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1a)$$

where I_e is the scattering by one electron.

This leads to the result that, in a direction inclined to the incident beam, the scattered beam shows fluctuations of intensity, the directions of the maxima and minima being governed by the values of $\sin x/x$. It is therefore clear that maxima occur when x is approximately equal to $5/2\pi$, $9/2\pi$, $13/2\pi$, etc., or more precisely when x is $2\cdot459\pi$, $4\cdot477\pi$, $6\cdot484\pi$, etc.

In the X-ray case referred to the angle φ_m corresponding to the first maximum was deduced from the diameter of the ring, and substitution was made in the formula

$x=(4\pi s/\lambda)\sin(\varphi/2)$ of the value $x=2\cdot459\pi=7\cdot72$; $\varphi_m=D/2L$

or

$$s=7\cdot72\lambda/4\pi\,.\sin(\varphi_m/2) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

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* Journal de Physique, p. 144 (1923).

The calculated values of s for the liquids used in the experiments came out of the order of 4 to 6 Å.U. Also the distance between the molecules of these liquids when calculated from the expression $d=1{\cdot}33\sqrt[3]{M/\varrho}$—i.e., assuming close packing of the molecules, was found to be in very good agreement with the corresponding value of s. This suggested that the scattering in these cases was from molecules, which are presumably close packed in the liquid, presenting a regularly spaced system of scattering centres. More evidence of the rings of maximum intensity is available, for X-ray scattering. For example, Hewlett* used an ionisation method to investigate the scattering from mesityline, using X-rays of wavelength 0·7 Å.U. He found a maximum at about 10°. Using equation (2), this gives the value of s about 5 Å.U. It seems therefore that the expression is one which is at any rate approximately true, and that it may be applied to the case of scattering from a large number of scattering centres.

It would appear that something on these lines might be used with advantage to explain the rings produced with celluloid, and it also seems desirable to see how far this point of view meets the case of the metal films in the Thomson experiments.

If we substitute for λ the value $\lambda=(h/m_0)\{150/P(e/m)\}^{\frac{1}{2}}$ in the expression (2) above, we have

$$s=\{7{\cdot}72h\sqrt{150}/4\pi m\sin(\tfrac{1}{2}\varphi)\}\div\sqrt{P(e/m)}=\text{constant}/D\sqrt{P}\quad .\ .\ .\ .\quad (3)$$

and we see that $s=$a constant$/D\sqrt{P}$. In other words, the constancy of $D\sqrt{P}$ is demanded if we assume *scattering* of the mechanical waves associated with the electron, as distinct from diffraction.

This point of view seems a reasonable one, as analogy between X-rays and mechanical waves holds in the case of "reflection" at the face of crystals. The presumption is that other X-ray properties should be shown in the waves, and in particular that of scattering.

Unfortunately, the data for celluloid is not so complete as for the metal films, and therefore use is made of the data for aluminium and gold as an alternative to the treatment given by Thomson. If we make allowance for the correcting factor for the mass, and use the mean corrected value of $D\sqrt{P}$ as obtained for aluminium (434), we find on substitution

$$s=h\times 7{\cdot}72\sqrt{150}/m_0 4\pi\sin(\varphi/2)\sqrt{P(e/m)}=2{\cdot}22\text{ Å.U.}$$

Making similar substitution for gold, we obtain $s=2{\cdot}39$ Å.U. If we further follow Keesom and Smedt, and evaluate $d=1{\cdot}33\,.\,\sqrt[3]{M/\varrho}$, we find in the two cases $d=3{\cdot}61$ for aluminium and $d=3{\cdot}63$ for gold.

This treatment, therefore, gives the right order for the distance between the scattering centres, but the difference between s and d suggests that this is not a case of scattering by molecules. It is interesting to note that the accepted values of the distance between the atoms in molecules of fluorides of aluminium, gold, etc., as calculated from band spectra data is of the order of 1·7 to 2·3 Å.U. This naturally suggests that the scattering centres might well be the *atoms* in the films. The diameters of the rings on this assumption should be in the ratio :—1 : 1·82 : 2·63, etc. According to the theory of "reflection" at minute crystals with a random distribution, and taking the different faces, more rings should appear, and, as stated at the outset, Thomson finds these, and obtains an agreement to within 7 per cent. of the known spacings. It seems therefore that for such films the effect corresponds to a Debye-Sherrer picture, giving another analogy with X-rays. For substances like celluloid this process does not seem so likely, and it has been suggested for this case that the effect may again be due to diffraction at centres, as in the production of haloes.

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* **Hewlett, Phys. Rev., 20, 688 (1922).**

It is here suggested that in a case like this the rings may be due to scattering; the test between these two causes is to be had in the measurement of the intensities and the diameters of the rings. For diffraction the diameters are in the ratio 1 : 1·6 ; 2·3 : 3 for scattering the diameters are as stated above, viz., 1 : 1·82 : 2·63, and any regular arrangement of scattering centres should produce this result if we may make this additional analogy with X-rays.

To test this it seems desirable to have data on the transmission of electrons through some "amorphous" substance of known composition, and I understand that results of this type will shortly be available. It is of interest to note that in the case of rings with celluloid Thomson says that the diameters of the first and second rings are approximately as 1 : 2.

At the present an experiment is being carried out in which many of the consequences of the wave nature of electrons are under independent test. In brief, these experiments consist in sending a beam of electrons at a known angle on to a ruled diffraction grating in much the same way as has been done in the case of X-rays. It is hoped to make a preliminary announcement of the results of these experiments quite soon. Incidentally, it may be possible to test the conclusions given above by means of the apparatus.

In conclusion, I wish to take this opportunity to thank my colleague Dr. H. T. Flint for many helpful discussions and advice on this subject, and also to thank Prof. E. V. Appleton for the interest he has taken in discussing the problem which forms the subject of this note.

XXXVII.—BASIC MEASUREMENTS OF THE EFFECTIVE RESISTANCE OF CONDENSERS AT RADIO FREQUENCIES.

By D. W. Dye, *D.Sc.*, *F.R.S.*, the National Physical Laboratory.

Received June 22, 1928.

ABSTRACT.

Following a very brief introductory statement concerning losses in condensers at radio frequencies, a description is given of a special air condenser so constructed that a change in its capacity may be made without change in the losses. The method of using such a condenser to measure the losses in a variable air condenser is next described. Measurements made by means of the special condenser on the losses in a high-class standard variable air condenser are given, and it is shown that the losses at any reading and any frequency may be represented to a good approximation by a fixed resistance plus an effective resistance represented by a constant power factor in the insulating material and hence proportional to the reciprocal of the square of the capacity; a third component of the effective resistance is proportional to the reciprocal of the square of capacity and to the square of frequency and is analogous to that which would be given by a constant shunt resistance.

The paper concludes with an appendix giving calculations of the effective resistance of the conductors of the special condenser whence it is shown that this may be neglected.

THE establishment of a basis for the measurement of the losses in condensers at radio frequencies is not an easy matter when certainty greater than that represented by the assumption that a high class air condenser has zero loss is desired.

The accumulated measurements of effective resistance in all its aspects at radio frequencies has led to the opinion that the losses in even very good variable air condensers is not of negligible importance.

A knowledge of the losses in condensers is therefore of considerable importance, not only for its own sake, but also in relation to the whole question of effective resistance measurements on inductive coils, etc., whenever a method is used other than one which measures directly the energy dissipated in any apparatus.

There are two classes of measurement which in the past have pointed to the conclusion that uncertainties in the knowledge of the losses of the condensers used may have been the cause of the discrepancies found.

Thus, thermal methods of measuring effective resistance of inductive coils have been compared with measurements made by methods involving the whole of a resonant circuit containing a good variable air condenser. In such cases when all care has been used a residual difference has been found indicating that a small loss resides in the condenser.

Again, in the cases where inductive coils of design such that the losses can be theoretically calculated have been constructed and carefully measured, the results, when a thermal method has been used have tended to be smaller than when a circuit method was adopted. The accumulated errors associated with the chain of measurements and calculations of which the final link is a good condenser are, in the present state of the art, such that the differences ascribed to the condenser can hardly be considered a measurement of its losses, except in cases where such losses are several times that which a really high class condenser may have.

In spite of the present unsatisfactory accuracy judged from the standpoint of the condenser, there would seem to be hope that further carefully made measurements, using coils of calculable resistance which can be measured by thermal and other methods, would enable the losses in a good air condenser to be measured to an accuracy of 20 per cent. or 30 per cent.

The problem of condenser loss measurement has been directly attacked by a few experimenters, and a very short résumé of these may be permitted here for the sake of completeness.

In 1926 Callis* published an account of measurements of losses in condensers made by an ingenious method in which two nearly identical coils were used.

The principle is as follows: Resonance is established in an oscillatory circuit, including one of the coils and the variable air condenser under measurement. The effective resistance of the whole circuit is measured by a usual method. A second similar measurement is made using the other coil.

A third measurement is now made using both coils in series in the circuit located, so that their mutual inductance causes the total effective inductance to be the same as before. Resonance is maintained without change in the capacity, but the effective resistance of the circuit has been increased by that of the second coil only. The difference between the measured total resistances of the circuit in the two cases, therefore, gives a measure of the effective resistance of the second coil, involving no assumptions regarding the condenser losses. The latter are thereby determined by difference from one of the first two measurements.

The general consistency of the results obtained justify the method to the somewhat limited accuracy attained and desired in the type of condenser measured. One fact has, however, been lost sight of in the method. The total effective resistance of the circuit in the second case will have a part due to what may be called the mutual resistance of the coils. Since each coil is present in the magnetic field of the other additional eddy current losses will occur from this cause. In the actual coils used by Callis the wire was thin, so that their effective resistances were but little greater than their direct current resistances; the mutual resistance was therefore small also. On the other hand, the total resistance of the circuit was thereby so large relatively that only in the case of condensers with considerable losses could reliable measurements be made. The corrections on account of self capacities of the coils are also of importance.

It is possible that a good case could be obtained using coils of much lower resistance and then making definite measurements of the mutual resistance by observing the increase in resistance of the circuit when the second coil is brought into position but not connected up.

This difference should be one-half that which will exist when both coils, assumed similar, are included in the circuit. Further measurements along these lines would be worth making.

The direct attack on the condenser has also been made by two methods; one is a thermal method, and the other involves the use of a specially designed condenser. There seems little doubt that a solution of the problem of condenser losses can be met by the design and proper use of special condensers.

The difficulties of the thermal method will be seen from a consideration of the quantities involved in a typical case. A high class condenser of 500 $\mu\mu$F capacity might have a resistance of 0·03 ohm at a frequency of 10^6 cycles per second. Taking a root mean square applied voltage of 500 as the safe maximum, the current would be about 1·5 ampere and the losses about 0·07 watt. The condenser will, of necessity, have a considerable thermal capacity, causing very slow changes in temperature. Specially made mica condensers, by reason of their small volume and low thermal capacity, might however yield accurate results by a thermal method; such experiments would be worth trying.

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* C. D. Callis, Measurement of the Resistance of a Condenser at Radio Frequencies, Phil. Mag., Vol. 1, 7th ser., p. 428 (1926).

A method recently described in an important Paper by E. Offerman* makes use of a specially designed condenser, consisting of three plates A, B and C, so disposed that A and B have a pure capacity to one another, and are each supported on C, with insulating material between. Three capacities are thus formed, C_2 and C_3, each with losses due to the solid insulators, and C_1, which is considered perfect. The design is such that the three capacities are nearly equal.

The principle of the measurements is to observe the differences in losses of the two-terminal condensers formed by shorting each of the capacities in turn. Thus, when C_3 is shorted, C_1+C_2 has a loss due to C_2 equal to, say, R_2. C_1+C_3 has a loss R_3, and C_2+C_3 has a loss R_2+R_3. Hence, R_2 and R_3 are determined. The work carried out by the authors of this method, including numerous measurements on various types of condensers, together with the apparently highly sensitive means used in the actual measurements, indicate that the method is of considerable value.

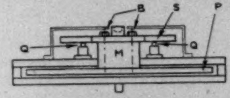

FIG. 1.—CONSTRUCTION OF CONDENSER.

The chief source of uncertainty lies in the unwieldy shape of the condenser, which was not screened. A modified design could doubtless be constructed to remedy this defect and enable measurements on really good condensers to be made. The results obtained indicate an uncertainty in power factor measurement of 2×10^{-5} corresponding to 0·006 ohm in a condenser of 500$\mu\mu$F at 10^6 cycles per second.

The necessity for the use of specially constructed condensers arises on account of the conductor losses in any normal type of variable condenser, otherwise, as will be shown later, it is quite valid to assume that the losses in a variable air condenser, with moderately large air gaps, are independent of the angular setting. On such an assumption there are no real difficulties in measuring the losses in another condenser if the standard is left connected to the circuit all the time.

The feature of the condenser about to be described is that the conductor losses within it may be considered quite negligible, and that a change in its capacity can be made without any change whatever occurring in the losses at constant voltage.

DESCRIPTION OF THE CONDENSER.

The design is very simple, as shown in Fig. 1. It consists of a thick brass plate P hung by means of a three-armed spider S upon three quartz rods Q, and located centrally between two other brass plates. The location is by hole, slot and plane. The attachment of the plate P is by means of the very thick central copper block M, which contains a mercury cup at the top. Three plates P of different diameters are provided, so that three different capacities may be obtained. The attachment of the plate P to the copper block is by means of three bolts B. When these are undone and removed the plate P rests upon the lower of the two outer plates, and the residual capacity of the spider and block to the enclosure containing it is the only capacity of the condenser. This capacity is about $18\mu\mu$F. It is now necessary to discuss the correct method to use such a condenser in order to obtain accurate measurements of the effective resistance of another variable air condenser by the method of substitution.

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* E. Offerman, Valve Method of Measurement of Losses in Condensers at High Frequencies, Zeit. für Hochfreqy., Vol. 26, p. 152 (1925).

The assumption made is that the standard condenser consists of a small fixed capacity with constant losses at a given frequency, and that a large fixed capacity can be added without causing any increase in the losses.

In considering the arrangements of the circuits for measuring the unknown condenser, it is convenient to represent the circuit as in Fig. 2.

The standard condenser is shown at K, with the capacity of the spider, etc., shown as K_1 shunted by S. The leads from the points of division a, b to this condenser are considered to have an indeterminable resistance r_k, and this branch carries current I_k. The right-hand branch contains the unknown condenser C, with equivalent series resistance R. An additional small condenser C_1, with shunt resistance T, is connected as shown in order to balance K_1 as far as capacity is concerned. This branch carries current I_c. The total current I is measured by thermo-junction and heater, and in the main circuit resistance can be added as usual for the purpose of resistance measurement by the ordinary resistance variation method. The coil L completes the circuit. It is assumed that a quantity R can be measured, which is the difference in the effective resistance of the whole circuit, referred to the current through the thermo-junction and heater, when C replaces K. If we consider the losses in the circuit between the points a, b for the two cases we have :

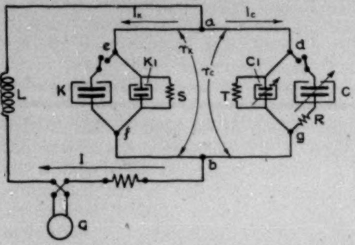

FIG. 2.—SCHEME OF EFFECTIVE IMPEDANCES IN COMPARISON OF CAPACITIES.

(1) Test condenser C in circuit and K disconnected.

$$\text{Losses}=I_k^2 r_k+E^2/S+I_c^2 r_c+E^2/T+I_c^2 RC^2/(C+C_1)^2$$

(2) Standard condenser K in circuit and C disconnected.

$$\text{Losses}=J_k^2 r_k+E^2/S+J_c^2 r_c+E^2/T.$$

On the assumption that $C=K$ and $C_1=K_1$ we have

$$I_k=IK_1/C_t;\quad I_c=I(C+C_1)/C_t$$

$$J_k=I(K+K_1)/C_t;\quad J_c=IC_1/C_t$$

where $C_t=C+C_1+K_1$ and $I=I_c+I_k=J_c+J_k$

On substitution it will be found that

$$\Delta R = 1/C_t^2 \times [(C^2+2CK_1)r_c - (K^2+2KK_1)r_k + C^2R]$$
$$= (C^2+2CK_1)r_c - (K^2+2KK_1)r_k + C^2R/C_t^2.$$

If $r_c = r_k$ this reduces to

$$\Delta R = RC^2/C_t^2$$

whence

$$R = \Delta RC_t^2/C^2$$

It will be seen therefore that, in order for the substitution method to give results not involving leads, it is necessary for C_1 to be made equal to K_1 and r_k to equal r_c.

EXPERIMENTAL PROCEDURE.

The condenser described was set up in a circuit as in Fig. 2 for the purpose of measuring the effective resistance of one of the best standard variable air condensers of the laboratory.

In order to satisfy the conditions outlined above, the leads *ad*, *ae* were very thick copper rods with one bent end of each dipping into mercury cups in a copper block at " *a*." They were equal in length, and that rod going to K had its bent end directly dipping into the mercury cup on the spider supporting the plate P of K. In a similar manner the leads *bg* and *bf* were equal thick copper rods connected to the screens of the respective condensers.

The small capacity C_1 made equal to K_1 was a small screened variable air condenser permanently connected across the ends *d*, *g*.

Experiments were made using each of the three plates P in turn, so that measurements were made at three different settings of the condenser under test. The measurements were made at a number of frequencies in each case.

Various methods of observation were tried, and much difficulty was experienced in attaining the accuracy desired on account of the small proportion of the resistance difference sought compared with the total resistance of the circuit.

The method finally adopted was that of simple resistance variation specially adapted and carried out to ensure the utmost possible accuracy in the measurement of the resistance difference resulting from the substitution of one condenser for the other.

Owing to unavoidable small differences in induced electromotive force when one condenser is substituted for the other in measurements of this kind, it is not sound simply to adjust a low range variable resistance to give equality of deflection of the current-indicating instrument upon changing one condenser for the other.

It is essential to measure the resistance of the circuit each time. By the use of an auxiliary small resistance, however, and by a proper procedure in observing, it is possible to throw the primary responsibility upon direct observation of a small difference of deflection, and not upon the ratio of two considerably different deflections.

The current measurements were made using a vacuum thermo-heater and galvanometer. The galvanometer scale was at about 5 metres distance, and readings could be taken to an accuracy of 0·2 mm. on a scale 2 metres long. The readings were taken on a time basis as from the instant of closing the galvanometer circuit, so as to eliminate the effects of zero drift of the galvanometer spot. The procedure was as follows: the standard condenser was first connected in circuit at "*e*" with no added resistance, resonance was carefully obtained, the deflection being adjusted to a value θ_0, not very different from a reading of 1000 on the equally divided scale. A suitable resistance was chosen and inserted so as to bring the deflection down to approximately 500 on the scale$=\theta_1$. The galvanometer was got into a cyclic state by inserting and shorting the added resistance alternately into and out of the circuit a number of times.

Alternate readings of θ_0 and θ_1 were then taken, and the deduced value of θ_1, corresponding to a standard value of $\theta_0=1000$, was determined.

The same procedure was repeated after interchange of the condensers, performed by unbolting the plate *P* and connecting the insulated terminal of the test condenser *C* to the end "*d*" of the thick copper rod "*ad*."

A second deflection, θ_2, corresponding to the same standard value of θ_0, was obtained. The quantity $\theta_2-\theta_1$ was very small, and of course represents the whole of the difference sought. The experiment was therefore repeated a considerable number of times, sandwiching the series for one condenser between that for the other.

From the simple and well-known equations the quantity ΔR, referred to the circuit at the point where *I* is measured, may be deduced from the observations directly, by means of the derived equation

$$\Delta R=\Delta\theta/2\times\text{added } R\times\sqrt{\theta_0/\theta_1}\times 1/(\sqrt{\theta_0}-\sqrt{\theta_1})^2$$

for the case concerned, in which θ_1 and θ_2 are nearly equal and are measured on a uniformly divided scale.

The values of ΔR so obtained were then converted into their equivalent values expressed as the effective resistance of the condenser itself, using equation (1). In general, C_t was not very much greater than *C*, except for the case of the smallest of the plates *P* of the standard condenser.

The three plates used gave capacities of approximately 500, 300 and $180\mu\mu$F respectively.

The measurements on the variable condenser under test were therefore made at readings corresponding to these values.

Each value of the variable condenser was measured over the range of radio frequencies comprised between the limits of about 1500 and 50 kilocycles per second.

The results when plotted on evenly divided scales of "series resistance" and the reciprocal of frequency respectively gave lines which were curved slightly in the direction indicating a rate of increase in *R* greater than the rate of increase of $1/n$, such as would be caused by a combination of a loss represented by a constant power factor in the insulation material of the condenser, together with that represented by a constant shunt resistance.

When plotted in terms of the reciprocal of C^2, however, straight lines are obtained of which the intercept on the resistance axis gives the series resistance; and the slope is nearly but definitely not quite proportional to $1/n$.*

The three lines corresponding to the three capacities tested converged to values of *R* very near to one another. The actual value was very small, being about 0·007 ohm.

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* In ordinary condensers the losses are such that, on the scales used, the intercept on the resistance axis is very nearly zero.

It is of interest to see whether the values of effective resistance obtained are consistent with the assumption that the losses in a variable condenser can be considered as made up of those represented by

(1) A constant series resistance,

(2) A constant power factor in the insulation,

(3) A constant shunt resistance.

It must be clearly understood that the experimental results by no means prove this, but only that they are not very inconsistent with this assumption.

The effective series resistance of the condenser at any capacity and at any frequency would, on the above assumption, be equal to

$$R_a + \varphi C'/C^2\omega + 1/SC^2\omega^2$$

where R_a is the constant series (conductor) resistance,

φ is the power factor of the insulation,

C' is the capacity contributed by the insulation,

and S is the effective constant shunt resistance.

A suitable value to assume for $\varphi C'$ in order to fit the curves is $7{\cdot}5 \times 10^{-15}$; this, on the assumption that C' is of the order $20 \mu\mu$F, would represent a power factor in the quartz insulation itself of about 4×10^{-4}, a not unreasonable amount.

The value of S in order to fit the change proportional to the square of the frequency is about $2{\cdot}5 \times 10^9$ ohms.

This value gives for the third constituent of effective series resistance the quantity $4 \times 10^{-10}/C^2\omega^2$.

The expression for the effective series resistance of the condenser at any capacity and any frequency becomes

$$R = 0{\cdot}007 + 7{\cdot}5 \times 10^{-15}/C^2\omega + 4 \times 10^{-10}/C^2\omega^2$$

Three curves have been calculated from this expression for the three capacities of 494, 314 and 184 $\mu\mu$F respectively corresponding to the actual capacities measured.

They have been calculated and plotted over the range of frequencies experimentally explored.

These curves, together with the experimental points, are shown in Fig. 3. Owing to the large range of resistance and frequency covered, the curves have been drawn to logarithmic scales.

It will be seen that fair agreement is obtained except at the highest frequency used on the two smaller capacities.

Later measurements carried out by a very sensitive bridge arrangement not yet published have shown that the results represented by the curves represent the losses to a very close approximation.

In view of the smallness of the power factor of the condenser, the possibility of representing the losses to the degree of accuracy obtained, in the form of a generalised equation, is of considerable value.

In considering these curves, it should be realised that the total power factor is very small. Thus, at 10^5 cycles per second, on the curve for the capacity of 314 $\mu\mu$F, the resistance is seen to be 0·14 ohm, from which

$$\varphi_e = 2\times10^5\times314\times10^{-12}\times14\times10^{-2} = 2{\cdot}75\times10^{-5}$$

Expressed as a phase angle, this quantity is equal to about 5″ of arc. In a circuit in which the whole power factor is 0·003 the condenser losses in such a case are about 1 per cent. of the total, so that an uncertainty of 10 per cent. in these losses would represent a quantity which is just about at the limit of measurement in the present state of the art of effective resistance measurement—i.e., one part in a thousand.

If the losses are expressed as power factors, then the results calculated by the equation (3) are as given in the following table :—

TABLE.

Frequency, cycles per second.	Power factor, parts in 10^6.		
	494 $\mu\mu$F	314 $\mu\mu$F	184 $\mu\mu$F
2×10^6	59·0	51·2	57·8
1×10^6	37·2	37·4	48·5
5×10^5	26·3	30·5	45·0
2×10^5	19·8	27·6	43·9
1×10^5	18·6	27·2	45·0
5×10^4	18·7	28·6	48·0
2×10^4	22·2	34·3	58·0

These results are shown by the dotted curves of Fig. 3. It will be seen that the power factor at a particular setting of the condenser is not very variable with respect to frequency over a considerable belt, but that it rises continuously for changes in each direction outside this belt.

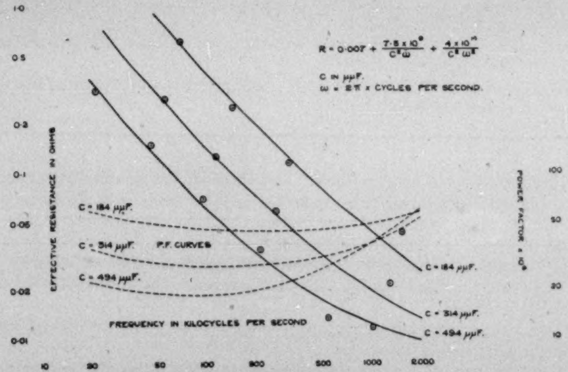

FIG. 3.—EFFECTIVE RESISTANCE AND POWER FACTOR CURVES OF STANDARD VARIABLE AIR CONDENSER.

The general consistency of the measurements made and the smallness of the losses so found on the high class variable air condenser tested would appear to indicate that the use of a special air condenser, constructed in accordance with the principles and design of that described, can, in conjunction with a good variable air condenser measured by it, form a basis for effective resistance measurements of condensers and inductance coils to an accuracy at least as great as that which may be obtained by other methods at present available.

In the appendix which follows, calculations have been made of the probable maximum effective conductor resistance of the condenser based upon formulæ developed by Prof. Fortescue. It will be seen that the losses due to this cause are insignificant.

I am indebted to Messrs. Martin and Vigoureux for assistance in taking the observations, and to the latter for making the calculations given in the Appendix.

The work described in this Paper forms part of a programme carried out for the Radio Research Board, and is published by permission of the Department of Scientific and Industrial Research.

APPENDIX.

Calculation of Effective Resistance of the Conductors of the Condenser.

The calculations are based on the assumption that the current flow in the plates of the condenser follows the same law of propagation as that in a plane conducting surface of thickness large compared with the effective penetration of the current.

In such a case the effective resistance of an element of the path of length ds and width b is given by

$$dR = \rho m ds/b \quad \text{. . . . .} \quad (1)$$

where ρ is the resistivity, ds and b are in the same units, and m is a factor for the penetration, having the dimensions of the inverse of a length. It is given by the well-known expression

$$m = 2\pi\sqrt{\mu n/10^9 \rho}$$

where n is the frequency in cycles per second and μ is the permeability in such units that for a vacuum it has the value unity.

The frequency conditions are such that m is of the order 60, so that the effective penetration is only 1/6 mm. We must assume, therefore, that the current is confined to a very thin surface layer, and hence that the current flowing out from the lower surface of the insulated plate has first to travel radially outwards over the upper surface and round the edge.

Returning to equation (1), it is clear that the resistance of an annular element of radius r and width dr, is, with respect to the current actually traversing it, equal to

$$dR = \rho m dr/2\pi r \quad \text{. . . . .} \quad (2)$$

The total current traversing any element will vary continuously with the radius. In order to express the effective resistance in terms of the total current flowing, we must therefore multiply dR by I_r^2/I_0^2 where

I_r represents the current at radius r
and I_0 is the total current.

We can now deal with each portion of the condenser separately.

Referring to Fig. 1,

Let r_1 be the radius at which current enters the insulated plate system at the top of the copper block;
- r_2, the radius at which current leaves the copper block and enters the upper surface of the insulated plate P;
- r_3, the radius of plate P;
- r_4, the radius of hole in upper outer plate;
- r_5, the radius of upper and lower outer plates; and
- r_6, the radius at which current leaves lower outer plate.

For the upper flat portion of the copper block of effective resistance R_1 we have for the current I_0, which flows radially from r_1 to r_2,

$$R_1=\frac{\rho_1 m}{2\pi}\int_{r_1}^{r_2}\frac{1}{r}dr=A_1\log_e(r_2/r_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the cylindrical portion of the copper block of effective resistance R_2 we have for the current I_0 flowing down its length l at radius r_2,

$$R_2=k\,\rho_1 l/\pi r_2^2,$$

where $$k=856\sqrt{m_1}+0{\cdot}25,$$

and $$m_1=\frac{4}{3}\times10^{-8}\times\mu n r_2^2/\rho.$$

For the upper surface of the insulated plate of effective resistance R_3 and of inner and outer radii r_2 and r_3, we have

$$I_r^2/I_0^2=(2r_3^2-r^2)^2/(2r_3^2-r_2^2)^2,$$

where $$R_3=\frac{A}{B^2}\int_{r_2}^{r_3}(2r_3^2-r^2)^2dr/r,$$

$$R_3=(A/B^2)\ [4r_3^4\log_e r_3/r_2-2r_3^2(r_3^2-r_2^2)+(r_3^4-r_2^4)/4]$$

where $A=\rho m/2\pi$, and $B=(2r_3^2-r_2^2)$.

For the lower surface of the insulated plate we have

$$I_r^2/I_0^2=r^4/B^2,$$

where $$R_4=\frac{A}{B^2}\int_0^{r_3}r^3dr,$$

$$=(A/B^2)r_3^4/4$$

In a similar manner for the outer plates it will be found that the effective resistance R_5 of the under surface of the upper plate from radius r_4 to radius r_3,

$$R_5=(A/B^2)\,[(r_3^4-r_4^4)/4-r_4^2(r_3^2-r_4^2)-r_4^4\log_e(r_3/r_4)]$$

For the surfaces of the upper and lower outer plates lying outside the insulated plate, and comprised between the radii r_3 and r_5 the current has the constant value given by

$$I_r/I_0 = r_3^2/B,$$

whence

$$R_6 = (A/B^2)\,[2(r_3^2 - r_2^2)^2 \log_e r_5/r_3]$$

Finally, for the remainder of the upper surface of the lower plate, we have

$$R_7 = (A/B^2)\,[B^2 \log_e r_3/r_6 - B(r_3^2 - r_6^2) + (r_3^4 - r_6^4)/4]$$

The following constants have been taken and calculations made for two sizes of insulated plate :—

$$r_1 = 6 \text{ mm.},\ r_2 = 19 \text{ mm.},\ r_3 = 82 \text{ mm. and } 132 \text{ mm.}$$

$$r_4 = 21 \text{ mm.},\ r_5 = 140 \text{ mm.},\ r_6 = 6 \text{ mm.}$$

For the brass plates the value 9×10^{-6} has been taken for ρ. The calculations all refer to a frequency of 10^6 cycles per second :

$$\rho m/2\pi = A = 1 \times 10^{-4}.\quad \text{Case I, } B = 130, \text{ Case II, } B = 345.$$

The values of resistance of the various parts for the smallest and for the largest plates calculated in microhms are as in Table I.

TABLE I.

Part of circuit.	Case I (small plate).	Case II (large plate).
R_1	45	45
R_2	62	64
R_3	112	154
R_4	7	6
R_5	6	6
R_6	27	4
R_7	215	265
Total (microhms) ...	476	544

It is thus seen that at the frequency 10^6 cycles per second the calculated conductor resistance of the condenser is of the order 0·0005 ohm., which is smaller than can be observed in the present state of resistance measurement.

The effective resistance so calculated is proportional to the square root of the frequency over the range of frequencies concerned.

XXXVIII.—A NOTE ON THE ACOUSTIC PRESSURE AND VELOCITY RELATIONS ON A CIRCULAR DISC AND IN A CIRCULAR ORIFICE.

By A. G. WARREN, *M.Sc.*, *M.I.E.E.*, *F.Inst.P.*

Received June 15, 1928.

ABSTRACT.

It is shown that at low frequencies the pressure at the edge of a rigid diaphragm, vibrating in a hole in a thin wall, is $2/\pi$ times the pressure at the centre (the case of the moving-coil loud-speaker). In the converse case of an acoustic wave impinging upon a circular orifice, the velocity increases towards the edge (the case of a resonator orifice). Were the medium non-viscous, an infinite velocity would be attained at the periphery.

IN a recent paper, Richardson[*] drew attention to the fact that in the orifice of an acoustic resonator the velocity increases towards the boundary. In the discussion which followed, the present writer suggested that an " annular effect " of this nature might be expected, in view of certain analogous effects which occur in the moving-coil loud-speaker. The explanation then suggested was substantially correct, although a defective recollection of the exact conditions obtaining at the edge of a vibrating disc led the writer to doubt at the time whether the explanation was complete.

It is proposed, therefore, to re-state briefly the conditions of both of these problems. Let us consider the conditions obtaining in the semi-infinite space on one side of an extensive wall A, of negligible thickness, in an aperture in which a diaphragm B vibrates harmonically (Fig. 1). If B is rigid we have the case of the loud-speaker. The case of the orifice is obtained when B is extremely light and extremely flexible ; we have then to determine the motion of its parts when a sound wave falls upon the wall.

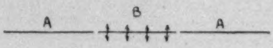

FIG. 1.

Case (a), B Rigid.

The conditions are : The normal velocity (harmonic) v ($=-d\varphi/dn$, where φ is the velocity potential) is constant over the disc, and $d\varphi/dn=0$ over the wall.

It is easy to show that at the centre of the disc

$$\varphi=(iv/k)(1-\cos kR+i\sin kR) \qquad (1)$$

and the pressure

$$p=a\rho v(1-\cos kR+i\sin kR) \qquad (2)$$

(where $k=\omega/a$, $\omega=2\pi\times$frequency, a is the velocity of sound, ρ the density and R the radius of the disc.)

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[*] The Amplitude of Sound Waves in Resonators, Proc. Phys. Soc., May 11 (1928).

Similarly, at the edge of the disc

$$\varphi=(iv/2k)\{1-J_0(2kR)+i\,.\,K_0(2kR)\} \quad \ldots\ldots \quad (3)$$

and

$$p=\tfrac{1}{2}a\,\rho v\{1-J_0(2kR)+i\,.\,K_0(2kR)\} \quad \ldots\ldots \quad (4)$$

The pressure has two components, one in phase with the velocity, responsible for the acoustic output, and one in phase with the acceleration, expended in overcoming inertia forces. At low frequencies (wave-length great compared to diameter of disc) we may write

$$p_{(\mathrm{centre})}=a\,\rho v(k^2R^2/2+ikR) \quad \ldots\ldots \quad (5)$$

$$p_{(\mathrm{edge})}\;=a\,\rho v(k^2R^2/2+ikR\,.\,2/\pi) \quad \ldots\ldots \quad (6)$$

The acoustic pressure is uniform over the disc, but the inertia pressure falls towards the edge. It is clear that as the frequency falls the inertia pressure becomes increasingly great compared with the acoustic pressure (for instance, for a disc of 10 cms. radius at a frequency of 27·2, the acoustic pressure is 0·00125 $a\,\rho v$, while the inertia pressure at the centre is 0·0500 $a\,\rho v$, and at the edge 0·0318 $a\,\rho v$). For frequencies at which we may assume that the phase lag in transmission over the face of the disc is negligible, we may write

$$p_{(\mathrm{centre})}=ia\,\rho vkR \quad \ldots\ldots \quad (7)$$

and

$$p_{(\mathrm{edge})}\;=ia\,\rho vkR\,.\,2/\pi \quad \ldots\ldots \quad (8)$$

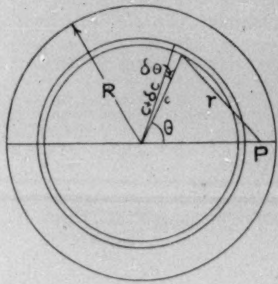

FIG. 2.

limited equations which may be derived by direct integration.

It is seen that the pressure at the edge of the disc is 0·617 of the pressure at the centre.

Case (*b*), *B Flexible, a Sound Wave Impinging upon it.*

Owing to the symmetry on the two sides of the aperture, it is clear that the motion of the air in the aperture must be normal to the plane of the wall. Hence the conditions are: φ constant over the aperture, $d\varphi/dn=0$ over the wall, and φ constant at infinity.*

Let the velocity at a radius c be $u=f(c)$. Then the potential at a point P distant a from the centre (Fig. 2) is

$$\varphi=-\frac{1}{\pi}\int_0^R f(c)\int_0^\pi \frac{e^{-ikr}}{r}\,d\theta\,.\,dc \quad \ldots\ldots \quad (9)$$

where

$$r=\sqrt{a^2+c^2-2ac\cos\theta}.$$

To satisfy the conditions, $f(c)$ must be found such that φ is independent of a. The general solution is almost impossible. When, however, the frequency is low, we may make similar approximations to those made in case (*a*)—i.e., write $e^{-ikr}=1$. The first integral then becomes an elliptic function ; even then considerable labour is involved in showing that

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* Rayleigh, Theory of Sound, Vol. 2, p. 176.

$$u=f(c)=vR/\sqrt{K^2-c^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

is a solution.

Fortunately for low frequencies, a much simpler solution is possible, based upon a method used by Rayleigh* in determining the conductivity of an elliptical orifice. In fact, the result here obtained is latent in Rayleigh's analysis of that case.

The general acoustic problem is the same as that of the determination of the distribution of electric charge upon a circular disc whose potential varies harmonically. When the radius of the disc is comparable with the wave-length the solution of this problem is difficult. It is simple, however, when the disc is small ; the distribution is then the same as in the static case. This is simply determined as follows : The distribution of charge on an isolated sphere is uniform, and no force is exerted at any point within it. Consider the vertical diameter to shorten so that the sphere becomes a spheroid (Fig. 3). Clearly, there will still be no force exerted at an internal point if the horizontal distribution of charge remains unaltered. Finally, let the sphere collapse to a disc, and we see that the charge density at a point distant *a* from the centre is proportional to cosec α, where cos $\alpha=a/R$. The velocity distribution in an acoustic orifice is similar.

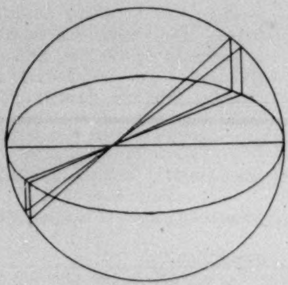

FIG. 3.

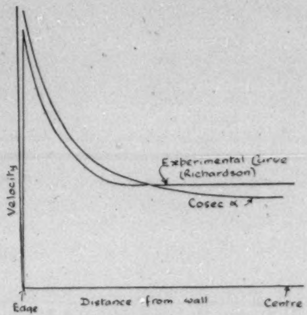

FIG. 4.

This indicates that an infinite velocity is required at the edge of the orifice to make φ uniform. Viscosity, of course, renders the velocity zero at the extreme edge, and limits the maximum velocity attained.

It is interesting to compare the theoretical distribution of velocity, neglecting viscosity, with one of Richardson's curves (Fig. 4).

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* Ibid.

DISCUSSION.

Dr. E. G. Richardson: The theorems of Rayleigh mentioned by Mr. Warren in regard to the " conductivity " of and flow through an orifice are based on static and continuous current electrical analogies. They contain no frequency term and should be equally applicable to continuous flow through an orifice. The infinite velocity at the edge which Rayleigh's equations give is the stumbling-block that Helmholtz and others encountered in theoretical hydrodynamics for the flow past a sharp edge. The fact is that alternating flow does give a high velocity at the edge of the " boundary layer " along the wall, whereas continuous flow through such an orifice shows a gradually falling velocity from the centre to the edge. It may be possible to describe the former motion by the introduction of inertia terms such as those which Mr. Warren has demonstrated for the case of the plate.

Author's reply: I think it is hardly correct to say that the theorems of Rayleigh commented upon in the paper are based upon static and continuous current electrical analogies. At low frequencies the analysis of the orifice is the same as that of the determination of the potential in a particular electrostatic case. (As Rayleigh says, the problems are *mathematically the same.*) Between the two problems, one static and the other harmonic, there is, however, no very obvious physical analogy. All the equations and reasoning in the paper are applicable only to the acoustic (harmonic) case; the variables are instantaneous values in a harmonic variation, and cannot be interpreted otherwise. The essential feature of the problem is a time variation; then, when the frequency is low (the velocity being necessarily high), the inertia, or out-of-phase pressure is the only component. To attempt to reduce the frequency sufficiently to cover steady flow renders all the equations indeterminate. With the case of continuous flow I am not particularly concerned, but I cannot see any inconsistency between the facts of such flow and the facts of harmonic flow experimentally determined by Dr. Richardson and suggested by Rayleigh's analysis. No doubt a comprehensive treatment would cover both cases, but it is only the harmonic case which was under discussion.

I do not gather the import of Dr. Richardson's final sentence. I hoped that I had shown that the problems of the plate and orifice were fundamentally similar, and that high velocities near the edge of the orifice would be expected. The apparent difference of treatment of the two cases was the result of an inability to solve equation (9) in general terms.

XXXIX.—VARIATIONS IN THE FINE STRUCTURE OF H_α.

By G. E. HARRISON, *Ph.D.*

Received May 26, 1928.

ABSTRACT.

The present paper describes the results of an attempt to study the intensity changes taking place in the fine structure of *Hα* with alteration of the bore of the discharge tube at constant gas pressures and current densities. The large variations in the electric field surrounding an emitting particle are concluded to account for the observed changes in the intensity distribution which amounted to 100 per cent. The doublet separation, however, appeared to be independent of these changes, the mean value obtained being 0·304 cm.$^{-1}$.

INTRODUCTION.

THE work of Hansen,[*] Houston[†] and Gehrcke and Lau[‡] has shown that the well-known fine structure components of the Balmer lines of hydrogen are subject to intensity variations which depend upon the gas pressure and current-density within the discharge tube. In these investigations, however, the changes observed were produced by varying the pressure and current within a tube of fixed diameter.

The observations made by Wood[§] and by Merton[‖] have revealed the great influence exerted on many spectral intensity distributions by the proximity of the radiating gas to solid surfaces, such as the walls of the tube. The present paper describes an attempt, therefore, to study the intensity changes in the fine structure taking place at fixed pressures and current densities, with alteration of the bore of the discharge tube. The results reveal the predominating effect of this variation upon the intensity distribution, and are of further interest in view of the fact that the old quantum mechanics appears to offer an inadequate explanation of the observations.

EXPERIMENTAL.

A discharge tube two feet long bent twice at right angles was employed. The central portion of the tube consisted of three pieces of capillary of 0·1, 0·16 and 0·2 cm. bore respectively, each about two inches long, and joined end to end. A McLeod gauge was connected to this tube, so that observations of the gas pressure could be made. Commercial hydrogen was admitted from a reservoir attached to the apparatus.

A Fabry-Perot interferometer used as an étalon was employed in the present experiments. The rear plate was carried upon a sliding block, the position of which was controlled by a good screw. It was found that temperature changes producing variations in the distance between the plates considerably affected the diameter of the interference fringes. The instrument was therefore enclosed in a double-walled metal cover, consisting of four sides and a detachable top. The space between

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[*] Hansen, Ann. d. Phys., 78, p. 588 (1925).

[†] Houston, Astro. Phys. Journ., 64, p. 81 (1926).

[‡] Gehrcke and Lau, Ann. d. Phys., 65, p. 575 (1921).

[§] R. W. Wood, Phil. Mag. (6), 44, p. 538 (1922).

[‖] Merton, Proc. Roy. Soc. (A), 97, p. 307 (1920).

these walls was filled with water at room temperature, while the outer surfaces were covered with felt. The temperature changes within this enclosure corresponded to a rate of change of ring diameter of 1 per cent. per hour. Provision was made at either end of the cover for the insertion of a tube, the first one carrying the collimation system, and the other the photographic objective and dark slide.

The collimating tube carried a fixed slit 1″ × 0·06″ at the one end, opposite which the discharge to be studied could be placed. A Wratten F filter transmitting radiations between 6100 and 7600 Å. was mounted at the other. As the H_a line was always considerably stronger than the lines of the secondary hydrogen spectrum, the transmitted radiation was found to be adequately monochromatic. The collimation lens was of 5 cm. focal length, set to converge a narrow cone of light upon the rear interferometer plate. As described by Fabry and Buisson,* the parallelism between these plates was improved by mounting a fixed circular diaphragm of about 0·6 cm. in diameter immediately in front of them.

It may be shown that the distance between the fringes due to two radiations separated by a spectral range of 0·14 Å. is a maximum when the interval between the silvered surfaces of the interferometer plates is 0·75 cm. The fringes due to the respective radiations then lie exactly half-way between one another and the instrumental corrections are a minimum.† This interval was kept constant throughout the experiments.

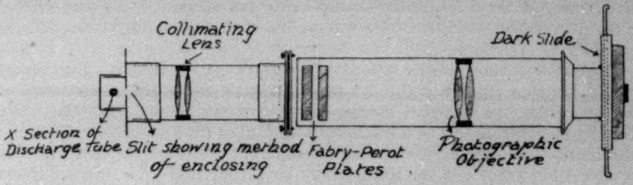

FIG. 1.—DIAGRAM OF OPTICAL APPARATUS.

A Beck isostigmat photographic objective of 15 cm. focal length was mounted in the second tube in such a position that parallel rays were focussed in the plane to be occupied by the photographic plate. The dark slide was constructed so that two comparison exposures could be made upon a single plate. The general arrangement of the apparatus is shown in Fig. 1.

A large induction coil fitted with a rotary break was used to excite the discharge tube. The primary potential was 40 volts, and the secondary currents varied from 1 to 10 milliamperes. At the higher currents the rectification was very incomplete. A mechanical rectifier which was found to be efficient and of negligible resistance was therefore fitted to the mercury break.

The intensity distribution in the fine structure was measured by a photo-electric microphotometer. The constructional detail of this instrument was similar to that described by Koch.‡ In the present measurements, however, the area of illumination upon the photographic plate under investigation was limited in length by the curvature of the interference fringes. It did not, therefore, exceed 0·00002 sq. cm., or about 1/50th of that employed in previous microphotometric measurements. The photo-electric current which was of the order of $0{\cdot}2 \times 10^{-10}$ ampere was obtained by observing the rate of growth of charge upon a pair of Dolezalek electrometer quadrants. This was controlled by increasing the capacity of the electrometer with a suitable fixed condenser. Correction was made for the dark conductivity of the photo-electric cell, which was found to vary from day to day, and also to decrease after the cell had been rested for several hours. Consistent results were obtained with this microphotometer when due precaution had been taken with the electrostatic insulation and the removal of water vapour from the surfaces of the cell and the electrometer.

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* Astro. Phys. Journ., 28, p. 169 (1908).

† C. Van Cittert, Ann. d. Phys., 77, p. 379 (1925).

‡ Ann. d. Phys., 39, p. 705 (1912).

The photo-electric current was calibrated with a photographic wedge and found to be accurately proportional to the intensity of the light incident upon the cell. The density of the photographic image was deduced from the relationship

$$D=\log_{10}\left(\frac{\text{Intensity of light incident upon plate}}{\text{Intensity of light transmitted}}\right).$$

The intensity of the fine structure components was assumed to be proportional to the density of the respective fringes recorded upon the photographic plate for a given time of exposure.

Photometric observations were made along a ring diameter. The minimum recorded between consecutive orders of interference was taken as the zero. The intensity of the fine structure components was then obtained from the resulting curves by subtracting this zero intensity from that of the respective maxima. The half-intensity line was deduced for each component and the mid-points calculated. The correction employed by Hansen for the partial fusion of two radiations of approximately equal intensity was applied, and the " true separation " of the mid-points obtained.

Although photographs of the fine structure were taken over a pressure range from 1·1 to 0·09 mm. Hg, and at current densities ranging from 13 to 86 milliamperes/sq. cm., the intensity ratio I_s/I_L for the two observed components of H_α (where I_s, I_L are the intensities of the short-wave and long-wave components respectively) were quite different from those obtained by Hansen. The long-wave component was always much the stronger. Thus, in the tube of 0·16 cm. bore, the intensity ratio I_s/I_L increased from 0·49 to 0·64, as the gas pressure was lowered from 0·93 to 0·097 mm. Hg. These results led to the conclusion that the diameter of the discharge tube had a considerable influence upon the intensity distribution in the fine structure.

To investigate this dependence, two further discharge tubes were constructed. These were exactly similar in design to the first one. The central portions, however, consisted of three pieces of capillary tubing joined end to end, of internal diameters 0·27, 0·34, 0·45 cm., and 0·5, 0·6 and 0·7 cm. respectively. Comparison photographs were taken at similar gas pressures and current densities to those employed in the first experiments.

For the widest bore (0·7 cm.) an intensity distribution was obtained in which the short wave component was considerably enhanced. In agreement with Hansen's results, this component was the stronger at low pressures. The intensity distributions in the case of tubes of about 0·5 cm. diameter, a specimen of which is shown in Fig. 2, were intermediate between those already discussed.

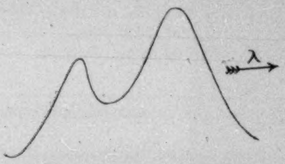

Fig. 2.—Intensity Distribution in the Fine Structure *H*α.

Tube bore 0·45 cm., gas pressure 1·02 mm. Hg, current-density 11·8 m. amps/sq. cm.

As a further test of the effect of the bore of the discharge tube upon the fine structure another system of tubes was constructed. This apparatus consisted of a central portion bounded by two tubes of 0·5 and 0·6 cm. internal diameter, fixed axially and in communication with a third tube of 0·9 cm. bore. The space between the central tube and the outer one was about 0·1 cm. A discharge could be established in the inner tube or in the interspace between the inner and outer portions independently. In the former the intensity distribution at a given pressure was similar to that obtained for the wider bores in the previous experiments, the intensity ratio (I_s/I_L) being about 0·97 at a pressure of 0·1 mm. Hg. In the latter case the ratio was about 0·6 at the same pressure and current density. It should be stated, however, that, owing to the imperfect axial alignment of the inner and outer tubes, the discharge took the path of least cross-section, and thus only about one-third of the interspace was excited. The results, nevertheless, afford further evidence of the effect of the proximity of the tube walls upon the fine structure.

Table of Results.

Tube width, cm.	Current density, m.amps./sq. cm.	Pressure, mm. Hg.	Intensity ratio (I_s/I_L)	Doublet separation, $\delta\nu$ cm.$^{-1}$
0·162	33·0	0·93	0·49	0·305
	34·9	0·37	0·53	0·309
	34·0	0·097	0·64	—
0·203	21·1	0·94	0·57	0·309
	22·4	0·36	0·66	0·304
	21·7	0·099	0·70	0·306
0·272	30·3	0·56	0·68	0·301
	29·8	0·89	0·77	0·297
0·344	20·3	1·01	0·66	0·298
	19·0	0·57	0·69	0·304
	18·3	0·087	0·81	0·297
0·45	11·8	1·02	0·67	0·306
0·49	36·4	1·10	0·69	0·301
	35·6	0·55	0·76	0·297
	28·9	0·153	0·89	0·307
0·68	18·4	1·10	0·73	0·304
	18·3	0·55	0·76	0·300
	14·3	0·153	1·07	0·305

Mean separation of the mid-points of the components ($\delta\nu$)$=0·304\pm0·005$ cm.$^{-1}$.

Specimen photographs of the fine-structure of H_a showing the large variations in the intensities of the two components are shown in the accompanying prints. The component of longer wavelength is situated nearer the centre of the interference pattern in each order.

CONCLUSION.

(*a*) *Experimental.*

(i) The intensity ratio (I_s/I_L) increased with the diameter of the discharge tube. If I_R represents the reciprocal of this ratio, then at a given gas pressure and current density the observations appeared to be approximately satisfied by the formula

$$I_R = a/d^2 + c$$

d being the internal diameter of the discharge tube and a and c constants depending upon the pressure and current density.

(ii) In a given tube operated at a constant current density, the intensity ratio (I_s/I_L) increased as the pressure decreased. This result is in qualitative agreement with Hansen's observations, but the variations in the intensity distribution are much larger.

(iii) The mean value obtained for the doublet separation was 0·304 cm.$^{-1}$. Re-measurement of Schrum's plates by Janicki* yielded the value 0·302 cm.$^{-1}$, while the same investigator obtained the value 0·306 cm.$^{-1}$ from the plates taken by Gehrcke and Lau.†

On the other hand, Hansen, Houston and Kent, Taylor and Pearson‡ all obtained the higher value of 0·316 cm.$^{-1}$, these results all being deduced from photometric measurements.

(iv) The doublet separation remained practically constant throughout the whole series of experiments. For a given tube, it appeared that the separation of the doublet was greater when the current density was reduced, in agreement with the observations of Hansen and Houston. This variation was, however, within experimental error.

(v) While Hansen observed an asymmetry only in the short wave component, in the present experiments the asymmetry was in general common to both of the observed radiations.

(*b*) *Theoretical.*

The bore of the discharge tube evidently exerts a predominant effect upon the intensity distribution in the fine structure. This is attributed to a variation of the electric field surrounding an emitting particle in the positive column brought about in the three following ways :—

(i) The general electric field in the positive column is accounted for by the space charge and the charge carried upon the walls of the discharge tube. Any change in the diameter of the tube will therefore bring about a modification in both of these variables, and the change will be most evident in the case of narrow tubes. The field in the neighbourhood of a radiating particle will therefore be largely increased by changing from a wide to a narrow bore.

(ii) The work of R. W. Wood has familiarised us with the strong catalytic action of the glass walls of a discharge tube. This results in the formation of

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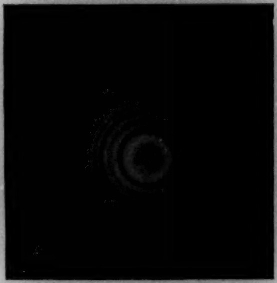
* Ann. d. Phys., 76, p. 561 (1925).

† Zeit. f. Phys., 35, p. 1 (1925).

‡ Nature, Vol. 119, Jan. 29 (1927).

TUBE BORE 0·20 CM., GAS PRESSURE 0·36 MM. HG., CURRENT DENSITY 22·4 M.AMPS./SQ.CM.



TUBE BORE 0·68 CM., GAS PRESSURE 0·153 MM. HG., CURRENT DENSITY 14·3 M.AMPS./SQ.CM.

To face page 304]

molecular hydrogen by re-combination of the dissociated atoms. A high concentration of molecular hydrogen is therefore to be expected upon the tube walls resulting in a strong inter-molecular field. The proximity of this field to an excited particle depends upon bore of the discharge tube.

(iii) The mean free path of a hydrogen atom at 0·5 mm. Hg is about 0·6 cm. This will be considerably decreased in the case of the narrow tubes employed in the present experiments. This will result in an increased number of collisions, and thereby an increased electric field surrounding a radiating particle.

Variation in the electric field in these three ways, with alteration of the bore of the discharge tube, is concluded to produce the predominant intensity changes observed in the fine structure.

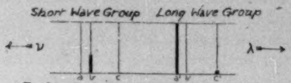
If the gas pressure rises within a discharge tube the general electric field is also increased. The pressure change may, however, be due to two different effects.

A mechanical increase of pressure represented by the admission of gas into the discharge tube will raise the concentration of charge within the tube, and thus increase the general electric field. Alternatively, an increase of pressure may be due to molecular dissociation. The more molecules dissociated the greater the concentration of free particles, and thus the higher the pressure. This type of pressure change will give rise to an increase of the electric field through variation of the number and kind of free charges.

An increase of current-density brings about an increased concentration of free charges, which in turn will lead to a more intense field within the discharge tube.

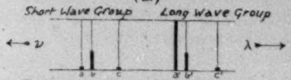
Combining the Bohr quantum theory of the emission of spectral lines with the relativity variation of mass with velocity, Sommerfeld has calculated the position of the six possible fine structure components for H_α shown as a, b c, a', b', c' in Fig. 3.

H. A. Kramers* found that the Bohr selection principle limited the number of "allowed components" to three in a field free space. The intensity distribution is that obtained in Fig. 3(a). Extending this investigation, Kramers calculated that, for a field of 100 volts/cm., the intensity distribution was modified to that shown in Fig. 3(b).



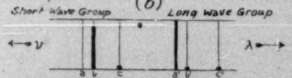
The fine-structure of H_α in a field-free space according to the Kramers Sommerfeld theory

(a)



The fine structure of H_α in an electric field of 100 volts/cm according to the Kramers-Sommerfeld theory

(b)



The fine structure of H_α according to the Sommerfeld-Unsöld calculations

(C)

FIG. 3.

Sommerfeld and Unsöld† have recently extended the analogy between the hydrogen spectrum and that of the alkalis. They have thus re-calculated the position and intensity of the allowed five structure components, using the alkali-like numbering of the quantum levels, together with the intensity rules due to Burger and Dorgelo. The results are indicated in Fig. 3(c).

Kramers' calculations based upon the original Sommerfeld theory lead to a

* Zeit. f. Phys., 3, p. 221 (1920).

† Zeit. f. Phys., 36, p. 259 (1926).

doublet separation not less than 0·329 cm.$^{-1}$, a value which is not supported by any recent photometric measurement. It appears, as pointed out by Houston, that this theory is inadequate.

The intensity distribution obtained by Sommerfeld and Unsöld undoubtedly agrees more closely with that found in practice. The doublet separation (0·320 cm.$^{-1}$) again appears to be too large, while the calculated intensity ratio (0·8) is supported by experimental evidence only under certain discharge conditions.

The observed asymmetry of the short wave component is in contradiction to all three calculations. It may safely be concluded that this asymmetry is due to the excitation of the c component. This agrees with Hansen's conclusions, and with the observations of Kent, Taylor and Pearson. But in order to account for this asymmetry the intensity of the c component must be concluded to be appreciable compared with a and b, certainly greater than that given by any of the three calculations.

Hansen supposes that the pressure variations in the intensity ratio are due to changes in the b' component, brought about by alteration of the electric field. The Kramers-Sommerfeld calculations would lead to the conclusion that while the intensity of a' is not materially altered by variation in the electric field, the intensity of c' is small compared with that of a'. Following Hansen, therefore, variations in the intensity ratio taking place with tube bore or gas pressure, which in the present experiments reached 100 per cent., are to be attributed to changes in the intensity of b'. Such large variations in a single component would presumably lead to changes in the doublet separation, and this is not supported by any experimental evidence. It is probable, therefore, that the intensity changes accompanying a variation of the electric field in the neighbourhood of an emitting particle take place in components of both groups, certainly in more than a single fine-structure component.

The variations in the intensity ratio of the fine structure doublet are concluded, therefore, to be due to the changes of the electric field accompanying a change of tube bore, gas pressure or current density. That the present theory is incapable of affording an adequate explanation of the phenomenon observed is not surprising when one considers the character of the fields of rapidly changing strength and direction, which act upon any emitting particle in a discharge tube. The observed gas pressures and current densities are, moreover, only averages over the whole tube. The actually field in the neighbourhood of an emitting particle is probably very different at any given instant from that calculated from these observations.

The most recent developments in atomic physics show that the model by means of which the fine structure has hitherto been calculated must be abandoned. The calculation must be made with quantum mechanics instead of classical mechanics, together with the modification due to spinning electrons. Heisenberg and Jordan* have shown that, as a result of these compensating modifications, exactly the same Sommerfeld possible fine structure levels are obtained. The difference between the results derived from the new and old calculations must therefore be confined to the intensities of the allowed components. Whether the new theory will explain the experimental facts is, as yet, a matter for conjecture.

In conclusion, I would like to thank Professor S. W. J. Smith, F.R.S., for the ready facilities he has provided for the carrying out of this work, and Mr. M. C. Johnson, M.A., for his helpful advice and criticism. I am also indebted to Mr. G. O. Harrison for the construction of the discharge tubes.

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* Zeit. f. Phys., 37, p. 263 (1926).

XL.—A SIMPLE GOLD-LEAF ELECTROMETER FOR HIGH-FREQUENCY MEASUREMENTS.

By SATYENDRANATH RAY, *M.Sc.(Cal.), B.Sc.(Eng.)(Lond.).*

Received June 8, 1928.

(*Communicated by* DR. D. OWEN.)

ABSTRACT.

A gold-leaf electrometer of the attraction type has been designed to supply the need for a simple instrument that can be used to measure voltages down to a few volts in high-frequency circuits, and which permits of precise indications of exact resonance in a simple circuit containing inductance and capacity.

The fixed element is a metal plate covered with a thin sheet of mica. The moving element is a narrow strip of gold leaf stiffened with a very light, tubular glass fibre extending almost up to the point of suspension of the leaf. The lower end of the glass fibre serves as the pointer. Simple means are provided for varying the sensitivity between wide limits.

The instrument will detect voltages as low as 4 volts and has a low capacity (2 to 4 micro-micro-farads). It also possesses the advantages of quick response and negligible power consumption.

Instances of its application are given.

§1. PRELIMINARY.

THE instrument here described was designed to meet the necessity which exists in certain high frequency tests of detecting and measuring the current in, or voltage across, a part of a circuit.

An instrument for this purpose should conform as closely as possible to the following requirements :—

1. Adequate sensitiveness.
2. Quick response—i.e., low period combined with large damping of the moving element.
3. Low power consumption.
4. Readings independent of frequency.
5. Minimum interference with the constants of the circuit into which it is introduced.

It should also be as simple and portable as possible.

The consideration that electrostatic instruments are inherently free from frequency errors and low in power consumption determined the choice of type of instrument.

The simplest arrangement of an electrostatic instrument consists of a fixed element close to a moving one which it attracts. For simplicity and constancy gravity control is incomparably the best. In an instrument so controlled the force between the two elements is, for a given potential difference, proportional to the rate of change of capacity of the system with displacement. This capacity is, therefore, necessarily variable, and as such variation must be kept as low as possible to satisfy condition (5) above, the couple of attraction must also be small. To retain adequate sensitiveness the restoring couple, which is due to gravity, in its turn must be small ; hence the weight of the moving system must be as small as possible.

After trial of thin foils of different metals, gold leaf was chosen for the combined advantages of lightness and perfect flexibility at the hinge when freely suspended.

The flexibility of the leaf as a whole is, however, a disadvantage, and it was accordingly decided to stiffen it with the lightest glass fibre that could be conveniently drawn and handled, and that would yet cause the gold leaf to move practically as a rigid body. In this way perfect definiteness of reading for a given potential difference was secured, with not too great a sacrifice of sensitivity.

§2. Description of Instrument.

Referring to Fig. 1, the instrument as finally constructed consists of a cylindrical case *A* closed by two windows, the back window of ground glass and the front one of clear glass. The case has two concentric grooves *HH* cut in it which can move along two semi-circular runners *RR*.

The fixed element is a brass plate *P* 20 mm. long and 5 mm. wide, covered with a mica film 0·01 mm. thick, attached with shellac varnish. This mica slip serves to protect the moving element from direct contact with the plate. Where the case is of metal the plate is in metallic connection with the case.

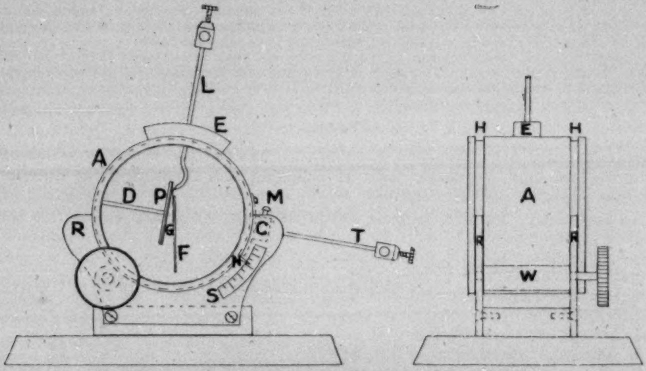

Fig. 1.—Front and Side Elevations of Instrument.

The moving element is a strip of gold leaf *G* about 1 mm. wide and 15 mm. long, attached to a thick brass wire *L* which passes through an ebonite block fixed on the case, and carries an external terminal. The leaf is stiffened with a fine tubular glass fibre *F* about 0·05 mm. in diameter and 25 mm. in length, weighing about 0·08 mgm. The total weight of the moving part is thus approximately 0·12 mgm. The gold leaf, which hangs vertically when at rest, is very nearly, but not quite, in contact at its point of suspension with the mica slip covering *P*. The lower part, owing to the tilt of the plate, is farther away from it. The case can be turned in the grooves and runners by the aid of a friction wheel *W*. By this means the angle of tilt between the plate and the leaf when at rest can readily be adjusted according to the sensitivity required ; the value of the tilt in degress being read off on the scale S by means of a pointer N fixed to the case. The instrument is held in the runners by two semicircular spring clips not shown in the drawing.

The rod T serves as the terminal for the plate side of the instrument. It slides in a collar C fixed to the case, so that when pushed home and tightened with a set screw M it serves as a clamp for the leaf during transport.

Another model has been constructed with a fibre case, and in this the terminal for P is on the continuation outside the case of the rod D, the leaf terminal being on L as before.

§3. CHARACTERISTICS OF INSTRUMENT.

Sensitivity, Range and Calibration.

The sensitivity and range can be widely varied according to the thickness of glass fibre and the width of gold leaf used. With any one instrument the sensitivity can be adjusted within wide limits by altering the tilt of the plate.

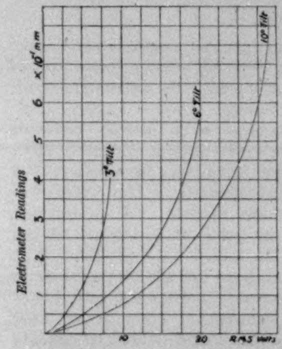

FIG. 2.—CALIBRATION CURVES.

Voltages as low as ·4 volts can be observed with a tilt of 3 degrees. For such indication unaided visual observation is sufficient with ordinary illumination of the back window.

For accurate measurement the instrument may be used with a low-power microscope (magnification, say, 10 diameters) ; with eyepiece scale divided into tenths of a millimetre deflections may be estimated to 1/100 mm. Calibration curves for the instrument as described in Section 2 are given in Fig. 2. From these it will be seen that with a tilt of 6° a potential difference of 15 volts can be measured to 1/5th of a volt—that is, with an error of less than 1·5 per cent.

Over the lower portion of the range of voltage the graph for any given angle or tilt approximates to a square-law calibration, but over the upper portion the curve rises more steeply. This latter represents the most sensitive region, and beyond this the point of instability is reached, at which the leaf is drawn up against the plate.

In the early stages of the work considerable trouble was experienced owing to the gold leaf adhering to the mica facing on the plate, from which it was hard to dislodge it without damage. After trial of various materials, it was found that fine talc powder dusted lightly on to the plate prevented stiction. This device works very well even in damp weather, only slight tapping being necessary, even at the worst of times, to free the gold leaf should it be drawn up to the plate.

The calibration is very constant, and no zero error is perceptible even after months of continuous work. This is the great advantage secured by the choice of a control which is almost solely due to gravity. Calculation shows that the restoring moment due to the flexure of the gold leaf at the hinge is not more than 1/200 of that due to gravity.

The instrument, like all sensitive idiostatic instruments, is preferably calibrated with alternating voltages, thereby eliminating the trouble due to contact electromotive forces when direct voltages are used. In the present instrument, with gold against brass, this electromotive force varies from day to day, but is of the order of 1·5 volts. This effect is, of course, without influence on alternating-voltage measurements.

Period and Degree of Damping.

The period of the moving system is well under half a second. The air damping is considerable, and causes the instrument to be almost dead beat. Actually, in the model described, on applying a voltage the pointer goes past the new resting-point but once, and that by less than 1/25 of the final deflection.

Capacity.

In the instrument with the brass case the capacity varies from 4·5 $\mu\mu$F at a tilt of 3° to 4·0 $\mu\mu$F at a tilt of 20°. In the model with a fibre case the capacities in these two positions are 2·0 and 1·5 $\mu\mu$F respectively.

§4. Example of Application.

To illustrate the behaviour and advantages of this leaf electrometer a brief account of its use in obtaining resonance curves may be given. Owing to the form of the calibration curves, the instrument readings plotted against another variable (e.g., condenser readings in a simple resonant circuit) give a curve with a distinctly more pronounced peak than even the square of the voltage plotted similarly. This effect becomes the more marked the nearer the approach to the point of instability of the leaf. With the milled head provided it is quite easy to adjust the tilt of the plate so that at the maximum voltage the leaf is near the verge of instability. When under these conditions the instrument is used to indicate resonance in a simple tuned circuit, or the crevasse in the quartz piezo-electric resonator circuit, sharp cusps appear on the curves, and the condenser settings for resonance can be reproduced with a precision limited usually only by the accuracy with which the condenser scale can be read. This is illustrated by the full curve given in Fig. 3.

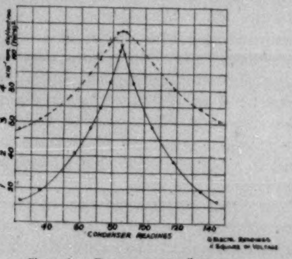

Fig. 3.—Resonance Curves.

For example: A variable condenser of maximum capacity 74 $\mu\mu$F, with dial divided into 180°, was used in parallel with a 250 $\mu\mu$F condenser. The resonance setting on the scale of the smaller condenser could be repeated to within 1/5th of a degree (the limit of error of reading of the scale). The resonance capacity was, therefore, determinable with an error of only 1 in 4,000.

Owing to the very quick response of the leaf, it is impossible to miss the peak, in the one case, or the crevasse in the other, even when the condenser dial is being turned rapidly.

The entire freedom of this instrument from time-lag, and the practically negligible power consumption constitute very appreciable advantages over the thermo-element and micro-galvanometer usually employed in resonance-circuit measurements.

There is, moreover, a third advantage in such measurements arising from the combination of superior sensitivity with small power consumption, namely, that a much lower degree of coupling to the exciting circuit is possible. To illustrate this the following data may be quoted. In a circuit oscillating at $10^6 \sim$, with a $200 \mu\mu F$ condenser in circuit, the instrument will indicate resonance sharply when the resonance voltage is 6 volts. This corresponds to a current of only 7·5 milliamperes in the circuit. If the high frequency resistance of such a circuit be taken as 8 ohms, the introduction of a heater element flattens the resonance curve, and, assuming its resistance to be 4 ohms, the maximum current to be detected falls to 5 milliamperes; or, alternatively, for the same current the coupling has to be made 50 per cent. stronger.

In conclusion, acknowledgments are due to the authorities of the Sir John Cass Technical Institute for facilities provided, and grateful thanks to Dr. D. Owen for his interest and guidance all through the work.

XLI.—THE STRUCTURE OF THE INDUCTION SPECTRA OF RARE GASES—ARC SPECTRA OF ARGON, KRYPTON AND XENON.

By Adolfo T. Williams, *Sc.D.*, Professor of Physical Chemistry, Instituto de Fisica, Universidad de La Plata.

Received May 12, 1928.

Abstract.

The purpose of this paper is to establish the usefulness of the excitation by induction method to separate different orders of the spectra of rare gases. The arc spectra are examined, constant separations and groups of lines deduced from Bloch and Dejardin's results, and the regions occupied by different series are established, and also the correspondence between theoretical terms and Meissner's terms in the arc spectrum of argon.

I. Arc Spectrum of Argon.

THE groups of lines deduced from Bloch and Dejardin's numerical data⁽¹⁾ do not agree with the series recently established by Meissner⁽²⁾. We have classified with Meissner's terms two new lines, 3354·3 ($1s_3$–$6p_8$) and 3355·4 ($1s_3$–$6p_9$), the residuals ($O-C$) in wave numbers being 0·81 and 1·21.

The following multiplets are established :—

	(s_3) b^3P_0	(s_4) b^3P_1	(s_5) b^3P_2
b^3P-c^3D			
$(p_4)c^3D_1$	4191·03	4054·50	3947·50
$(p_3)\ c^3D_2$		4044·42	?
$(p_9)\ c^3D_3$			4200·69
$b^3P-c^3\bar{P}$			
$(p_1)\ c^3\bar{P}_0$	?	3979·71	
$(p_2)\ c^3\bar{P}_1$	4181·88	4045·96	3948·98
$(p_6)\ c^3\bar{P}_2$		4266·29	4158·59

	(p_4) c^3D_1	(p_3) c^3D_2	(p_9) c^3D_3
c^3D-d^3P			
(d_6) d^3P_0	6818·26		
(d_5) d^3P_1	6754·30	6827·24	
(d_3) d^3P_2	6684·73	6756·10	6013·68
$c^3D-d^3\overline{D}$			
(d_2) $d^3\overline{D}_1$	6499·10	?	
(s_1'') $d^3\overline{D}_2$	6155·23	6215·95	5581·83
(d_1') $d^3\overline{D}_3$		?	5727·13
c^3D-d^3F			
(s_1'''') d^3F_2	6105·64	6165·11	5540·90
(d_4) d^3F_3		6722·90	5987·29
(d_4') d^3F_4			6032·13

	d^3P_0	d^3P_1	d^3P_2
c^3S-d^3P			
(p_{10}) c^3S_1	5650·70	5606·73	5558·71

The new designations of Meissner's terms in accordance with Hund's theory are indicated.

II. ARC SPECTRUM OF KRYPTON.

We have found 49 constant separations, some of which have already been mentioned by Paulson,(3) and two groups of lines.

	λ	Δν	λ	Δν	λ	ΣΔν
A_1	5823·7	460·7	5993·8	648·8	6236·4	1109·5
A_2	6236·4	461·1	6421·1	649·9	6700·8	1111·0
B_1	5827·1	648·7	6056·1	822·8	6373·8	1471·5
B_2	5993·8	648·8	6236·4	822·8	6573·8	1471·6

In this spectrum are other groups of lines in which the order of constant separations is inverted, or the values $\Sigma\Delta\nu$ are obtained in two or more different manners. Examples :—

	λ	Δν	λ	Δν	λ	ΣΔν
C_1	4362·7	193·8	4399·9	324·7	4463·7	518·5
C_2	4399·9	324·7	4463·7	192·5	4502·4	517·2
D_1	5827·1	477·1	5993·8	171·6	6056·1	648·7
D_2	5893·8	171·6	6056·1	477·2	6236·4	648·8

III. Arc Spectrum of Xenon.

The constant separations obtained are 195 in number. The groups of lines are :—

	λ	Δν	λ	Δν	λ	ΣΔν
A_1	7119·2	...	...	...	6655·8	977·8
A_2	6849·8	...	...	...	6419·8	977·6
A_3	6668·6	...	...	...	6259·8	979·0
A_4	4923·1	...	...	...	4697·0	977·7
A_5	4916·5	...	...	...	4691·0	977·5
A_6	4829·7	919·4	4624·3	58·1	4611·9	977·5
A_7	4734·1	...	...	...	4524·7	977·2
A_8	4109·7	...	...	...	3951·0	977·1
A_9	3823·7	921·6	3693·5	55·8	3685·9	977·4
B_1	4923·1	977·7	4697·0	117·3	4671·2	1095·0
B_2	4734·1	977·2	4524·7	116·4	4501·0	1093·6
B_3	3823·7	977·4	3685·9	117·5	3670·0	1094·9
C_1	4923·1	977·7	4697·0	530·6	4582·7	1508·3
C_2	4829·7	977·5	4611·9	534·1	4501·0	1511·6
D_1	6503·9	809·2	6178·6	1371·9	5695·7	2181·1
D_2	6487·5	808·5	6164·1	1368·6	5688·4	2177·1
D_3	6112·4	810·2	5823·9	1372·5	5392·7	2182·7
E_1	7119·2	977·8	6655·8	2168·8	5816·0	3146·6
E_2	4844·3	981·8	4624·3	2163·0	4203·7	3144·8
E_3	4829·7	977·5	4611·9	2162·7	4193·5	3140·2
F_1	4844·3	764·7	4671·2	3684·0	3985·2	4448·7
F_2	4792·6	758·9	4624·3	3684·0	3951·0	4442·9
G_1	6727·7	...	...	...	4829·7	5839·7
G_2	6472·6	...	...	...	4697·0	5839·0
G_3	5716·0	...	...	...	4285·8	5836·4
G_4	5163·8	...	...	...	3967·6	5836·9
G_5	4923·1	810·7	4734·1	5027·7	3823·7	5838·4
G_6	4697·0	810·2	4524·7	5028·0	3685·9	5838·2
G_7	4671·2	809·3	4501·0	5029·1	3670·0	5838·4
G_8	4285·8	...	...	...	3549·8	5836·3

Some of these groups have already been found by Sommer(4).

The complete list of constant separations and irregular groups is published in number 3, Vol. 4, of the " Contribuciones al Estudio de los Ciencios (serie Matematicofisica) " (La Plata).

IV. THE VALIDITY OF BLOCH AND DEJARDIN'S METHOD.

The arc spectrum of argon described by the authors mentioned is not complete. Many lines classified by Meissner are absent, and others belong to the spark spectra (simple ionized atom principally), according to the results obtained by Bloch and Dejardin. The spectra of the two other gases are very probably similar.

An interesting conclusion can, however, be deduced from the data of Bloch, the series established by Meissner and the early observations of Merrill(5). The following table summarises probable regions occupied by different series :—

—	$1s_n - mp_i$	$2p_i - \begin{matrix} ms \\ md \\ ms' \end{matrix}$	$1s_n - 2p_i$
Argon	3354–4510	5187–6870	6964→
Krypton (probably) ...	4274–4502	5475–6906	7590→
Xeuon (probably) ...	3549–4285	5392–7119	8230→

The values of m are : For mp, 3 to 6 ; for ms, 3 to 6 ; for md, 4 to 8 ; and for ms', 4 and 5.

REFERENCES.

(1) Bloch and Dejardin, Ann. Physique (10), Vol. 2, p. 461 (1924).

(2) K. W. Meissner, Zeit. für Physik., Vol. 37, p. 238 (1926) ; Vol. 39, p. 172 (1926) ; and Vol. 40, p. 839 (1927).

(3) E. Paulson, Ann. der Physik., Vol. 45, p. 419 (1914).

(4) L. A. Sommer, Zeit. für Physik., Vol. 13, p. 85 (1923).

(5) P. W. Merrill, Scient. Papers, B. of Standards, No. 345 (1919).

XLII.—AN INTEGRATING PHOTOMETER.

By M. G. Bennett, *M.Sc.*, University of Bristol.

Received June 25, 1928.

Abstract.

The paper is a description of a type of integrating photometer designed to measure the total amount of light arriving at a point on the earth's surface, from the sun and sky, irrespective of the position of the sun or of the distribution of cloud. Some measurements made with it, shewing the variation of direct sunlight and of indirect sunlight (arriving from clouds, &c.) throughout a typical day, are given as an illustration of its use.

Preliminary Remarks.

In the problem of visibility as it concerns the meteorologist an important factor is the amount of light falling on the object viewed and on the atmosphere between it and the observer. Obviously, this quantity will very largely control the visibility of the object and the amount of haze in the atmosphere. Further, it will vary between wide limits with the type of weather, amount of cloud, and the time of day, etc. In examining meteorological records, the author has been struck by the lack of data on this question, and the instrument described in this paper was designed to provide a simple method of obtaining them.

What is required is a photometer which will measure the total amount of light coming from the sky to a point on the earth's surface, irrespective of the position of the sun or of the distribution of cloud. Further, the instrument should be robust, simple to manipulate, and cheap, as routine measurements are to be made with it at more than one observatory.

Description of Photometer.

Figs. 1A and 1B represent front and side elevations of a section through the photometer. The novel feature of the instrument is the light integrating part of it, which consists of two opal glass bulbs A and B. The inner bulb B is mounted on an opal glass tube fitted into a cork in the mouth of the outer bulb A. Through this tube a patch of the interior of the inner bulb (shown by a short dotted line) can be viewed. Consider a beam of light incident on the outer bulb A at an angle θ with the horizontal. A certain amount of this will be diffusely transmitted into the interior of A. The illumination at any point in the interior, for a certain size and material of bulb, will be a function of the position of the point and of the angle θ. Similarly a certain proportion of this light will be diffusely transmitted into the interior of the inner bulb B. The brightness of the patch in the interior of B with which we are concerned, will depend upon the position of B and on the distribution of light inside A (that is, on the angle θ). Thus it is fairly obvious that if B were near the top of A the patch in question would be brighter for $\theta=90°$ than $\theta=0°$ or 180°, assuming the beam to be of constant intensity; if B were now brought near the bottom of A, the brightness for $\theta=90°$ compared with the value for $\theta=0°$ or 180° would be reduced. No attempt at a mathematical analysis has been made, but it has been found experimentally that for bulbs of given size and material, it is possible to find a position of *B* such that the brightness of the patch in question is independent of the angle θ. This is shown in Fig. 2 in which the brightness produced by a beam of constant intensity incident at varying angles θ is plotted as a polar function of θ. If *A* receives light from all quarters of the hemisphere of sky the brightness of this patch will be proportional, therefore, to the total intensity of the incident light irrespective of its distribution.

This brightness is measured by comparison with the light from the lamp *L*.

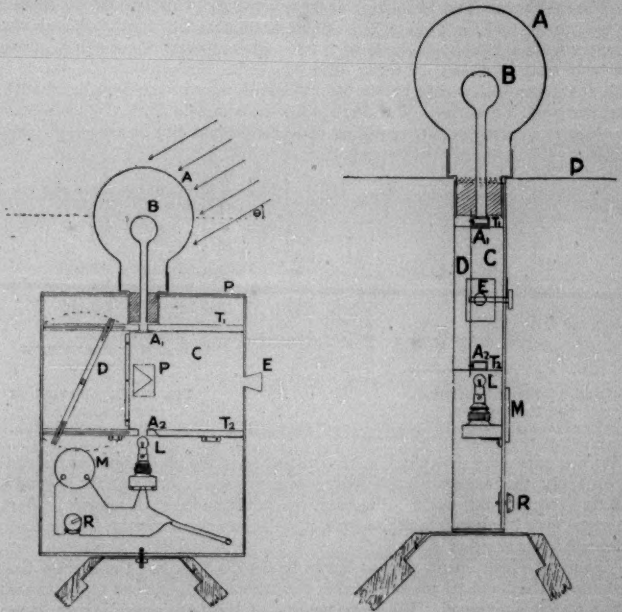

FIG. 1A. FIG. 1B.

DIAGRAMS OF THE PHOTOMETER.

The two illuminations enter the chamber *C* through the apertures A_1 and A_2, and fall on the two sides of the plaster of paris prism *P* which is viewed through the eyepiece *E*. The two sides are matched by means of optical wedge filters which slide in lengths of rectangular tubing T_1 and T_2 over the apertures A_1 and A_2. Their positions are controlled by the rotating arm *D* which engages in pins cemented to the wedges projecting through slots in the tubing. The axis of *D* carries a pointer which moves over a scale on the outside of the instrument. The lamp L is controlled by means of the rheostat R (which has an external handle) and ammeter M. A flexible lead is brought outside to an accumulator. Space is allowed inside for dry batteries, however, if it is desired to make the instrument more portable. The chamber C is lined with black velvet to avoid internal reflections, and over the aperture A_1 is cemented a piece of Flavazine T No. 16 Wratten filter which brings the illuminations to be matched to the same colour. A sheet of iron P painted dead black is placed at the base of the bulbs to minimize the effect of upward reflections from the earth and surroundings.

The instrument has, of course, to be calibrated. This is done by setting up the instrument and the plate of a Macbeth Illuminometer side by side at various distances from a 1,000 c.p. pointolite lamp, and comparing the readings obtained. The range may be altered by using either one of the optical wedges alone or both. With the upper wedge only in use big intensities may be measured and with the lower one small intensities. With both in use the complete double range is included, but there is a corresponding crowding of the scale and loss in accuracy. Typical calibration curves are shown in Fig. 3. (In the case of the photometer to which these curves apply, the range when the upper wedge was used alone was far beyond what is likely to occur in this work. It is, therefore, not included in this graph.) Further, the range may be altered to any degree, more or less permanently, by the insertion of neutral filters at either A_1 or A_2.*

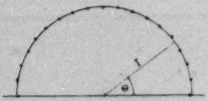

FIG. 2.—POLAR SENSITIVITY OF PHOTOMETER.

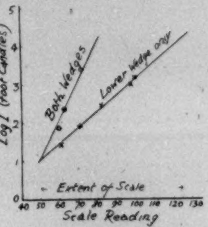

FIG. 3.—CALIBRATION OF PHOTOMETER.

To use the instrument, all that has to be done is to adjust the current through the lamp to the standard amount used in the calibration, and then rotate the arm D until a match is obtained. The scale reading of the position of D then gives the total illumination falling on A.

As an example of the use to which this photometer has been put, the following measurements are given of the variation of illumination on the earth's surface throughout a particular day. Fig. 4 shows the observations taken on the tower of the H. H. Wills Physics Laboratory, University of Bristol, on 20th June, 1928. The amount of cloud and opacity of the atmosphere was continually changing throughout the day. On the occasions marked *A* to *F*, the sun was shining brightly. The upper dotted line around these points may be taken as roughly representing the light which would have arrived if the sun had been shining all day. On the occasions *U* to *Z* the sun was obscured by cloud. The lower dotted line represents the amount of light arriving at the observation point indirectly from the sun via the clouds and haze in the atmosphere. The difference between these two lines represents, therefore, the value of the direct sunlight throughout the day. Such results will obviously depend very much on the type of weather.

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* The photometer in reality measures the total flux falling on the sphere A. For the present purpose it was more convenient to express the results in terms of foot-candles, the unit in which the illumination of a perfectly diffusing plate placed horizontally at A would be measured. This can readily be converted into lumens by multiplying by πr^2, where r is the radius of the sphere A.

While the instrument in its present form is sufficiently accurate for the purpose for which it is designed, some improvement is necessary if it is to be used for the measurement of small changes in intensity. It may be pointed out that the opal glass integrator, in itself, is of sufficient accuracy to be applied to much more delicate work. That this is the case is shewn in the following way. The distribution curve in Fig. 2 was not actually obtained with the prism-wedge system used in the present form of the instrument, but with a photometer of much higher sensitivity, namely, a Macbeth illuminometer. The smoothness of the curve is highly satisfactory. The limitations of the present instrument are, therefore, largely determined by the photometric part of the apparatus. Considerable increase in sensitivity is here possible, such, for instance, as may be obtained by the use of wedges of smaller density gradient.

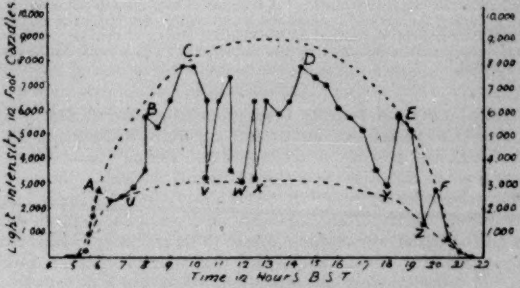

FIG. 4.—VARIATION OF DAYLIGHT AT BRISTOL, JUNE 20, 1928.

This work has been carried out in the H. H. Wills Physical Laboratory of the University of Bristol, and I wish to thank Prof. A. M. Tyndall for his kind interest in the work.

XLIII.—SOME RECENT DEVELOPMENTS OF STATISTICAL MECHANICS.

Lecture by J. E. LENNARD-JONES, The University, Bristol.

Received June 29, 1928.

ABSTRACT.

Some fundamental changes have been introduced into statistical theory since the publication of a paper by Bose in 1924. The characteristic features of the new statistics, including that of Bose and Einstein, and the later one of Fermi and Dirac, are pointed out in this lecture and compared with the classical statistics. A number of recent applications of the new ideas are described, including the work of Thomas and Fermi on the distribution of electrons in atoms, and the work of Sommerfeld and others on the electron theory of metals. An account is also given of some recent results obtained by the author and H. J. Woods on the distribution of electrons in a metal.

§1. INTRODUCTION.

AN unexpected turn has recently been given to statistical theory which adds considerably to its interest, and makes important additions to its scope; so much so that it is now possible, as Thomas(1) and Fermi(2) have shown, to derive certain average properties of the atom by statistical methods. The new departure dates from the publication by Bose(3) in 1924 of a new method of determining Planck's radiation formula. The importance of this work depends not so much on the results attained as on the initial assumptions found to be necessary. Bose modified one of the basic steps in statistical theory—viz., that which is concerned with *a priori* probability. In order that his innovation and others which have been stimulated by his work may be the more fully appreciated, we shall first review some of the main features of classical statistical theory.

The main problem of statistical theory is to determine what are the most probable or the normal properties of an assembly, or, as Fowler(4) would express it, what are the *average* properties of an assembly. The scope of statistical theory is thus limited to the equilibrium properties of an assembly, and is not concerned with such properties as viscosity, diffusion, which only become evident when conditions are non-uniform. The first necessary step is to decide what events are equally likely, or (to put it somewhat crudely) what is the unit of probability.

§2. LIOUVILLE'S THEOREM.

The argument is simplified by reference to generalised space. Suppose that $2s$ co-ordinates are necessary to describe uniquely the position and momenta of each particle (or system), and that the assembly consists of N such systems. (In most of the applications in this Paper we shall consider point systems defined by 3 positional co-ordinates and 3 velocity co-ordinates, so that $s=3$.) A point in a $2Ns$ generalised space then provides a full description of the instantaneous positions and momenta of the whole assembly, and the path of this point gives the life history of the assembly. Not all paths are possible owing to certain conditions imposed on the assembly, such as constancy of total energy. These restricting conditions confine the path to certain surfaces in the $2Ns$ space. It is not, of course, possible to determine the path of the representative point, for this would involve a complete solution of all the dynamical equations of motions of the whole assembly simultaneously, and would require us to know at any given time the co-ordinates of all the individual molecules of the gas. The problem of statistical theory is to find as much information as possible about the path in the generalised space without actually knowing it in detail. In this connection a theorem due to Liouville is of fundamental importance. Instead of concentrating on the life history of any selected assembly, Liouville considered a large number of identical assemblies started from all conceivable initial conditions, so that the generalised space could be regarded as filled with representative points. He was able to show that, provided the systems obey the ordinary dynamical laws, the density of these points remains constant as they each move along their paths. The result is of considerable importance, as it shows us that there is no tendency for the points to congregate in special regions in the generalised space.

If, therefore, an assembly when left to itself always possesses a certain property P after a sufficient lapse of time, then the above theorem entitles us to assert that the property must be true of the whole of the generalised space.

The argument at this stage is usually supplemented by an assumption, known as Maxwell's assumption of continuity of path, which is tantamount to the assertion that the average properties of any selected assembly taken over a long interval of time is the same as the average taken over a large number of systems. Although this assumption plays a central part in the theory, it has never actually been justified, and this step must be regarded as the least satisfactory in the whole development of statistical theory. However, some such step is necessary if the properties of a single assembly are to be inferred from the behaviour of a large number of assemblies.*

In this way the following answer is reached to the question which we raised above as to what is to be regarded as the unit of probability: *All points in the generalised space are to be regarded as equally likely.*

§3. The Classical Statistics.

To illustrate the application of this result to statistical theory we may consider a special problem. We seek the most probable distribution of energy among the N members of an assembly, endowed with translational energy only, given that the total energy is E. For this purpose it is an advantage to consider not only the generalised space mentioned above, but also a phase space of 6 dimensions, x, y, z, p_x, p_y, p_z, the first 3 being the usual positional co-ordinates and the second 3 the co-ordinates of momenta. A single point in this space then describes uniquely the co-ordinates and components of momenta of any *one* member, and N points describe uniquely the whole system; in fact, an array of N points in the phase space gives (so to speak) a " snap-shot " of the assembly, and may be called a *complexion*. For each such array there is one point in the generalised space of $2Ns$ dimensions. (For convenience, we shall now refer to the $2s$ space as the *phase* space, and the more general $2Ns$ space as the *hyper* space.) Now it is clear that many interchanges may be made among the array of N points without altering its distribution as a whole—in other words, many microscopic changes may be made without altering the complexion macroscopically. Every such interchange, however, affects the position of the representative point in the hyper space owing to the re-arrangement of its coordinate (just as the point 1, 2, 3 in ordinary space is different from the point 2, 1, 3). The greater the number of possible interchanges the greater the number of points in the hyper space, and the greater the probability that such a complexion will occur. It is in the manner of counting interchanges in a complexion that the theory has been modified, and we must, therefore, explain in detail the method adopted in the classical theory.

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* **For a discussion of this and other difficulties in statistical theories, the reader is referred elsewhere(5) (6).**

The classical method supposes the phase space divided up into a large number of cells, labelled 1, 2, . . . r . . . of equal extension ω. A distribution of N points in the phase space may then be described in terms of the number of points in each cell, say a_1 in cell 1, a_2 in cell 2, and so on. The number of possible interchanges which may be made among the N points without making any perceptible difference to the complexion is

$$W=N\,!/a_1!a_2!\ldots, \qquad \ldots\ldots\ldots\ldots \quad (1)$$

for this is the number of ways of allotting a_1 points to cell 1, a_2 to cell 2, out of a total of N points. This is, therefore, the number of representative points in the hyper space of $2Ns$ dimensions. The complexion for which this number is a maximum may be regarded as the most probable, or the normal distribution of the assembly. In order to find the most probable distribution we must find that for which W is a maximum, subject to the conditions that

$$\sum_r a_r=N, \qquad \ldots\ldots\ldots\ldots \quad (2)$$

and

$$\sum_r a_r\varepsilon_r=E, \qquad \ldots\ldots\ldots\ldots \quad (3)$$

where ε_r is the energy associated with the r^{th} cell. In the simple case we are considering

$$\varepsilon_r=\frac{1}{2}m(\dot{x}_r^2+\dot{y}_r^2+\dot{z}_r^2)=\frac{1}{2m}(p_x^2+p_y^2+p_z^2)$$

(where p_x, p_y, p_z are the components of momentum).

Using a theorem due to Stirling that

$$\underset{a\to\infty}{Lt}\ a\,!=(2a\pi)^{\frac{1}{2}}(a/e)^a, \qquad \ldots\ldots\ldots\ldots \quad (4)$$

we then easily find that

$$\frac{\delta W}{W}=-\sum_r\{1+\log a_r\}\delta a_r, \qquad \ldots\ldots\ldots \quad (5)$$

For a maximum, therefore, we have

$$\sum_r\{1+\log a_r\}\delta a_r=0, \qquad \ldots\ldots\ldots\ldots \quad (6)$$

together with

$$\sum_r\delta a_r=0, \qquad \ldots\ldots\ldots\ldots \quad (7)$$

and

$$\sum_r\varepsilon_r\delta a_r=0, \qquad \ldots\ldots\ldots\ldots \quad (8)$$

from (2) and (3) above. These equations are simultaneously satisfied if we write

$$\sum_r\{1+\log a_r+\lambda+\mu\varepsilon_r\}\delta a_r=0, \qquad \ldots\ldots\ldots\ldots \quad (9)$$

where λ and μ are Lagrangian undetermined multipliers. If this equation is true for all variations of δa_r, we must have

$$1+\log a_r+\lambda+\mu\varepsilon_r=0 \qquad (10)$$

whence

$$a_r=e^{-1-\lambda-\mu\varepsilon_r}=Ae^{-\mu\varepsilon_r} \qquad (11)$$

where A has been written for $e^{-1-\lambda}$ and is as yet an undetermined constant. To determine the constant μ, we use the familiar expression for the entropy

$$S=k\log W, \qquad (12)$$

where, of course, $W_{\text{max.}}$ is now understood. By making a variation in the total energy E and putting $\delta E/\delta S=T$, the temperature, we have

$$\delta S=\frac{\delta E}{T}=k\frac{\delta W}{W}=-k\sum_r\{1+\log a_r\}\,\delta a_r=-k\sum_r\{-\lambda-\mu\varepsilon_r\}\,\delta a_r=k\mu\delta E,$$

so that

$$\mu=1/kT, \qquad (13)$$

The constant A is determined from the condition (2),

$$N=A\Sigma e^{-\varepsilon_r/kT}, \qquad (14)$$

the summation being taken over all cells of the phase space. In order to effect this summation it is convenient to group together the terms which have approximately the same energy. To do this, we note that the amount of phase space for which the energy is less than a fixed amount ε is given by an integral of the type

$$I=\iiiint\!\!\iint dx\,dy\,dz\,dp_x\,dp_y\,dp_z, \qquad (15)$$

taken over the phase space for which

$$\frac{1}{2m}(p_x{}^2+p_y{}^2+p_z{}^2)\leqslant\varepsilon, \qquad (16)$$

The integration with respect to x, y, z is over the whole permissible range of these variables—~~that is,~~ over the whole volume of the containing vessel—and accordingly results in a factor V. The integral with respect to p_x, p_y, p is over a sphere of radius $(2m\varepsilon)^{\frac{1}{2}}$, and, therefore,

$$I=V\frac{4}{3}\pi(2m\varepsilon)^{3/2}, \qquad (17)$$

Similarly, the amount of phase space for which ε lies between the limits ε_r and $\varepsilon_r+d\varepsilon_r$ is

$$dI=V4\pi(2m\varepsilon_r)d(2m\varepsilon_r)^{1/2}=V(2m)^{3/2}2\pi\varepsilon_r{}^{1/2}d\varepsilon_r, \qquad (18)$$

If we suppose the cells to be of extension ω, we get for the number of cells with energy between ε_r and $\varepsilon_r+d\varepsilon_r$

$$V(2m)^{3/2}2\pi\varepsilon_r{}^{1/2}d\varepsilon_r/\omega.$$

Grouping now all the terms in the summation (14) of energy between ε_r and $\varepsilon_r+d\varepsilon_r$, we have

$$N=\{A\Sigma V(2m)^{3/2}2\pi\varepsilon_r{}^{1/2}d\varepsilon_r/\omega\}\,e^{-\varepsilon_r/kT} \qquad (19)$$

On writing A' for A/ω, and replacing the summation by an integral, as we obviously can if the cells have been chosen small enough, we have

$$N=V(2m)^{3/2}2\pi A'\int_0 \varepsilon_r^{1/2}d\varepsilon_r e^{-\varepsilon_r/kT}=V(2m)^{3/2}2\pi A'\,.\,\tfrac{1}{2}\sqrt{\pi}\,.\,(kT)^{3/2}. \quad . \quad . \quad (20)$$

so that

$$A'=N(kT)^{-3/2}/V(2m\pi)^{3/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The number of atoms in the gas with energy between ε and $\varepsilon+d\varepsilon$ is then

$$\{2N(kT)^{-3/2}/\sqrt{\pi}\}e^{-\varepsilon/kT}\varepsilon^{1/2}d\varepsilon\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

or in terms of velocities

$$4\pi(m/2\pi kT)^{3/2}e^{-(1/2)mc^2/kT}c^2dc,$$

which is the usual Maxwellian Law.

§4. BOSE-EINSTEIN STATISTICS.

The step in this development which has now been modified is in the calculation of W, equation (1). The classical method tacitly assumes that each member of the assembly has an identity which is recognisable in counting possible interchanges. In dealing with light quanta by the statistical methods, Bose(3) saw that the distinction between one quantum and another is lost, and that some other method of counting complexions must be devised. He considered light quanta as particles of energy $\varepsilon(=h\nu)$, with momenta $h\nu/c$ in the direction of its motion, so that

$$p_x^2+p_y^2+p_z^2=(h^2\nu^2)/c^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

A 6 dimensional phase space can be constructed as in the previous case to describe the position and momenta of the light particles. The amount of phase space in which ε lies between ε_r and $\varepsilon_r+d\varepsilon_r$ is given by (18), and so the amount of phase space in which ν lies between ν_r and $\nu_r+d\nu_r$ is

$$dI_r=V4\pi h^3\nu_r^2d\nu_r/c^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Bose divided his phase space into cells of extension h^3 (without explaining why*), where h is the usual Planck's constant, so that the number of cells in the above region is

$$V4\pi\nu_r^2d\nu_r/c^3.$$

In dealing with light quanta where the fact of polarisation has to be taken into account, it is necessary to multiply this number by 2, so that we have

$$C_r=8\pi V\nu_r^2d\nu_r/c^3, \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

as the number of cells appropriate to the $d\nu_r$ region.

In counting complexions, Bose concentrates on the cells and not on the particles, and supposes that any selected cell can contain any number of quanta with equal probability. If there are C_r cells in a given region, say the region in which the energy lies between ε_r and $\varepsilon_r+d\varepsilon_r$, the total number of ways of distributing a_r particles among them is

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* As a partial explanation of the choice of h^3 as the extension of the cells, we note (1) that it has the right dimensions, (2) that the ordinary quantum conditions $\oint pdq=nh$ divide up the phase space into cells of extension h^3.

$$(C_r+a_r-1)!/a_r!(C_r-1)!, \quad \ldots \ldots \ldots \quad (26)$$

and the total number of ways of arranging the whole assembly is

$$W=\prod_r (C_r+a_r-1)/a_r!(C_r-1)!.$$

In the case of quanta we cannot impose the condition that the number is fixed as with material particles, because light quanta in an assembly can be created and destroyed, though the energy condition is to be imposed as usual—viz.,

$$\sum_r a_r\varepsilon_r=E, \quad \ldots \ldots \ldots \quad (27)$$

a_r being the number of quanta of frequency ν_r.

Using Stirling's theorem as before, we have

$$\log W=\sum_r \{(C_r+a_r-1)\log(C_r+a_r-1)-a_r\log r-(C_r-1)\log(C_r-1)\},$$

and making W a maximum subject to (27),

$$0=\sum_r \{\log(C_r+a_r-1)-\log a_r\}\delta a_r, \quad \ldots \ldots \quad (28)$$

$$0=\sum_r \varepsilon_r\delta a\,, \quad \ldots \ldots \ldots \quad (29)$$

so that

$$-\log(C_r+a_r-1)+\log a_r+\mu\varepsilon_r=0 \quad \ldots \ldots \quad (30)$$

We, therefore, have

$$\frac{C_r+a_r-1}{a_r}=e^{\mu\varepsilon_r},$$

that is

$$a_r=\frac{C_r}{e^{\mu\varepsilon_r}-1}=\frac{V8\pi\nu_r^2 d\nu_r}{c^3}\cdot\frac{1}{e^{h\nu_r/kT}-1}, \quad \ldots \ldots \quad (31)$$

remembering that a_a and C_r are both large.

The density of radiation of frequency ν is then given by

$$\rho_\nu d\nu=a_r h\nu_r/V,$$

$$\rho_\nu=\frac{8\pi h\nu^3}{c^3}\cdot\frac{1}{e^{h\nu/kT}\quad 1},$$

which is Planck's formula.

The method of counting complexions is, therefore, justified by its success. Einstein⁽⁷⁾ thereupon applied exactly the same methods to an ideal monatomic gas, and sought the distribution which made the W of equation (26) a maximum subject to the conditions (27) and the additional one

$$\sum_r a_r=N. \quad \ldots \ldots \ldots \quad (32)$$

Instead of equation (30) we then get

$$-\log(C_r+a_r-1)+\log a_r+\lambda+\mu\varepsilon_r=0 \quad \ldots \ldots \quad (33)$$

so that

$$\log\{C_r+1)/a_r\}=\lambda+\mu\varepsilon_r$$

$$a_r=C_r/\{e^{\lambda+\mu\varepsilon_r}-1\} \quad \ldots \ldots \quad (34)$$

The constant μ is easily determined as in (13) to be $1/kT$, and λ is determined by summing the equation over all values of a_r. The appropriate value of C_r is obtained from equation (18) on dividing by h^3—that is

$$C_r = V(2m)^{3/2}\, 2\pi\varepsilon_r^{1/2} d\varepsilon_r / h^3 \quad \text{. . . . .} \quad (35)$$

We thus have

$$N = \frac{2\pi(2\pi)^{3/2}V}{h^3} \int_0^\infty \frac{\varepsilon^{1/2} d\varepsilon}{e^{\lambda+\varepsilon/kT}-1} \quad \text{. . . . . . .} \quad (36)$$

The equation (34) for a_r is the equivalent of Maxwell's distribution law in the Bose-Einstein statistics. The two are clearly approximately the same when λ is large, the unit term in the denominator being then negligible. Actually, λ is found to be large for ordinary gases at atmospheric pressure (for example, $e^\lambda = 6{\cdot}10^4$ for hydrogen). The divergence from the classical result only becomes perceptible as the density of the gas increases or its temperature decreases; but obviously any such effect would be masked by the increasing imperfection of the gas under such conditions.

The new distribution law, however, has one or two novel features. Thus the entropy of a gas (deduced from $S = k \log W$) tends to zero as the temperature decreases to zero, while the specific heat increases to a maximum, then diminishes to zero. The vanishing of the specific heat at absolute zero is in accord with Nernst's Heat theorem, but the maximum property has not been observed for any gas, and this fact tends to discredit the statistics, at any rate so far as atoms are concerned. This suspicion has been confirmed by a new line of reasoning, to which we refer in the next paragraph.

§5. THE FERMI-DIRAC STATISTICS.

In the theory of atoms a principle known as Pauli's Exclusion Principle has been found to be of wide validity. According to this, a unique description can be applied to any electron in an atom (or in a molecule, for that matter) in terms of quantum numbers. No two electrons have the same set of quantum numbers. The analogy of this in statistical theory is that no two electrons occupy the same cell of phase space. Instead, therefore, of supposing, as Bose and Einstein do, that the occurrence of any number of particles in a cell is an event of equal probability, Fermi(8) and Dirac 9) have independently come to the conclusion that a cell can contain one or none, and that each alternative is equally likely.

If we suppose, as before, that there are C_r cells in a given region, say that in which the energy lies between ε_r and $\varepsilon_r + d\varepsilon_r$, and a_r is the number of particles in that region, then the total number of ways of arranging them so that there is not more than one in any cell is

$$C_r!/a_r!(C_r - a_r)!,$$

and the total number of ways of arranging the whole N particles of the assembly among all the regions is

$$W = \prod_r C_r!/a_r!(C_r - a_r)!, \quad \text{. . . . .} \quad (37)$$

subject to the conditions (27) and (32). In this case

$$\log W = \sum_r \{C_r \log C_r - a_r \log a_r - (C_r - a_r)\log(C_r - a_r)\}, \quad \text{. . . . .} \quad (38)$$

and the same procedure as before leads to

$$\log(C_r - a_r) - \log a_r + \lambda + \mu\varepsilon_r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

for a maximum. We therefore find

$$a_r = \frac{C_r}{e^{\lambda+\mu\varepsilon_r}+1} = \frac{2\pi(2m)^{3/2}V}{h^3}\,\frac{\varepsilon^{1/2}d\varepsilon}{e^{\lambda+\varepsilon/kT}+1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

with $\mu = 1/kT$ as before. Instead of equation (36) to determine λ we find

$$N = \frac{2\pi(2m)^{3/2}V}{h^3}\int_0^\infty \frac{\varepsilon^{1/2}d\varepsilon}{e^{\lambda+\varepsilon/kT}+1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

It is curious that such a fundamental change in the statistics should result in such a simple change in the distribution law. The unit term in the denominator of (34) now becomes positive instead of negative.

The energy of such a system remains finite even at absolute zero. This is to be expected, since only one member is allowed to come to rest. The others fall into cells of as low an energy as possible in the phase space, and all cells up to a certain limit are filled. It is found, in fact, that at low temperatures

$$E = E_0 + aT^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where

$$E_0 = \frac{2\pi}{5}\;\frac{VGh^2}{m}\left(\frac{3N}{4\pi GV}\right)^{5/3}, \quad a = \frac{\pi mGV}{6h^2}(2\pi k)^2\left(\frac{3N}{4\pi GV}\right)^{1/3}, \quad . \quad . \quad . \quad (43)$$

so that the specific heat is proportional to the absolute temperature and vanishes at the absolute zero in accordance with Nernst's Heat Theorem. As there is a finite energy at absolute zero, so there is a finite pressure, given by

$$p = p_0 + \beta T^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

as could also easily be deduced from the relation

$$pV = 2E/3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

which is true for all statistical theories.

If λ is large and positive, the new distribution law (40) differs little from the Maxwellian. This is the case for all ordinary gases except at very low temperatures or very high pressures. Sommerfeld(10) has shown that the condition for proximity to or large divergence from the Maxwellian Law is

$$\left.\begin{array}{lll}\text{(i)} & \{(2\pi mkT)^{3/2}/h^3N\}V \gg 1 & \text{(approx Maxwellian, } \lambda \text{ large and positive),} \\ \text{(ii)} & \{(2\pi mkT)^{3/2}/h^3N\}V \ll 1 & \text{(non-Maxwellian, } \lambda \text{ large and negative),}\end{array}\right\} \quad (46)$$

The first condition is satisfied for ordinary gases at normal temperatures and pressures. When the second condition is satisfied, λ is large and negative, so that the exponential term in the denominator of equation (39) is small compared with unity. This means that for a large range of ε, $a_r = C_r$ approximately, and every cell is filled. The assembly is then said to be *degenerate*.

An example of degeneracy is provided by the free electrons in a metal even at normal temperatures. Assuming that there is one free electron for every atom of a metal, so that N/V is of order 10^{22} to 10^{23}, we then find for $T=300°$

$$\{(2\pi mkT)^{3/2}/h^3N\}V=2\cdot10^{-4} \text{ approx.},$$

which certainly satisfies the second of the above conditions. It is the small mass of the electron which differentiates it from ordinary gas atoms and leads to the above result.

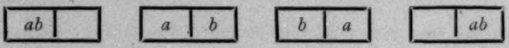
Sommerfeld calculates that the electrons have each an average energy at absolute zero of about 3 volts, and that the pressure exerted by the electrons is of the order 10^5 atmospheres.

One important consequence of the statistics as applied to electrons in a metal is that the contribution of the electrons to the specific heat is negligibly small. This disposes of a well-known difficulty in the electron theory of metals, and constitutes one of the most striking successes of the new statistics.

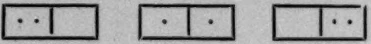
§ 6. COMPARISON OF THE STATISTICAL THEORIES.

The essential difference between the three statistics considered above may be illustrated by a simple example. Suppose that there were only two members of an assembly, and only two cells in the phase space. The characteristic feature of the classical statistics is that each member is assumed to have an identity, and each is as likely to be in one cell as in another. There are thus four possible arrangements as shown in the figure, and each is to be counted in assigning a weight to the complexion. In the Bose-Einstein statistics, however, the members of the assembly lose their identity, and attention is concentrated on the cell rather than on the individual members of the assembly. In the figure, therefore, the members are represented by indistinguishable points(11).

Classical :

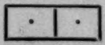


Bose-Einstein :



Each cell may have any number of points, and so the total number of possible arrangements is three. In the Fermi-Dirac statistics, on the other hand, not more than one representative point is to be in any one cell, with the result that in this simple case only one arrangement is possible. When there are a large number of cells, and comparatively few points to arrange in them, there is no essential difference between the three methods, because the likelihood of any cell containing more than one point is very small. It is when the number of points is comparable with the number of cells that the three methods lead to divergent results.

Fermi-Dirac :



Another method of discriminating between the three cases has been given by Brillouin(12). He makes the fundamental assumption that the presence of a particle in a cell diminishes the *a priori* probability of another particle in the same cell by a fraction β. If, therefore, there are C_r cells in a certain region and a_r particles in the region, then the number of cells available for an additional particle (the $(a_r+1)^{th}$ particle) is $(C_r-\beta a_r)$. The number of possible arrangements of the a_r particles in the C_r cells subject to this condition is then

$$C_r(C_r-\beta)\ .\ .\ .\ (C_r-\overline{a_r-1}\beta)=\beta^{a_r}\Gamma\left(\frac{C_r}{\beta}+1\right)\Big/\Gamma\left(\frac{C_r}{\beta}-a_r+1\right),$$

where Γ is the usual Gamma function. The total number of possible arrangements for the whole of the phase space is

$$W=N!\prod_r\beta^{a_r}\Gamma\left(\frac{C_r}{\beta}+1\right)\Big/a_r!\,\Gamma\left(\frac{C_r}{\beta}-a_r+1\right),$$

which is to be made a maximum subject to the usual conditions. Brillouin shows that all the statistical theories, classical, Bose-Einstein, and Fermi-Dirac are included in this general method; for the classical case, $\beta=0$, for the Bose-Einstein $\beta=-1$, and for the Fermi-Dirac $\beta=+1$.

The difference between the three formulæ for the number of particles with prescribed energies, equations (22), (34) and (40), lies in a function f, defined in the three cases by

Classical: $f=e^{-\lambda}e^{-\epsilon/kT}$, ... (47)

Bose-Einstein: $f=\{e^{\lambda+\epsilon/kT}-1\}^{-1}$, ... (48)

Fermi-Dirac: $f=\{e^{\lambda+\epsilon/kT}+1\}^{-1}$, ... (49)

This function f is the probability that a cell in the region, where the energy is ϵ, shall contain a particle. It is to be noted that though the λ's are constants, they are not necessarily the same in the three cases, even when the density and temperature are the same. They tend to become so for small concentrations and high temperatures, and then λ is large and positive. The distribution curves then tend to the usual Maxwellian form. Fig. 1 shows the three distribution curves (f plotted as a function of ϵ), for an assembly in which the new curves differ appreciably from the classical one; for the Fermi-Dirac case, f is approximately unity for a large range of energies.

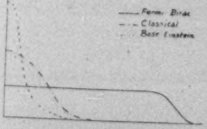


FIG. 1.—THE DISTRIBUTION CURVES IN THE CLASSICAL, BOSE-EINSTEIN AND FERMI-DIRAC STATISTICS.

In the Bose-Einstein statistics λ is essentially positive. Its limiting value is zero, and then f becomes infinite for zero energy. This means that all the representative points in the phase space have crowded into the cell of lowest energy; in other words, the members of the assembly are all at rest. This is the condition of extreme degeneracy. The assembly, therefore, passes from a classical assembly for large values of λ to a state of degeneracy when λ approaches zero. For small values of λ, *particles of low velocity are more frequent than in a classical assembly.*

Degeneracy in the Fermi-Dirac statistics occurs when λ is large and negative, and then f is constant and equal to unity for a large range of ϵ, after which it decreases exponentially like the Maxwellian law. All the cells of low energy in the phase space are then filled. *Particles of low velocity are less frequent than is indicated by Maxwell's law.*

§7. THE DISTRIBUTION OF ELECTRONS IN HEAVY ATOMS.

An interesting application of the ideas outlined above has been made by Thomas⁽¹⁾ and recently also by Fermi⁽²⁾. Thomas supposes the electrons in an atom to be degenerate in the sense of Fermi and Dirac. In this case, however, there is a new condition to be imposed—viz., that electrons shall not have sufficient energy to take them out of the field of the atom. This places a restriction on the amount of phase space available. Thus

$$\frac{1}{2m}(p_x^2+p_y^2+p_z^2)\leqslant eV, \quad \dots\dots\dots\dots \quad (50)$$

where V is the potential inside the atom. We here assume that relativity considerations can be neglected. For an electron whose positional co-ordinates lie between x, y, z, and $x+dx$, $y+dy$, $z+dz$, the amount of phase space is limited by a sphere in the p_x, p_y, p_z space of radius $(2meV)^{1/2}$, so that the total amount available is

$$\frac{4}{3}\pi(2meV)^{3/2}dxdydz.$$

The number of cells in this phase space is

$$\frac{4}{3}\pi(2meV)^{3/2}dxdydz/h^3.$$

If the position and momenta of the electrons could be defined by six co-ordinates, this phase space would be sufficient; but it is now found necessary to attribute to each electron a spin, and the axis of spin can point in one or other of two opposite directions. This requires a new co-ordinate to specify the position and motion of an electron, or, alternatively, we can suppose by an extension of the Pauli Exclusion Principle that not more than one electron of prescribed spin can exist in a cell of extension h^3 in the six-dimensional phase space. Two electrons can co-exist in the same cell provided they have opposite spins. The assumption of degeneracy in an atom is equivalent to assigning two electrons to each available cell of the phase, so that the number of electrons permitted within $dxdydz$ is

$$2\left\{\frac{4}{3}\pi(2meV)^{3/2}dxdydz\right\}/h^3. \quad \dots\dots\dots\dots \quad (51)$$

If ρ denotes the electric density, we have

$$\rho=\frac{8}{3}\pi e(2meV)^{3/2}/h^3. \quad \dots\dots\dots\dots \quad (52)$$

Now since the potential V is determined by the nuclear charge and the electron distribution round it, V is determined by Laplace's equation. Hence

$$\Delta^2V=-4\pi\rho=\frac{32}{3}\pi e(2meV)^{3/2}/h^3, \quad \dots\dots\dots \quad (53)$$

which is a differential equation to determine V. The constants of the solution are determined by the conditions that

$$\left.\begin{aligned} &Vr\to E \quad \text{as} \quad r\to 0, \quad \dots\dots\dots\dots \\ &V\to 0 \quad \text{as} \quad r\to\infty, \quad \dots\dots\dots\dots \end{aligned}\right\} \quad (54)$$

E being the charge on the nucleus.

Thomas has solved the equation by numerical methods, and has drawn up tables from which may be inferred the variation of ρ and V with r. The potential is found to be proportional to $E^{4/3}$, so that the distribution of electrons is the same in all atoms (on this theory) except as regards scale. The treatment is only approximate, as it does not indicate the existence of atomic levels, and only gives a "smoothed-out" distribution. Fig. 2 shows the radial density $4\pi r^2 \rho$ of Rb$^+$, obtained by this method as a function of r. The same figure includes the corresponding results for Rb$^+$ obtained by the more accurate (though more laborious) calculation of Hartree(13). The unit of length is the radius of the Bohr 1_1 orbit for the hydrogen atom. The ordinate is the radial density $4\pi r^2 \rho$, or the rate of change of effective nuclear charge Z.

Fermi(2) has recently considered the same problem independently, and has obtained similar results. He has further applied his results to calculate the angular momentum of the electrons in an atom. The number of electrons in any element $dxdydz$ is known from the above work, and the distribution of their velocities can easily be deduced, since all cells in the phase space determined by equation (50) contain two electrons, so that the fraction of electrons in the element with prescribed angular momentum P to $P+dP$ can easily be deduced. It is thus an easy matter to calculate the total number of electrons in the atom with the same prescribed angular momentum. Taking the usual relation between angular momentum and the azimuthal quantum number k—viz., $P=kh/2\pi$—Fermi is thus able to deduce the total number of electrons in an atom with a prescribed azimuthal quantum number, and thus the number of electrons in the *s*-, *p*-, *d*-, *f* . . . states, since for these states $k=1/2, 3/2, 5/2, 7/2$ . . . (in the old quantum theory). The result is compared with Stoner's(6) distribution in the various atoms of the periodic table. The theory indicates the atomic number for which the *s*-, *p*-, *d*-, *f*- electrons first begin to appear—viz., Z (atomic number)$=1, 5, 21, 55$. This is in accordance with accepted results that the *p*- terms begin at Boron($Z=5$), the *d*- terms in Scandium ($Z=21$) and the *f* terms in Cerium ($Z=58$).

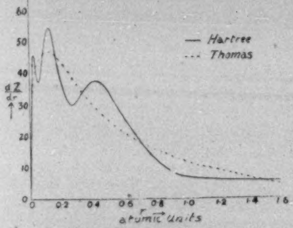


FIG. 2.—THE DISTRIBUTION OF ELECTRONS IN Rb$^+$.

Milne(14) has used the method and results of Thomas to calculate the total ionisation potential of a heavy atom $\Sigma\chi_r$, where χ_1, χ_2, . . . are the energies required to remove the first, second . . . electrons from the atom. He finds the simple result that

$$\Sigma\chi_r = 17Z^{7/3} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

for an atom of atomic number Z. Thus for iron ($Z=26$), $\Sigma\chi_r=34{,}000$ volts, and for silver ($Z=47$) $\Sigma\chi_r=136{,}000$ volts, while other more accurate methods(15) give 33,600 and 138,000 volts respectively. Such estimates of the total ionisation potential are useful in astrophysical investigations.

§ 8. THE SUSCEPTIBILITY OF METALS.

It is a well-known fact that many metals (especially the alkalis) exhibit only slight paramagnetic properties and the susceptibility is practically independent of temperature. It has hitherto been impossible to explain these facts either by supposing the magnetic properties to be due to the atomic cores or to the known magnetic moment of the valency electrons. All attempts to construct a theory on classical lines have led to a variation of susceptibility with temperature. Recently Pauli⁽¹⁶⁾ has attempted to explain the observed facts on the supposition that it is the moment of the valency electrons which play the central rôle, and that the new Fermi-Dirac statistics must be used instead of the classical.

In the presence of an external field, the axis of a magnetic moment can take up only certain quantised directions, defined by a quantum number m, which is a measure of the component of magnetic moment along the field. The number m can take only certain discrete values (either integers or half integers) defined by the limits $-j \leqslant m \leqslant j$, where j is the total magnetic quantum number. In applying these ideas to an electron gas, Pauli assumes as a rational generalisation of the Exclusion Principle that no two electrons can exist in the same cell of the phase space with the same quantum number m. The maximum number of electrons in a cell is thus equal to the number of possible values of m between $\pm j$. When the maximum number is present their net contribution to the magnetic moment is zero.

The energy of an electron is now made up of two parts, an energy of translation ε_r and a magnetic energy φ_m. The distribution of electrons in the region of the phase space for which ε lies between ε_r and $\varepsilon_r + d\varepsilon_r$ is not now given by equation (40), but by a modification of it—viz.,

$$a_{r,m} = \frac{V\,.\,2\pi(2m_0)^{3/2}}{h^3}\,\frac{\varepsilon_r^{1/2}\,d\varepsilon_r}{e^{\lambda+(\varepsilon_r+\varphi_m)/kT}+1}. \qquad (56)$$

We have here used m_0 to denote the mass of the electrons to differentiate it from the magnetic quantum number m.

The constant λ is obtained by summing the equation over all values of a_r (that is, by integrating over all values of ε_r) and equating the result to the total number of electrons N. The total number of electrons with magnetic energy φ_m is then

$$N_m = \frac{V\,.\,2\pi(2m_0)^{3/2}}{h^3}\int_0^\infty \frac{\varepsilon^{1/2}\,d\varepsilon}{e^{\lambda+(\varepsilon_r+\varphi_m)/kT}+1}, \qquad (57)$$

and the magnetic moment M of the whole assembly is given by

$$MH = -\sum_{m=-j}^{m=+j} N_m\varphi_m. \qquad (58)$$

Now the magnetic moment of an electron is equal to $eh/2\pi m_0 c$, and so

$$\varphi_m = -mH(eh/2\pi m_0 c). \qquad (59)$$

Substituting in the expression above, Pauli finds for the susceptibility (for weak fields)

$$\chi = (N/V)^{1/3}(2{\cdot}209)10^{-14} = n^{1/3}(2{\cdot}209)10^{-14}, \qquad (60)$$

n being the number of electrons per unit volume. This expression is independent of temperature as required by experiment, and moreover yields absolute values of susceptibilities in satisfactory agreement with the observed values. Pauli gives the following results—

	Na	K	Rb	Cs
$X_{\text{calc.}}$ $0 \cdot 10^7$	6·57	5·2	4·88	4·54
$X_{\text{obs.}}$ $0 \cdot 10^7$	5·8	5·1	0·6	−0·5

It is to be anticipated that the observed values will be less than the calculated, owing to the diamagnetic effect of the core. This will increase with the size of the core, and it is not surprising to find that it is sufficient in the case of Cs to neutralise the paramagnetic effect of the electrons and to render the metal as a whole diamagnetic.

Frenkel(17) in a paper which has just appeared, has given a simpler though less general method of arriving at the above formula for susceptibility. Frenkel's treatment is limited to conditions at the absolute zero.

The obvious defect in Pauli's work is the neglect of the interaction between electrons and atomic cores. The ordinary electrostatic attraction is never mentioned, though it seems fairly clear that this will modify the statistical distribution of electrons in the metal, just as in Thomas's treatment of electrons in an atom. The method is equivalent to considering an assembly of electrons in a closed box and neglecting their interaction except in so far as this is represented by the Pauli Exclusion Principle.

§ 9. THE ELECTRON THEORY OF METALS.

The work of Pauli on the statistics of electrons in metals has stimulated Sommerfeld(10) to consider the whole electron theory of metals in the light of the new statistics. We have already referred to some of Sommerfeld's results. It remains to discuss his investigations of electrical and thermal conductivities and allied phenomena, though it is beyond the scope of this review to enter into the new theories in any detail.

The distribution function f given in equation (49) is symmetrical in the momentum co-ordinates, which enter only through ε, so that particles with a prescribed energy are equally distributed in directions. When, however, conditions are unsymmetrical, as they are when an electrical field is applied in a given direction, or when there is a temperature gradient, the function f no longer gives the correct distribution law. If, for simplicity, we suppose the non-uniformity of conditions to be confined to one direction (say along the x-axis), f becomes changed from its initial value f_0 (say) to

$$f=f_0+\xi\chi, \qquad \ldots \ldots \ldots \qquad (61)$$

where χ is a function of ε only, and ξ is the component of velocity in the x-direction. Lorentz(18) has shown how to obtain f when f_0 is known, though he, of course, assumed the Maxwellian distribution for f_0. He showed, in fact, that when an electric field F is applied in the x-direction

$$f=f_0-\frac{l}{v}\left(\frac{eF}{m}\frac{\delta f_0}{\delta \xi}+\xi\frac{\delta f_0}{\delta x}\right), \qquad \ldots \ldots \ldots \qquad (62)$$

where l is the free path of the electrons (assumed to be a function of velocity). When f is once obtained, it is easy to deduce the net flow of electrons across a given plane under the influence of the external field, and similarly the net flow of energy owing to a temperature gradient.

Using the f_0 determined by the Fermi-Dirac statistics, equation (49), Sommerfeld is led to the following formulæ :—

—	Electrical conductivity (σ)	Thermal Conductivity (κ)	Ratio (κ/σ)
New theory	$\frac{8\pi}{3}\frac{e^2l}{h}\left(\frac{3n}{8\pi}\right)^{2/3}$	$\frac{8}{3}\pi^3\frac{lk^2T}{h}\left(\frac{3n}{4\pi}\right)^{2/3}$	$3{\cdot}3\left(\frac{k}{e}\right)^2T$
Classical theory ...	$\frac{4}{3}\frac{e^2ln}{(2\pi mkT)^{1/2}}$	$\frac{8}{3}\frac{lnk^{3/2}T^{1/2}}{(2\pi m)^{1/2}}$	$2\left(\frac{k}{e}\right)^2T$
Experiment	$\propto 1/T$	independent of T	$(3\,..\,3{\cdot}5)(k/e)^2T$

Except in so far as the free path l depends on the temperature (though the relation is left undetermined), Sommerfeld's formula for the electrical conductivity is independent of temperature, whereas actually the electrical conductivity of metals is inversely proportional to the temperature. The formula for thermal conductivity similarly fails to account for the observed facts by a factor T, but the Wiedemann-Franz law, as in most theories, is preserved. Moreover, the numerical factor in the ratio of κ/σ is closer to the observed value than that of any previous theoretical formula.

An attempt has been made by Houston(19) to improve Sommerfeld's theory by treating the electrons as waves in accordance with the ideas of de Broglie(20). As the mean velocity of the electrons according to the new statistics is of the order of a thousand kilometers a second, the equivalent wave-length is about 5Å., and corresponds to white X-rays. Instead of considering encounters between electrons and atoms in the usual way, Houston calculates the scattering of the electron waves by atomic centres, and takes into account the heat motion of the atoms in the same way that Debye(21) has done for X-rays. He finds that the electrical conductivity varies inversely as the temperature for normal temperatures and inversely as the square of the temperature at very low temperatures. The theory is, therefore, in accord with facts at high temperatures, and indicates a rapid increase in the conductivity at low temperatures, as is also required by observation. The theoretical increase at low temperatures is, however, not as great as that found experimentally. Nevertheless, the theory is more successful than any other in its interpretation of the dependence of electrical conductivity on temperature ; but, like all other theories, it fails to account for the phenomenon of superconductivity. This still remains a mystery.

An observed property of metals which would at first sight appear to be at variance with the new statistics is the velocity distribution of electrons emitted at high temperatures. This has been shown by Richardson(22) and others to be of the Maxwellian type, whereas the velocity distribution inside the metal, according to the new statistics, is completely different from this. (*See* Fig. 1.) The new theory, however, yields a formula for electron emission as satisfactory, if not more satisfactory, than the old theory. The two formulæ for the number of electrons emitted by a metal are

$$J \propto T^{1/2}e^{-W/kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(Classical)}, \quad (63)$$

$$J \propto T^2e^{-(Wa-Wi)/kT} \quad . \quad . \quad . \quad \text{(Sommerfeld)}, \quad (64)$$

where W is the energy of the electrons emitted. In the Sommerfeld formula a new term W_i appears. This represents the effect of the high energy of the electrons inside the metal, for, as was pointed out in a preceding paragraph, the average energy at absolute zero is of the order of 2 to 3 volts.

The appearance of the exponential in the Sommerfeld formula is due to the fact that the new velocity distribution curve, while approximately constant for a large range of energies, does eventually fall off exponentially, and it is only the high energy electrons which emerge from the metal. The new statistics is, therefore, successful in producing a satisfactory formula for electron emission; in fact, a formula of the type (64) had previously been suggested by Dushman(23) from other considerations, and had been shown to be in better agreement with the observations than (63).

Other phenomena dealt with by Sommerfeld include the Volta effect, Hall effect and various allied thermomagnetic phenomena; but we cannot enter into an account of these investigations here. For these the reader is referred to the original papers(24), (25).

§10. THE DISTRIBUTION OF ELECTRONS IN A METAL.

A defect in Sommerfeld's work is the neglect of the interaction of electrons and atomic cores in calculating the statistical distribution. No theory of metals can be regarded as adequate which does not include the assembly of electrons and atomic cores in one system to be treated as a whole. Recently the author, in collaboration with H. J. Woods,* has attempted to find the average distribution of electrons in a metal in the same way that Thomas(1) and Fermi(2) have done for isolated atoms. The mathematics for a three-dimensional array of atoms, however, proves intractable, and the problem has, therefore, been simplified by considering atoms as long and chain-like (like hydrocarbons, for instance) packed in a two-dimensional square array. The nuclei can then be regarded as line charges arranged in a square lattice, surrounded by enough electrons to make the system as a whole electrically neutral. We consider a section of this array bounded by two parallel planes at right angles to the nucleus and distant a apart.

In order to apply the condition of degeneracy, we suppose that the velocity of all the electrons parallel to the nuclei is the same, and is determined by a de Broglie wave of wave-length $2d$. The momentum in this direction is then given by

$$p_z = \pm h/2d. \qquad (65)$$

In the other directions, p_x and p_y are limited by the condition that the kinetic energy plus potential energy shall not be positive. The assumption that the system is degenerate, and therefore has 2 electrons in every cell of extension h^3 of the phase space, leads to a differential equation analogous to equation (53), obtained by Thomas and Fermi—viz.,

$$\frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} = k^2 V, \qquad (66)$$

where

$$k^2 = 16\pi^2 me^2/dh^2. \qquad (67)$$

This equation has to be solved subject to the conditions that $V \sim \log r$ near each nucleus, and that V is doubly periodic in the x, y plane. The solution of this equation leads to a pattern of equipotential lines (which are also lines of equal density) shown in Fig. 3. The nuclei are indicated by crosses.

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* To be published shortly.

Near a nucleus the equipotential lines are approximately circles. These are gradually deformed until a curve is reached shown by the continuous line which extends through the crystal. An electron on this line can travel about the crystal without doing any work. The equipotential lines enclose another system of closed curves which surround the centres of the squares. The electrons in these regions may be regarded as shared electrons, while the others are more definitely bound to a nucleus. The shared electrons form a lattice array which interpenetrates that of the nuclei. This provides some justification for the model of a metal, proposed by J. J. Thomson(26) and Lindemann(27), in which the valency electrons form a rigid lattice interpenetrating that of the atomic cores.

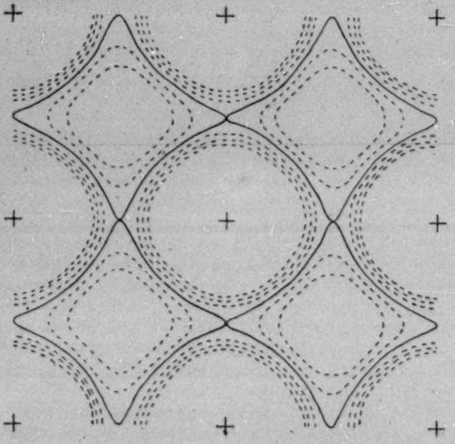

FIG. 3.—THE DISTRIBUTION OF ELECTRONS IN A METAL.

§11. THE STATISTICS OF GASEOUS MIXTURES.

An important extension of statistical theory has been made by Jordan(28) to assemblies of different particles (or systems), including cases when one set of particles obey one statistics and another set a different statistics. He has applied his results to a number of interesting astrophysical problems, including that of the equilibrium of a system of protons and electrons. In the latter he assumed with J. J. Thomson that electrons and protons can annihilate one another and set free a quantum of radiation. Bothe,(29) too, has considered mixed assemblies, and has obtained a general formula for the equilibrium constant in a chemical reaction.

Fowler(30) has dealt with these and similar problems in a comprehensive manner. In recent years Fowler has made enormous improvements in the mathematical technique of statistical theory. Instead of calculating the most probable states of an assembly, Fowler calculates average properties by summing over all parts of the generalised phase space, and in this connection is able to introduce a powerful theorem in complex integration. The ultimate results are, of course, the same as those obtained in the usual way, but there is considerable gain in elegance and in mathematical rigour. The method is capable of dealing with the new developments outlined above as well as with the theory of electrolytes, assemblies of ions and electrons as in giant stars, and other extensive applications. It would, however, take us too far even to review the achievements of the theory in such varied fields of inquiry.

REFERENCES.

(1) Thomas, Proc. Camb. Phil. Soc., **23,** 542 (1926).
(2) Fermi, Zeits. f. Phys., **48,** 73 (1928).
(3) S. N. Bose, Zeits. f. Phys., **27,** 384 (1924).
(4) Fowler, Phil. Mag., **44,** 450 and 823 (1922).
(5) Jeans, Dynamical Theory of Gases (1926).
(6) Herzfeld, Kinetische Theorie der Wärme (Braunschweig, 1925).
(7) Einstein, Sitzungsber. Preusz. Akad. Wiss, **22,** 261 (1924).
(8) Fermi, Zeits. f. Phys., **36,** 902 (1926).
(9) Dirac, Proc. Roy. Soc., A, **112,** 661 (1926).
(10) Sommerfeld, Zeits. f. Phys., **47,** 1 (1928), **47,** 43 (1928).
(11) Jordan, Naturwissenschaften, **15,** 636 (1927).
(12) Brillouin, Comptes Rend., **184,** 589 (1927).
(13) Hartree, Proc. Camb. Phil. Soc., **24,** 89 (1928).
(14) Milne, Proc. Camb. Phil. Soc., **23,** 794 (1927).
15) Hartree, Proc. Camb. Phil. Soc., **22,** 473 (1924).
(16) Pauli, Zeits. f. Phys., **41,** 81 (1927).
(17) Frenkel, Zeits. f. Phys., **49,** 31 (1928).
(18) Lorentz, Theory of Electrons, pp. 63-67, 266-273 (Leipzig and Berlin, 1909).
(19) Houston, Zeits. f. Phys., **48,** 449 (1928).
(20) de Broglie, Ann. d. Physique, **3,** 22 (1925) ; Thèses, Paris (1924).
(21) Debye, Verhandl, Dtsch. Phys. Gesell, **15,** 678 (1913) ; Ann. d. Phys., **43,** 49 (1914).
(22) Richardson, Electron Theory of Matter (1916).
(23) Dushman, Phys. Rev. (A), **21,** 623 (1923).
(24) Eckart, Zeits. f. Phys., **47,** 38 (1928).
(25) Houston, Zeits. f. Phys., **47,** 33 (1928).
(26) J. J. Thomson, Phil. Mag., **44,** 663 (1922).
(27) Lindemann, Phil. Mag., **29,** 127 (1915).
(28) Jordan, Zeits. f. Phys., **41,** 711 (1927).
(29) Bothe, Zeits. f. Phys., **46,** 327 (1928).
(30) Fowler, Proc. Roy. Soc., A, **113,** 432 (1926).

DEMONSTRATION ON SOME RECENT STUDIES IN ALTERNATING MAGNETISM AND SOME POSSIBLE APPLICATIONS.

By W. M. MORDEY, *M.I.E.E.*, *M.Inst.C.E.*, at the Bessemer Laboratory, Royal School of Mines.

THIS demonstration followed the lines of a Discourse given by the lecturer some years ago before the Royal Institution,* with some references to later developments.

In most of the experiments pulverised materials were used to enable Faraday's simple lines-of-force method of study to be used, and because of possible applications to the concentration of crushed minerals. Ordinary lines of force over a permanent magnet were shown with iron, nickel, cobalt, magnetite and pyrrhotite. Specular hæmatite when free from magnetite was unaffected. On trying the same materials over one pole of a small 1-phase a.c. bar magnet the lines of force of iron, nickel, cobalt and magnetite were the same as over the permanent magnet, but pyrrhotite differed; near the pole it formed ordinary radial lines of force, but when the cardboard on which the particles were sprinkled was lifted an inch or more above the pole there was a double effect—a halo caused by hysteretic repulsion was formed at a distance of several inches from it, the radial formation being weakened. On lowering the cardboard—i.e., increasing the field—the halo disappeared, the particles rejoining the radial lines of force, i.e., repulsion was changed to attraction.

With specular hæmatite a halo was formed in either a weak or a strong field, being larger with the latter. No radial lines of force were formed. Repulsion always prevailed over attraction.

With 1-phase multipolar magnets the coalescence of the halos round the poles formed distinct "equatorial" lines, which by Faraday's definition would denote diamagnetism; but he worked with direct current, and his definitions had reference to that only, and do not apply in the present case. The halos show no sign of mutual polar action between the particles.

In the field of 1-phase magnets there were thus three classes of effect:—

(i) With the more strongly magnetic substances of lines of force of the ordinary or direct current type were formed representing attraction.

(ii) With the moderately magnetic substance pyrrhotite both attraction and repulsion took place, the former represented by ordinary lines of force, and the latter by a halo, which increased as the field was weakened, or the cardboard raised.

(iii) With specular hæmatite, the least magnetic of the substances used, a halo formed was round the pole representing repulsion and there were no ordinary lines of force.

In multiphase fields with multipolar magnets several new phenomena were observed—e.g., even with good magnetic iron hysteretic repulsion from the poles took place with weak fields, while with strong fields attraction predominated, and the material was drawn towards the poles and propelled along over the poles in a direction depending on the phase relation.

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* **Proc. Royal Instn., Vol. 24, Part 1, pp. 267-288 (1923).**

The change from attraction to repulsion as the field was weakened was shown by placing the material on a thin board resting at one end on the magnet, and sloping upward so as to make the air gap gradually increase to several inches. The material was propelled up this incline by the multiphase action, spreading out laterally as it ascended and leaving bare the region over the poles. In strong fields what were called " planes of force " were formed by the building up of the material into fins by vertical repulsion from the poles, fed by the propulsion of material along the line of poles, the fins being separated longitudinally by lateral repulsion due to their magnetically equipotential condition.

But with specular hæmatite hysteretic repulsion always predominated over attraction, owing to the fact that its hysteresis coefficient was several hundred times greater than that of good iron, and its permeability probably not much over unity.

Practical mineral concentration was demonstrated by passing powdered ore down wooden shutes placed over long narrow multiphase magnets, the material as it descended separating into two streams, one the amenable constituent, and the other the inert gangue.

That these hysteretic materials were capable of exerting a considerable torque in multiphase fields was shown by the rapid spinning of a rough piece of pyrrhotite rock over the poles of a multiphase magnet, and by little model motors having rotors of various hysteretic substances without winding mounted on a spindle and placed in the field of a multiphase magnet.

There were no eddies, as the materials were either of very high specific resistance or were made practically non-conducting by fine division, and in some cases by mixing with an insulating cement.

In his Royal Institution paper the lecturer gave his reasons for attributing the repulsion to hysteresis. He sees no reason for changing that view.

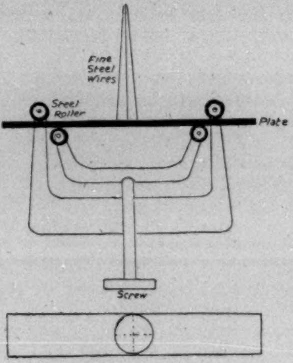
In that paper fuller explanations of some of the effects, for which there is not space here, are attempted.

DEMONSTRATION OF VARIOUS METHODS OF MEASURING POISSON'S RATIO.

By J. P. Andrews, *M.Sc.*

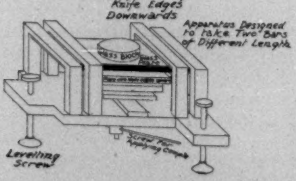
FOR this demonstration a number of pieces of apparatus designed to measure Poisson's Ratio by the method of bending were exhibited and explained. They consisted of copies, as close as possible, of actual experimental arrangements previously used in research, and included :

1. *A. Mallock's Apparatus,** in which the two anticlastic curvatures are measured by the motion of the ends of four spines erected around the centre of the horizontal surface of a bar bent by couples at its ends.


Fine Steel Wires
Steel Roller
Plate
Screw

MALLOCK'S APPARATUS.

2. *R. Straubel's Apparatus,*† an elaboration of Cornu's original interference method (1869), which was itself an extension of Newton's Rings to anticlastic instead of spherical surfaces. In Straubel's apparatus a complicated arrangement of knife edges is introduced to secure an even pressure on the glass bar.


Bridges Bearing Knife Edges Downwards
Apparatus Designed to take Two Bars of Different Length
Glass Block
Glass Plate
Levelling Screw
Screw for Applying Couple

R. STRAUBEL'S APPARATUS.

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* Phil. Mag. (1879).

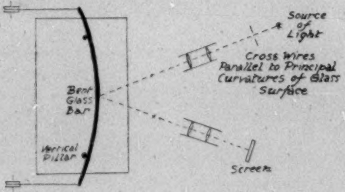
† Ann. d. Physik, 68, p. 369 (1899).

3. *I. Williams' Method,*[*] in which the transverse curvature is measured by the rotation of a mirror suspended by a Kelvin bifilar, which itself depends from the tops of two rods fixed respectively to the two sides of the bar used.

4. *H. Carrington's Method.*[†]—The spines of Mallock's experiment are replaced by mirrors upon small pillars; and in order to eliminate any correction due to the translatory motion of these mirrors, the pillars are of a definite, calculated length, and the mirrors inclined at a definite angle to the vertical.

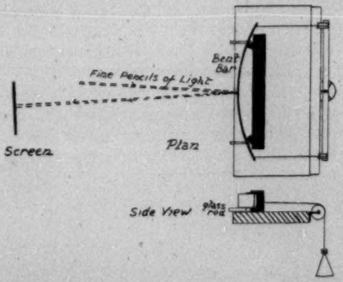
5. *H. T. Jessop's Method.*[‡]—An improvement of the Cornu method in so far as the glass bar under test may be bent in both directions to eliminate the effect of initial curvature.

6. *G. F. C. Searle's Method.*[§]—The distance between the focal lines in the image of an object seen by reflection in an anticlastic surface is employed here to determine Poisson's Ratio for glass.


Source of Light
Cross Wires Parallel to Principal Curvatures of Glass Surface
Bent Glass Bar
Vertical Pillar
Screen

G. F. C. SEARLE'S FOCAL LINE METHOD.

7. *The Method of A. Ferguson and J. P. Andrews* (1928).—Four fine pencils of light are reflected directly from the polished surface of a bent brass plate to replace the spines of Mallock's experiment. The deflection of the reflected pencils, as observed by reception on a distant screen, enable Poisson's Ratio to be determined.


Bent Bar
Fine Pencils of Light
Screen
Plan
Side View
glass rod

APPARATUS OF A. FERGUSON AND J. P. ANDREWS.

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[*] Phil. Mag., 24, p. 886 (1912).

[†] Phil. Mag., 41, p. 206 (1921).

[‡] Phil. Mag., 42, p. 551 (1921).

[§] Proc. Camb. Phil. Soc., 21, p. 772 (1922).

DEMONSTRATION OF A SIMPLE PRACTICAL APPLICATION OF THE PROPERTIES OF SELENIUM CELLS.

By GEO. P. BARNARD, The National Physical Laboratory.

THE experiment shown was a development of a demonstration given before the Society in 1926* by Prof. Hans Thirring. The light from a 1000 watt gas-filled lamp was interrupted by a large perforated metal disc (2 ft. in diameter) driven by a small d.c. motor. Circular holes, ¾″ in diameter, were drilled in the disc as follows:—Five concentric circles of radii 3⅜″, 5⅛″, 6⅞″, 8⅝″, 10⅜″ respectively were ruled on the disc. At equidistant distances along the circumferences of the five circles, 15, 16, 18, 20, 21 holes respectively were drilled. The ratios of the numbers of these holes correspond to the frequency ratios of the following notes in the natural diatonic scale—viz., E, F, G, A, B♭. A wooden shutter was set up close behind the metal disc, so that a selenium cell could be conveniently moved on to any of the five circles of holes.

A Fournier d'Albe type selenium cell having a light-sensitive area of approximately ¾″ × ¾″ was connected in series with a 50 volt battery to the primary of an inter-valve transformer, the secondary of which was connected to the input circuit of the detector valve of a 2-stage resistance-capacity coupled amplifier. A large loud-speaker was placed in the plate circuit of the last valve.

The selenium cell could thus be exposed in turn to intermittent illumination of five different acoustical frequencies, and the fluctuations in current through the cell, amplified as described, produced the corresponding notes in the loud-speaker.

By this means the first part of the National Anthem was played, and was clearly audible throughout the theatre. By the simple expedient of altering the speed of the motor driving the disc the fundamental tone could be changed.

The demonstrator pointed out the extraordinary quickness of response of the electrical resistance of selenium to intermittent illumination of acoustical frequency, and suggested that applications of selenium cells based on this would prove very efficient.

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* Proceedings, Vol. 39, p. 97.

ELECTRICAL METHODS OF HYGROMETRY.

AUTHORS' reply to discussion on Paper by Messrs. P. W. BURBIDGE and N. S. ALEXANDER (*see* page 153).

In reply to Professor Tyndall : The more rapid rise of the saturation curve in dry air (Fig. 6, curve IV) appears to be due directly to the greater mobility of ions in dry air. In reply to Dr. Griffiths : No experiments were performed to ascertain if the cotton exhibited any gradual change in its electrical properties at a definite humidity, over a long period of time.

We regret that at the time of publishing, we were not aware of the work done by Mr. F. P. Slater. We find, however, that in general the lag in the change of electrical conductivity with humidity may to a great extent be attributed to the time required for the air to reach equilibrium with the solution. This effect was not entirely eliminated even by rapid circulation of air by means of a fan. It was eliminated, using the apparatus of Fig. 3, by arranging for the change in humidity to be brought about merely by the mixing of the air in the large box with that in the hygrometer box, the solution having been removed. Lag was still found, however, and this we have attributed to the cotton itself. It is to be remembered that experimental conditions are of great importance ; for example, the walls of the containing vessel exercise a marked effect. Using the apparatus in Fig. 3, we found initially very large lag effects due to the wooden walls, which gave off water vapour or absorbed it from the air when a change in humidity was made. This was eliminated by covering the wood with paraffin wax. The apparatus used by Slater would apparently permit considerable lag in the rate of acquisition of equilibrium between the air and the solution.

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